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# Slot Allocation in a Multi-airport System under Flying Time Uncertainty\*

Chang LIU,<sup>1),2)</sup> Chao hao LIAO,<sup>3)</sup> Xu HANG,<sup>3)</sup> Yanjun WANG,<sup>1),2)†</sup> and Daniel DELAHAYE<sup>4)</sup>

<sup>1)</sup>College of Civil Aviation, Nanjing University of Aeronautics and Astronautics, Nanjing 211106, China

<sup>2)</sup>State Key Laboratory of Air Traffic Management System, Nanjing 211106, China

<sup>3)</sup>Air Traffic Management Bureau of Central-South China, Guangzhou 510405, China

<sup>4)</sup>ENAC Research Lab, Ecole Nationale de l'Aviation Civile, Toulouse 31055, France

Slot allocation in a single airport aims to maximize the utilization of airport-declared capacity under operational and regulation constraints, while that in a multi-airport system (MAS) has to take airspace capacity into account. This is due to the fact that the conflict of using the limited capacity of certain departure/arrival fixes in the terminal airspace could induce unnecessary flight delays. The uncertainty of flying times between the airport and congested fixes makes it even more complicated for slot allocation in a MAS. Traffic flow may exceed capacity when the flying times of flights change. In this paper, the authors propose an uncertainty slot allocation model for a MAS (USAM). The objective of the model is to minimize the total displacement of slot requests in the MAS while considering all of the capacity constraints, as well as the uncertainty of flying time. The constraints of departure/arrival fixes are formulated as chance constraints, and then the Lyapunov theorem is applied for reformulation. The USAM is applied in the MAS of the Guangdong-Hong Kong-Macao Greater Bay Area (GBA). Specifically, the impact of the uncertainty of flying times from five airports to airspace fix YIN is investigated. Results show that the total displacement would increase if the uncertainty of flying time was considered. The optimized schedule using the USAM, however, is more robust and can satisfy capacity constraints under various scenarios.

**Key Words:** Slot Allocation, Multi-airport System, Uncertainty Model, Chance Constraint

## 1. Introduction

Air transport demands at most busy airports around the world are significantly higher than the capacity that the airport can provide. Therefore, a slot allocation process has to be implemented before each scheduling season. In the slot allocation process, a slot coordinator allocates airport slots to the airlines. An airport slot gives the right to an airline to use airport infrastructures and services at a specific time. Slot allocation is an important means that can effectively balance air transport demand and airport capacity.<sup>1,2)</sup>

Slot allocation aims to distribute airport capacity (i.e., slots) to the airlines under various constraints, which is a resource-constrained allocation problem.<sup>3)</sup> Depending on the number of airports that are considered, slot allocation can be divided into slot allocation for a single airport, an airport network, and a multi-airport system (MAS). Early studies have focused on slot allocation for a single airport. In 2012, Zografos et al. proposed an integer programming model for a single-airport slot allocation problem.<sup>1)</sup> The model

considers slot allocation rules given in the Worldwide Slot Guidelines (WSG) and EU 95/93, with the objective of minimizing scheduled delays. The constraints of the model include airport capacity and flight turnaround constraints, and the concept of priority groups of slot requests is introduced. The results show that the model can significantly reduce airlines' scheduled delays.

Later, much attention in modeling slot allocation shifted gradually from single objective to multiple objectives.<sup>4)</sup> The objectives are no longer limited to minimizing the total displacement, but also include minimizing the maximum slot displacement, minimizing the number of slots outside the airlines' acceptable time windows, and fairness. In 2018, two bi-objective models to trade-off slot allocation efficiency and slot acceptability were developed.<sup>2)</sup> The first model minimizes the total displacement and the maximum acceptable slot displacement. The second model minimizes the total displacement and the number of slots outside of the airlines' acceptable time windows. The results show that the acceptance for slots by airlines can be effectively increased at a cost of a small amount of slot coordination efficiency. Fairness is introduced into the slot allocation model.<sup>5,6)</sup> Results show that reducing a small amount of slot coordination efficiency using multi-objective optimization can effectively improve other metrics of the model. There are also studies that consider all of the priority rules in the WSG for slot allocation.<sup>7–9)</sup>

The limitations of a single-airport slot allocation model are obvious. The main drawback is that the model ignores the continuity and consistency of flights in the origin and destination airports. For example, the allocated departure slot at an origin airport may be unused because the airline was



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†Corresponding author, ywang@nuaa.edu.cn

not able to obtain an arrival slot at the destination airport. This has driven researchers to locate the slot allocation problems from a single airport in an airport network. Airport network slot allocation models has been proposed by Pellegrini et al.<sup>10)</sup> and Castelli et al.<sup>11)</sup> The uncertainty of airport capacity has also been considered, and a two-stage stochastic programming model was developed.<sup>12)</sup> The challenge of airport network slot allocation is to find the solution to the model. This is because the scale of variables and constraints of a model are often too large. Thus, various algorithms have been developed for the airport network slot allocation model, such as heuristic algorithms,<sup>13)</sup> intelligent optimization algorithms,<sup>14)</sup> dual integer cutting,<sup>15)</sup> etc.

Over the decades, the constraints and dependencies between the airports in a MAS have become more prominent, mainly due to the conflict in using shared departure/arrival fixes and routes. The earliest work on investigating the operation of a MAS can be traced back to 2008. The U.S. Joint Development and Planning Office (JDPO) first defined a multi-airport system as a cluster of airports that are in close geographical proximity and interdependent for inbound and outbound air routes serving a metropolitan area.<sup>16)</sup> Traditionally, slot allocation does not have to consider airspace capacity. However, there may be conflicts between flights from different airports at a shared departure/arrival fix (waypoint) when allocating slots for each airport individually. It is therefore necessary to consider all of the airports at the same time when determining slot allocation in order to alleviate congestion and delay in the whole region.

Note that almost all of the slot allocation models are deterministic, and modeling slot allocation under uncertainty has not been investigated as much. As discussed above, shared resources (fix capacity) must be considered in slot allocation for a MAS. For each flight, the Estimated Time Over (ETO) a fix has to be calculated from the planned departure/arrival time (i.e., from the schedule) and the flying time between the airport and the fix. The flying time varies due to several factors, such as traffic flow management, weather, airport congestion, etc. Accordingly, the ETO is uncertain. This leads to the fact that traffic at the fix varies within a certain time window. The flying time uncertainty is of serious concern in practice because schedules of slot allocation are often sensitive to fluctuations in the fix traffic. To capture the uncertainty of flying time, the authors introduce a slot allocation model based on chance-constrained programming.

Chance-constrained programming was first proposed by Charnes and Cooper.<sup>17)</sup> The uncertain parameters in a chance-constrained model are considered as random variables with known distributions. In general, the formulation of a chance-constrained optimization problem ensures that the probability of meeting certain constraint(s) is above a predefined level.<sup>17,18)</sup> One group of studies for solving chance constraints is using stochastic simulation techniques, such as the Markov chain Monte Carlo model,<sup>19)</sup> and sampling-based particle control methods.<sup>20)</sup> A second group of methods transforms chance constraints into deterministic constraints based on the method of stochastic programming or distribu-

tionally robust optimization.<sup>21,22)</sup>

To capture the constraints from shared resources in slot allocation for a MAS, the authors first propose a certainty slot allocation model (CSAM). Next, the fix capacity constraints are modeled as chance constraints to consider the uncertainty of flying times. By leveraging stochastic programming, the chance constraints are further reformulated as deterministic constraints. In this way, an uncertainty slot allocation model (USAM) was developed. This model was then tested using real data from the MAS of the Guangdong-Hong Kong-Macao Greater Bay Area (GBA). The rest of the paper is organized as follows. Section 2 presents a certainty slot allocation model and an uncertainty slot allocation model. An equivalent formulation is proposed such that the USAM can be solved using the Gurobi solver. Section 3 explains the numerical experiments and analyzes the results. Finally, Section 4 summarizes the main findings and presents recommendations for future research.

## 2. Model

### 2.1. Model description

The overall objective of the model is to minimize the total displacement of given slot requests/flights (i.e., between the original and the modified schedule) under various constraints, such as airport declared capacity, fixes' capacity, the operating rules, and the uncertainty of flying times on the routes. The following assumptions are made:

- Only flights departing from or arriving at the airports in the MAS are considered; while overflying flights are not.
- The probability of flying time of an airport-fix pair is equal to its frequency.
- The distributions of flying times for flights are independent.
- Slot request cannot be rejected.

The model uses the following as inputs: (i) the flight schedule of each airport (slot requests); (ii) airport capacities and fix capacities; (iii) flight routes; and (iv) flying times between airports and fixes. The output is the optimized flight schedule.

### 2.2. Certainty model formulation

First, the CSAM is formulated as a MIP model that consists of a set of  $K$  airports and a set of  $Q$  fixes. Table 1 lists the notations of the model. For each flight  $i \in F$ ,  $\omega_i^{\text{arr}}/\omega_i^{\text{dep}}$  denotes its original arrival/departure time respectively. Decision variable  $x_{i,n}^{\text{arr}}/x_{i,n}^{\text{dep}} = 1$  indicates that the flight  $i$  is scheduled to arrive/depart no earlier than slot  $n$ ; otherwise  $x_{i,n}^{\text{arr}}/x_{i,n}^{\text{dep}} = 0$ . Then  $\delta_{i,n}^{\text{arr}}$  or  $\delta_{i,n}^{\text{dep}}$  is used to determine whether or not flight  $i$  arrives/departs at slot  $n$ . The uncertainty parameters in the model are the flying times between airport  $k$  and fix  $q$  (i.e.,  $\tau_{k,q}^{\text{arr}}/\tau_{k,q}^{\text{dep}}$ ), which are considered as random variables following some distributions. In the following, the CSAM formulation is first presented as Eq. (1).

$$Z_c = \min \sum_{i \in F} \Delta_i \quad (1a)$$

Table 1. Model notation and description.

Notation	Description
<b>Sets</b>	
$T$	Set of slots in a day, $n = 1, 2, 3, \dots \in T$
$T_w$	Set of time windows in the day, $t = 1, 2, 3, \dots \in T_w$
$F$	Set of flights in the MAS
$K$	Set of airports in the MAS
$Q$	Set of fixes in the MAS
$\mathbb{F}$	Set of connecting flights operated by the same aircraft
$F_k^{\text{arr}}/F_k^{\text{dep}}$	Set of arrival/departure flights in airport $k$ , $k \in K$
$F_q^{\text{arr}}/F_q^{\text{dep}}$	Set of arrival/departure flights passing fix $q$ , $q \in Q$
$N_t$	Set of slots in time window $t$
<b>Parameters</b>	
$t_m$	The last slot that a flight can be delayed
$S$	The number of scenarios in each airport-fix route
$\alpha$	Violation probability
$con_{i,j}^1/con_{i,j}^2$	Minimum/Maximum turnaround time for connecting flights, $(i, j) \in \mathbb{F}$
$y_{i,n}^{\text{arr}}/y_{i,n}^{\text{dep}}$	Binary parameter, where 1 indicates that flight $i$ would have been scheduled to arrive/depart no earlier than slot $n$ , while 0 otherwise, $i \in F, n \in T$
$\omega_i^{\text{arr}}/\omega_i^{\text{dep}}$	Original arrival/departure slot of flight $i$ , $i \in F$
$k_i^{\text{arr}}/k_i^{\text{dep}}$	The destination/origin airport of arrival/departure flight $i$ , $i \in F$
$C_{k,t}^{\text{arr}}/C_{k,t}^{\text{dep}}/C_{k,t}^{\text{all}}$	Number of arrival/departure/total flights limit in airport $k$ at time window $t$ , $k \in K, t \in T_w$
$D_{q,t}$	The number of total flights limit passing fix $q$ at time window $t$ , $q \in Q, t \in T_w$
$\tau_{k,q}^{\text{arr}}/\tau_{k,q}^{\text{dep}}$	Random parameters of arrival/departure flying time between airport $k$ and fix $q$ , $k \in K, q \in Q$
<b>Indirect decision variables</b>	
$\Delta_i$	The slot adjustments of flight $i$ , $i \in F$
$\delta_{i,n}$	Binary variables, where 1 indicates flight $i$ is scheduled to arrive/departure at slot $n$ , $i \in F, n \in T$
$NK_{k,t}^{\text{arr}}/NK_{k,t}^{\text{dep}}$	The number of arrival/departure flights in airport $k$ at time window $t$ , $k \in K, t \in T_w$
$NQ_{q,t}^{\text{arr}}/NQ_{q,t}^{\text{dep}}$	The number of arrival/departure flights passing fix $q$ at time window $t$ , $q \in Q, t \in T_w$
<b>Decision variables</b>	
$x_{i,n}^{\text{arr}}/x_{i,n}^{\text{dep}}$	Binary variables, where 1 indicates flight $i$ is scheduled to arrive/departure no earlier than slot $n$ after optimization, while 0 otherwise, $i \in F, n \in T$

s.t.

$$\Delta_i = \begin{cases} \sum_{n \in T} |x_{i,n}^{\text{arr}} - y_{i,n}^{\text{arr}}|, & i \in F^{\text{arr}} \\ \sum_{n \in T} |x_{i,n}^{\text{dep}} - y_{i,n}^{\text{dep}}|, & i \in F^{\text{dep}} \end{cases} \quad (1b)$$

$$\delta_{i,n}^{\text{arr}} = x_{i,n}^{\text{arr}} - x_{i,n+1}^{\text{arr}} \quad \forall i \in F^{\text{arr}}, \forall n \in T \quad (1c)$$

$$\delta_{i,n}^{\text{dep}} = x_{i,n}^{\text{dep}} - x_{i,n+1}^{\text{dep}} \quad \forall i \in F^{\text{dep}}, \forall n \in T \quad (1d)$$

$$x_{i,1}^{\text{arr}} = 1 \quad \forall i \in F^{\text{arr}} \quad (1e)$$

$$x_{i,t_m}^{\text{arr}} = 0 \quad \forall i \in F^{\text{arr}} \quad (1f)$$

$$x_{i,1}^{\text{dep}} = 1 \quad \forall i \in F^{\text{dep}} \quad (1g)$$

$$x_{i,t_m}^{\text{dep}} = 0 \quad \forall i \in F^{\text{dep}} \quad (1h)$$

$$x_{i,n}^{\text{arr}} \geq x_{i,n+1}^{\text{arr}} \quad \forall i \in F^{\text{arr}}, \forall n \in T \quad (1i)$$

$$x_{i,n}^{\text{dep}} \geq x_{i,n+1}^{\text{dep}} \quad \forall i \in F^{\text{dep}}, \forall n \in T \quad (1j)$$

$$\sum_{n \in T} (x_{j,n}^{\text{dep}} - x_{i,n}^{\text{arr}}) \geq con_{i,j}^1 \quad \forall (i, j) \in \mathbb{F} \quad (1k)$$

$$\sum_{n \in T} (x_{j,n}^{\text{dep}} - x_{i,n}^{\text{dep}}) \leq con_{i,j}^2 \quad \forall (i, j) \in \mathbb{F} \quad (1l)$$

$$NK_{k,t}^{\text{arr}} = \sum_{n \in N_t} \sum_{i \in F_k^{\text{arr}}} \delta_{i,n}^{\text{arr}} \quad \forall t \in T_w, \forall k \in K \quad (1m)$$

$$NK_{k,t}^{\text{dep}} = \sum_{n \in N_t} \sum_{i \in F_k^{\text{dep}}} \delta_{i,n}^{\text{arr}} \quad \forall t \in T_w, \forall k \in K \quad (1n)$$

$$NQ_{q,t}^{\text{arr}} = \sum_{n \in N_t} \sum_{i \in F_q^{\text{arr}}} \delta_{i,n+\tau_{k_i,q}^{\text{arr}}} \quad \forall t \in T_w, \forall q \in Q \quad (1o)$$

$$NQ_{q,t}^{\text{dep}} = \sum_{n \in N_t} \sum_{i \in F_q^{\text{dep}}} \delta_{i,n-\tau_{k_i,q}^{\text{dep}}} \quad \forall t \in T_w, \forall q \in Q \quad (1p)$$

$$NK_{k,t}^{\text{arr}} \leq C_{k,t}^{\text{arr}} \quad \forall t \in T_w, \forall k \in K \quad (1q)$$

$$NK_{k,t}^{\text{dep}} \leq C_{k,t}^{\text{dep}} \quad \forall t \in T_w, \forall k \in K \quad (1r)$$

$$NK_{k,t}^{\text{arr}} + NK_{k,t}^{\text{dep}} \leq C_{k,t}^{\text{all}} \quad \forall t \in T_w, \forall k \in K \quad (1s)$$

$$NQ_{q,t}^{\text{arr}} + NQ_{q,t}^{\text{dep}} \leq D_{q,t} \quad \forall t \in T_w, \forall q \in Q \quad (1t)$$

The objective of the CSAM is to minimize the total slot displacement as given by Eqs. (1a) and (1b). The relationships between  $\delta_{i,n}^{\text{arr}}/\delta_{i,n}^{\text{dep}}$  and  $x_{i,n}^{\text{arr}}/x_{i,n}^{\text{dep}}$  are shown in Eq. (1c) and Eq. (1d). Equations (1e)–(1h) ensure that each flight has at least one slot assigned.  $x_{i,1}^{\text{arr}}/x_{i,1}^{\text{dep}} = 1$  makes sure that no flight can arrive/depart earlier than the beginning of the day, while  $x_{i,t_m}^{\text{arr}}/x_{i,t_m}^{\text{dep}} = 0$  ensures that every flight has arrived/departed at the end of the scheduling day. Equations (1i) and (1j) ensure that only one slot is assigned to a flight. Equations (1k) and (1l) impose the turnaround time restriction on the two connected flights. Equations (1q)–(1s) represent airport capacity constraints, which limit the amount of arrival and departure traffic of the airports (i.e., Eq. (1m) and Eq. (1n)) within a certain time window. The time window refers to a period of time that begins at a specific time and ends at a later time. For example in Fig. 1, time window  $t$  denotes the time period from 21:45 to 22:00. The lengths of a time window can be 15 min or 60 min, i.e.,  $|N_t|$  is respectively taken as 3 and 12. Equations (1o) and Eq. (1p) calculate traffic at a fix  $q$  from/to airport(s) in time period  $t$ . Equation (1t) ensures that the total traffic at a fix is lower than its capacity.

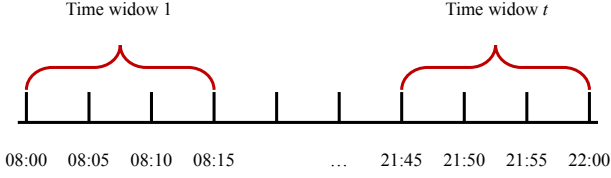


Fig. 1. Example of the time window.

### 2.3. Uncertainty model transformation

In Eq. (1o) and Eq. (1p), the arrival and departure traffic passing a certain fix in a certain time window depends only on the decision variables. However, in practice, the airport-fix flying times are not deterministic values, but random ones following some distributions. Therefore, for a specific set of decision variables, the arrival and departure traffic of a certain fix in a certain time window is also a random variable. Here, the chance constraints are used to reformulate the certainty problem. Let  $\alpha$  be the violation probability of the chance constraint, showing the degree of violation that the decision-maker may take. In this way, Eq. (1t) is reformulated as Eq. (2).

$$P\left(NQ_{q,t}^{\text{arr}} + NQ_{q,t}^{\text{dep}} \leq D_{q,t}^{\text{all}}\right) \geq 1 - \alpha \quad (2)$$

$$\forall t \in T_w, \forall q \in Q$$

**Theorem 1** When the number of flights that pass fix  $q$  in time window  $t$  is large, then chance constraint Eq. (2) is approximately equivalent to Eq. (3)

$$\frac{D_{q,t} - \mu_{q,t}}{\sigma_{q,t}} \geq \Phi^{-1}(1 - \alpha) \quad (3)$$

where  $\Phi$  is the standard normal distribution, and  $\mu_{q,t}$  and  $\sigma_{q,t}$  denote the expectation and standard deviation of traffic (number of flights) at fix  $q$  in time window  $t$ .

**Proof 1** The left-hand side of Eq. (2) can be reformulated as Eq. (4) according to Eq. (1o) and Eq. (1p).

$$P\left(\sum_{n \in N_t} \sum_{i \in F_q^{\text{arr}}} \delta_{i,n+\tau_{k_i,q}^{\text{arr}}}^{\text{arr}} + \sum_{n \in N_t} \sum_{i \in F_q^{\text{dep}}} \delta_{i,n-\tau_{k_i,q}^{\text{dep}}}^{\text{dep}} \leq D_{q,t}^{\text{all}}\right) \quad (4)$$

where  $\delta_{i,n+\tau_{k_i,q}^{\text{arr}}}^{\text{arr}}$  and  $\delta_{i,n-\tau_{k_i,q}^{\text{dep}}}^{\text{dep}}$  can be considered as random variables following some distribution (i.e. depending on  $\tau_{k_i,q}^{\text{arr}}$

and  $\tau_{k_i,q}^{\text{dep}}$ ). Let  $b = |N_t| \cdot \left(|F_q^{\text{arr}}| + |F_q^{\text{dep}}|\right)$  denote the number

of flights that pass fix  $q$  in time window  $t$ . According to the Lyapunov theorem, when  $b$  is large, the sum of the random variables approximately follows a normal distribution. In practice, the number of flights at a key fix in the MAS is usually between 400 and 500. Accordingly, the Lyapunov central limit theorem can be used here,<sup>23)</sup> expressed as Eq. (5).

$$\sum_{n \in N_t} \sum_{i \in F_q^{\text{arr}}} \delta_{i,n+\tau_{k_i,q}^{\text{arr}}}^{\text{arr}} + \sum_{n \in N_t} \sum_{i \in F_q^{\text{dep}}} \delta_{i,n-\tau_{k_i,q}^{\text{dep}}}^{\text{dep}} \quad (5)$$

$$\sim N(\mu_{q,t}, \sigma_{q,t}^2)$$

As a result, the constraint of Eq. (4) can be reformulated as

Eq. (6)

$$P\left(\frac{NQ_{q,t}^{\text{arr}} + NQ_{q,t}^{\text{dep}} - \mu_{q,t}}{\sigma_{q,t}} \leq \frac{D_{q,t} - \mu_{q,t}}{\sigma_{q,t}}\right) \quad (6)$$

$$= \Phi\left(\frac{D_{q,t} - \mu_{q,t}}{\sigma_{q,t}}\right)$$

The proof of Theorem 1 is completed.

According to the Lyapunov theorem, the expectation and standard deviation of traffic are calculated as follows,

$$\mu_{q,t} = \sum_{n \in N_t} \sum_{i \in F_q} \mu_{i,q,n}, \quad \sigma_{q,t} = \sqrt{\sum_{n \in N_t} \sum_{i \in F_q} \sigma_{i,q,n}^2} \quad (7)$$

where  $\mu_{i,q,n}$  and  $\sigma_{i,q,n}^2$  denote the expectation and variance of  $\delta_{i,n+\tau_{k_i,q}^{\text{arr}}}^{\text{arr}}$  or  $\delta_{i,n-\tau_{k_i,q}^{\text{dep}}}^{\text{dep}}$ . Because the flying time in this research is integers in minutes,  $\mu_{i,q,n}$  and  $\sigma_{i,q,n}^2$  can be expressed as Eq. (8) and Eq. (9),

$$\mu_{i,q,n} = \begin{cases} \sum_{s=1}^m p_{k,q}^{\text{arr}(s)} \cdot \delta_{i,n+\tau_{k_i,q}^{\text{arr}(s)}}^{\text{arr}}, & i \in F^{\text{arr}} \\ \sum_{s=1}^m p_{k,q}^{\text{dep}(s)} \cdot \delta_{i,n-\tau_{k_i,q}^{\text{dep}(s)}}^{\text{dep}}, & i \in F^{\text{dep}} \end{cases} \quad (8)$$

$$\sigma_{i,q,n}^2 = \mu_{i,q,n} - \mu_{i,q,n}^2 \quad (9)$$

where  $\tau_{k_i,q}^{\text{arr}(s)} / \tau_{k_i,q}^{\text{dep}(s)}$  denotes the  $s$ th realization of flying time between airport  $k_i$  and fix  $q$ , and  $p_{k,q}^{\text{arr}(s)} / p_{k,q}^{\text{dep}(s)}$  for corresponding probabilities. Accordingly, the USAM can be reformulated as Eq. (10).

$$Z_u = \min \sum_{i \in F} \Delta_i \quad (10)$$

s.t.

Eqs. (1b)–(1s), Eq. (3), and Eqs. (7)–(9)

## 2.4. Solution approach

The model is implemented using Python and the Gurobi solver. The model is run on Windows 10 with a 64-bit operating system equipped with 16 GB RAM and a i7 processor.

## 3. Case Study

### 3.1. Experiment setup

In this paper, slot allocation in the GBA MAS is investigated. The MAS includes five airports: Guangzhou Baiyun International Airport (ICAO code: ZGGG), Shenzhen Baoan International Airport (ZGSZ), Zhuhai Jinwan Airport (ZGSD), Macau International Airport (VMCC) and Huizhou Pingtan Airport (ZGHZ). Since the flights at Hong Kong International Airport (ICAO code: VHHH) are isolated from other airports, VHHH is not considered in the MAS. Figure 2 shows the structure of the airspace of the MAS. It can be seen that the airspace of the MAS is complex. Several airports have to use a shared departure/arrival fix.

The flight schedule on December 21, 2019 is used as input for the model. Statistically, a total of 2,969 flights were planned for the MAS, with 1,494 arrival flights and 1,475 de-



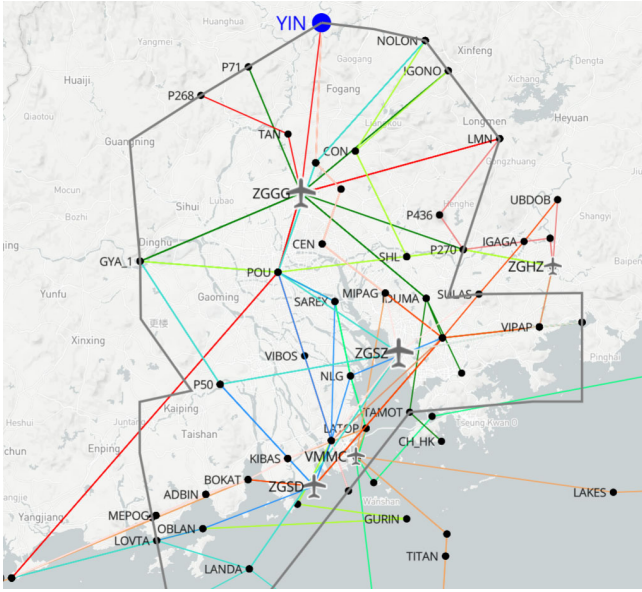


Fig. 2. Airports and departure/arrival routes in the GBA MAS.

parture flights. The departure fix, YIN, in the MAS is chosen for investigating flying time uncertainty. (The geographical location of YIN is highlighted in blue at the top of Fig. 2.)

The reasons are as follows:

- (1) A total of 462 flights passed through YIN on that day. The traffic passing through YIN ranks third, and the traffic accounts for a relatively high percentage among all of the fixes in the MAS.
- (2) YIN is jointly used by the five airports in the MAS. Accordingly it is representative to choose this fix for the optimization considering flying time uncertainty in the MAS.
- (3) The number of flights during some peak hours are over the capacity. Effectively solving the flying time uncertainty of this fix will provide a reference for other fixes. The box plot of the number of flights scheduled for main fixes in every 15 minutes (i.e., denoted as 15-min traffic) is shown in Fig. 3.

### 3.2. Parameters setup

In this paper, the length of each slot is set to 5 min. The airport capacity is set according to the declared capacity given by the civil aviation authority; the fix capacity is set by referencing an air-traffic controllers' subjective assessment. The time granularity of capacity is 15 min or 1 hr. The minimum turnaround time of flights is set to 30 min and the maximum is set to 180 min based on civil aviation measures in China. The probability distribution of flying times from airports to YIN is obtained by fitting the actual historical flight operation data recorded in December 2019. The frequency distribution of flying times from ZGGG to YIN for departing flights is shown in Fig. 4. It can be seen that the flying times from ZGGG to YIN follow an obvious form of multi-peak distribution.

For the uncertainty constraints, the flying times from each airport to YIN were calculated. According to the historical distribution, two cases ( $m = 2$ ) with equal probability are ob-

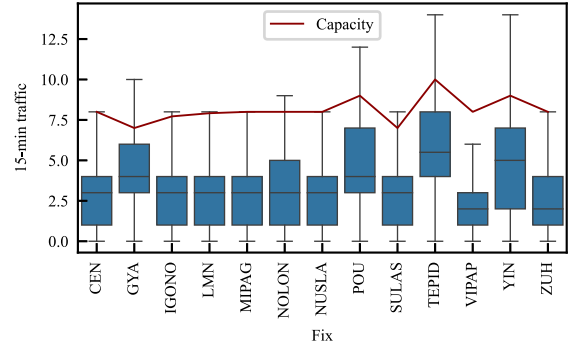


Fig. 3. Boxplot of 15-min traffic at main fixes.

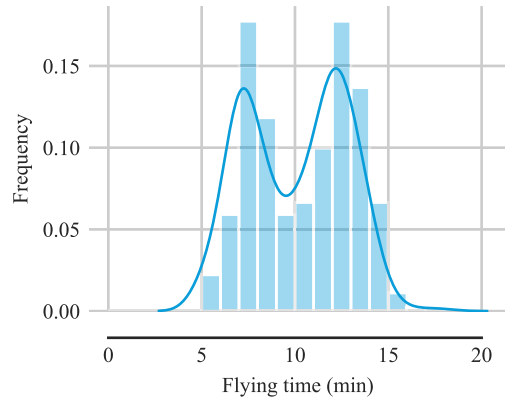


Fig. 4. Flying time distribution of departure flights from ZGGG to YIN.

Table 2. The flying times from different airports to YIN.

Airport	Set of flying times (5 min)	Median
VMMC	{4, 5}	5
ZGGG	{2, 3}	3
ZGHZ	{4, 5}	5
ZGSD	{5, 5}	5
ZGSZ	{4, 4}	4

tained. Each case corresponds to a value of flying time. Notice that  $m = 2$  is reasonable because the flying times from most airports to YIN vary within 10 min. For example in Fig. 4, the flying times of most flights are scattered between 5 and 15 min. For the certainty constraint, the flying times are taken as the median of the historical flying times. The range of flying times for the uncertainty constraint and the certainty condition is shown in Table 2. Note that the flying times from ZGSD and ZGSZ are the same in the two cases. Because the flying times are relatively concentrated, there is only one value for flying times when two cases are considered.

### 3.3. Computational time

The computational times under different violation probabilities  $\alpha$  are presented in Fig. 5. First, the computational time is under 20 min when  $\alpha$  ranges are from 0.2 to 0.5. The relatively short computational time indicates that it does not require too much computational cost to practically apply the USAM. Second, the computational time increases when the violation probability  $\alpha$  decreases. This is proven by understanding that a smaller value of violation probability  $\alpha$  makes the constraints tighter, thereby requiring more time to find the optimal solution. Third, the relationship between the

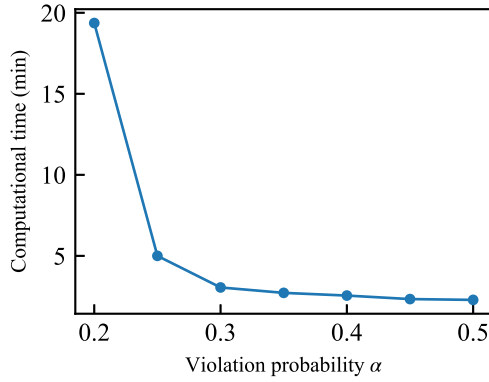


Fig. 5. The relationships between computational time and violation probability.

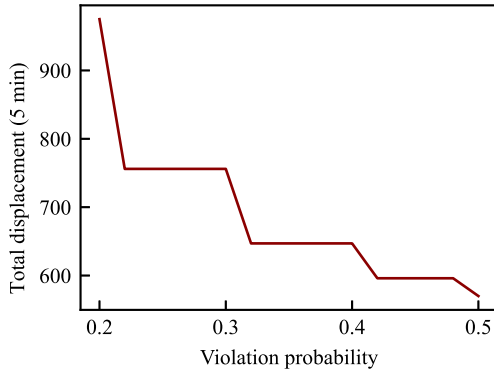


Fig. 6. The relationship between total displacement and violation probability.

computational time and the violation probability  $\alpha$  is not linear. Specifically, the change in computational time slows down when the violation probability increases. That is to say, a small reduction of violation probability when  $\alpha$  is small may require more computational effort to obtain the solution.

### 3.4. Slot displacement

Next, the violation probability impact on total displacement is examined. The results are shown in Fig. 6. It can be seen that the total displacement decreases in a step-like manner when the probability of violation increases. This is because the number of flights within 15 min is not a continuous variable. To compare the results of the CSAM and the USAM, three different uncertainty allocation schemes with different violation probabilities are used. A fourth scheme with the certainty of flight times is defined as the control group. The total displacement corresponding to the four different schemes is shown in Table 3.

In the certainty scenario, the flying times from each airport to the fix are constant values. Accordingly, the ETO is determined using only the flight's departure/arrival time. Therefore, a flight will only impact the traffic within a specific time window of the fix.

Under uncertainty constraints, the ETO is determined using both the departure/arrival time and the flying time. This may lead to the situation that a single flight may affect more than one time window of the fix. Therefore, considering flying time uncertainty may make the model more conservative,

Table 3. Total displacement of the four schemes.

Schemes	Displacement (5 min)
Scheme I (certainty)	570
Scheme II ( $\alpha = 0.4$ )	647
Scheme III ( $\alpha = 0.3$ )	756
Scheme IV ( $\alpha = 0.2$ )	975

Table 4. Flying time (5 min) under different experimental scenarios.

Airport	$S_1$	$S_2$	$S_3$	$S_4$	$S_5$	$S_6$	$S_7$	$S_8$
VMMC	5	4	5	4	5	4	5	4
ZGGG	3	3	2	2	3	3	2	2
ZGHZ	5	5	5	5	4	4	4	4
ZGSD	5	5	5	5	5	5	5	5
ZGSZ	4	4	4	4	4	4	4	4

Table 5. The number of flights exceeding capacity in each scenario.

Schemes	$S_1$	$S_2$	$S_3$	$S_4$	$S_5$	$S_6$	$S_7$	$S_8$
Scheme I	0	2	8	8	3	3	10	9
Scheme II	0	1	1	1	0	0	0	0
Scheme III	0	0	0	0	0	0	0	0
Scheme IV	0	0	0	0	0	0	0	0

while the total displacement increases as the probability of violation decreases.

In theory, the violation probability should be as small as possible. However, it is necessary to consider the computational time (Section 3.3) and total displacement (Section 3.4) due to different violation probabilities. This gives room to decision-makers to trade off the violation probability and allocation cost. Here, it was found that  $\alpha = 0.4$  can basically satisfies the capacity constraints in various scenarios, and  $\alpha = 0.3$  almost satisfy the capacity constraints in all of the scenarios. Therefore, a smaller value of  $\alpha$  may no longer be necessary. Otherwise, it would increase unacceptable allocation costs.

### 3.5. Traffic at YIN

The purpose of considering flying time uncertainty is to enhance the robustness of the optimization results. The modified flight schedule should satisfy the capacity constraint of fixes with different flying times for different operation scenarios. To examine the effect of flying time uncertainty on fix traffic, a total of eight experimental scenarios ( $S_1$  to  $S_8$ ) are set up, in which the flying times from the airport to the fix are different (Table 4). Notice that the experimental scenarios in Table 4 contain all of the scenarios under  $m = 2$  for a certain departure flight to YIN. Next, the traffic at YIN under all eight scenarios is analyzed in detail.

The four schemes are tested under different experimental scenarios. Whether or not the traffic of different optimization plans when using different experimental scenarios exceeds the fix capacity was investigated. It was found that the results of Scheme III and Scheme IV can perfectly cover all possible scenarios. Schemes I and II may have traffic exceeding the capacity of the fix. In different scenarios, the number of flights that exceeded the capacities of Scheme I and Scheme II are shown in Table 5.

For  $S_1$ , the traffic optimized using both the CSAM and USAM did not exceed the capacity of the fix, because the fly-

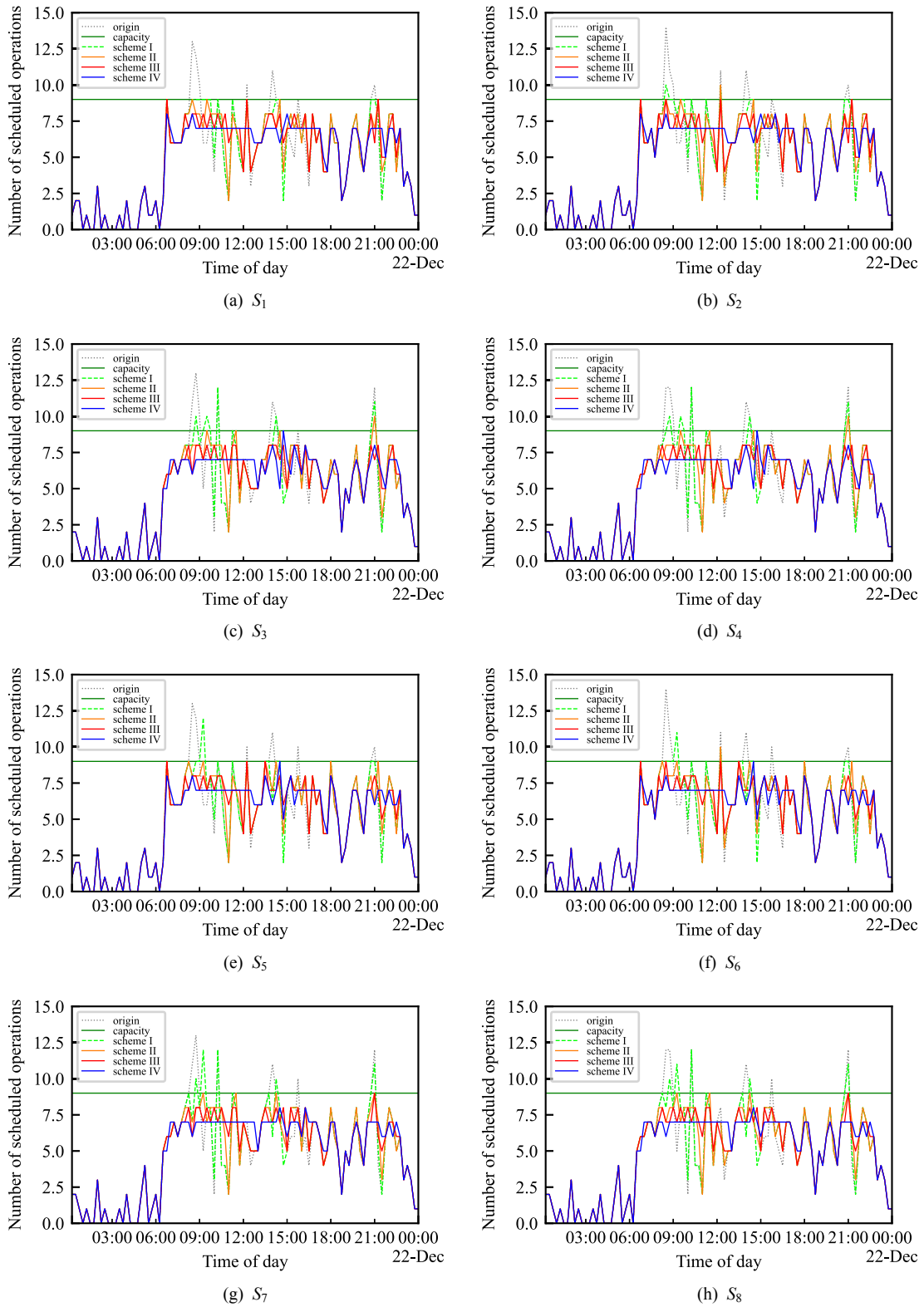


Fig. 7. Traffic at YIN under different scenarios.

ing times under this scenario are the same as that of Scheme I. However, the traffic exceeds the capacity in the other seven scenarios from  $S_2$  to  $S_8$  using CSAM. However, the allocation results of Scheme II only slightly exceeded the capacity in a few cases. Even if the traffic exceeds the fix capacity, the number of flights exceeding capacity was relatively small. The number of flights exceeding capacity, to

some extent, indicates the robustness of the allocation results. Specifically, the smaller the number is, the more robust the results are. From this perspective, it was proven that the robustness of the USAM is much better than that of CSAM.

Next, the flight traffic at YIN under different scenarios (shown in Fig. 7) was investigated. There are three peaks at YIN in the original schedule (i.e., 8:00–9:00, 14:00–



15:00, and 21:00–22:00). As expected, these peaks are smoothed. That is, fewer flights are scheduled in peak time periods, and more flights are rescheduled to other time periods.

It is worth noting that although the traffic after allocation is smoother (i.e., traffic in peak time periods is apparently reduced), it is still not evenly distributed throughout the day. In other words, there still exist peak periods. Intuitively speaking, when the flight flow is evenly distributed throughout the day, the delay will be small. However, a smoother flight distribution will cause much more displacement than the solution produced using USAM. On the contrary, USAM reduces some flow peaks when compared to the original schedule. This is more realistic than a completely flat flow distribution. It can also consider airline preferences and potential passengers' demands.

As stated in the discussion above, the results of CSAM (Scheme I) cannot meet the capacity constraint perfectly. Under some scenarios, the traffic peak is only reduced slightly. Sometimes, CSAM even causes a negative effect. For example, in scenarios  $S_4$ ,  $S_7$ , and  $S_8$ , the traffic peak becomes even more serious. For these scenarios, the time period of traffic peaks is even longer, and the exceeding numbers of flights are higher. This suggests that if the uncertainty of flying times is not considered, the effect of slot allocation may be counterproductive in some scenarios, causing more flights to be delayed.

For the USAM, traffic at YIN under different schemes is also different. The traffic peaks are reduced to different degrees. In contrast, traffic under Scheme II is basically below capacity. For Schemes III and IV, traffic at the fix is always below capacity. There is even capacity unused, which depends on the violation probability. For example, in  $S_4$  from 8:00 to 12:00, traffic is less than 8 and 7 for Scheme III and Scheme IV, respectively. This leaves room for traffic adjustment at the tactical stage.

#### 4. Conclusion

This paper presents an uncertainty slot allocation model (USAM) for a MAS that considers the uncertainty of flying times between airports and departure fixes. The overall objective is to efficiently use both airport capacity and airspace capacity. The authors propose an uncertainty slot allocation model in which the uncertainty of flying times is modeled by chance constraints. The chance constraints are transformed into deterministic constraints, thereby enabling the model to be solved using the Groubi method. The model is applied to a MAS in the Guangdong-Hong Kong-Macao Greater Bay Area. Two conclusions are made from the experimental study. First, the slot allocation cost (i.e., total displacement) grows as the violation probability decreases. Second, the robustness of the optimized schedule can be improved significantly using a smaller violation probability. The results suggest that the USAM can provide better solutions to slot management bodies at the strategic stage. Furthermore, the trade-off between scheduling costs and the robustness of

the optimized schedule was examined. It was found that 0.3 is an appropriate level for violation probability for slot management in the MAS of GBA.

There are several limitations in the work conducted here that require further investigation. First, the granularity of a slot is 5 min, while the actual flying times between airports and fixes are within 1 min. To use finer granularity for airport slots, the computational cost would increase dramatically. Second, the chance constraints were transformed based on the distribution of airport-fix flying times. An approximate solution may be obtained by leveraging the Lyapunov theorem. Future work may generate uncertainty sets and develop a robust optimization model. This research provides both a toolbox and an impetus for future work to improve the operation efficiency of a MAS. The proposed approach could also be extended to pre-tactical or tactical air-traffic flow management.

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Keumjin Lee  
Associate Editor