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An adapted Fast Marching Tree for contrails mitigation: A short-and long-range flight study

Céline Demouge*, Andréas Guitart, Gaspard Dannet and Daniel Delahaye †

February 13, 2024

Abstract

This paper addresses the problem of finding a trajectory for an aircraft minimizing its total environmental impact. In particular, CO₂ and contrails are considered. The Fast Marching Tree method is known to be efficient to find trajectories avoiding obstacles. This method is adapted to the case of soft obstacles, here areas favorable to contrails. This study is done in 2D and for the cruise phase of airliners. Two case studies on short- and long-haul flights are presented. The given results show trajectories minimizing the overall impact according to the given metric by trading off between CO₂ and contrails. Moreover, they are obtained with a low computation time allowing one to consider extensions. Finally, an improvement concerning the sampling is proposed, reducing the computation time and cost.

1 INTRODUCTION

Air transportation is responsible for 1.59% of radiative forcing due to human emission of CO₂, and 5 % of total radiative forcing due to human activity [1]. Fuel optimization is thereby a critical issue for environmental and economic reasons. This is a subject that has been studied extensively in the literature because of the need of efficient models to optimize trajectories. However, the environmental impact is not limited to CO₂. Among these other effects, greenhouse gases such as NO_x can be mentioned, but also other phenomena like contrails. They form behind aircraft when the air is humid and cold enough. They can persist and transform into cirrus clouds, leading to radiative forcing. This phenomenon is increasingly studied in the literature. Many uncertainties remain on their prediction and impact but it is established that the overall effect of this phenomenon is warming.

Contrail areas should be avoided as much as possible. However, if the CO₂ cost of avoidance is too high, it is not worthwhile to avoid contrails. Moreover, unlike thunderstorm areas, safety is not an issue when crossing contrail areas: they are not “hard”

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obstacles and are therefore considered as “soft” obstacles. In a free-flight context [2], this boils down to finding a trajectory that minimizes CO₂ while avoiding such areas as much as possible.

This paper addresses the problem of trajectory optimization in presence of contrails by using the Fast Marching Tree (FMT*) algorithm. This work follows a study done on wind-optimal trajectory generation using FMT* [3], demonstrating the efficient adaptability of FMT* for wind and great circle distance considerations. In addition, previous works have shown the performance of FMT* in environments constrained by hard obstacles. We therefore propose an adaptation of this method to the case of contrails, considered as soft obstacles.

This paper is organized as follows: Section 2 presents previous works on contrails avoidance trajectory generation. Section 3 proposes a mathematical model, and a resolution method. Finally, Section 4 shows the first results of the proposed algorithm on two study cases (Berlin-Madrid and Paris-Los Angeles) and presents an improvement of the algorithm.

2 PREVIOUS RELATED WORKS

This section presents previous works related to contrails avoidance methods and sampling-based path planning algorithms.

2.1 Contrail avoidance methods

Contrails form at the rear of aircraft in humid and cold areas, typically at cruising altitudes. Under certain conditions, detailed in the sequel, they persist and turn into clouds called cirrus (named hereafter Contrails Induced Cirrus - CIC). They can have either a parasol effect or a greenhouse effect. The later prevails and therefore, globally, these CIC have a warming effect on the climate [1].

Making a big detour to avoid contrails would increase fuel consumption and thereby CO₂ emissions too much, and would therefore not be environmentally efficient. Thus, contrails should be considered as soft obstacles. This subsection first presents the different metrics related to contrails that can be found in the literature. Then, different methods used in the literature are presented.

2.1.1 Contrail metrics

The goal is to minimize the total environmental impact of a trajectory, then metrics are needed to put CO₂ and non-CO₂ effects on the same scale.

The *Global Warming Potential* (GWP) is used as a conversion factor to balance the impact of a greenhouse gas and that of CO₂. It is based on the comparison of a pulse impact of a gas to 1 kg of CO₂ and integrated over a time horizon H . GWP is therefore a function of H . For contrails and CIC, it is defined on a per CO₂ kg emission basis. More details about GWP can be found in [4], and Table 1 gives some examples over three time horizons.

Table 1: GWP for contrails with different time horizon.

	$H = 20$ years	$H = 100$ years	$H = 500$ years
$\text{GWP}_{\text{contrail}}(H)$	0.74	0.21	0.064
$\text{GWP}_{\text{CIC}}(H)$	2.2	0.63	0.19

Other metrics, based on more complex models than those mentioned before, have been developed [5]. They are more realistic, but by using a more complex model, the trajectory generation can be more complicated and may require more computation time.

2.1.2 Methods used in the literature

Several methods have been used for contrail mitigation trajectory design.

Since the temperature is a crucial point in contrail formation, flying at lower altitudes can be a very efficient mean for mitigating contrails. In [6], authors show that reducing by 2000 ft the cruise altitude, radiative forcing due to CO_2 effects increases by 1%, but non- CO_2 effects decrease by 18%.

In [7], optimal control is used to compute wind-optimal trajectories for cross-polar flights while avoiding regions favorable to persistent contrail formation. The results show gradual reduction of flight time in contrail areas with fuel consumption increase. In [8], a Mixed Integer Linear Programming (MILP) approach is used, resulting in some cases to 48% reduction of persistent contrails, with only 0.5% supplementary fuel consumption. Metaheuristics such as genetic algorithm are also used [9]. Finally, deterministic graph methods are used, for instance in [10], Rosenow *et al.* use an A* algorithm coupled with a simulation framework. These deterministic methods have really good results but are based on some established grid, which, in the free-flight context, can be a problem. Indeed, a dense graph leads to a solution near to the optimal in the free space but with a high computation time. At the opposite, a less dense graph leads to larger sub-optimality but with small computation time.

Sampling-based methods do not appear in the literature review, but are effective in the case of optimal-wind trajectory computations and obstacle avoidance. This paper therefore proposes to adapt them to the case of contrails.

2.2 Sampling based methods

First introduced by Lavalley *et al.* [11], these methods were then improved in [12] and [13] to become very fast and asymptotically optimal. The three main methods are Probabilistic RoadMaps (PRM), Rapidly-exploring Random Tree (RRT), and Fast Marching Tree (FMT*) [14]. Due to their efficiency, these methods are widely used in robotics. Recently, they have been adapted to aircraft trajectories design [15, 16]. The principle of these methods is to generate a graph by random sampling. The sampling can be initially generated as in FMT* and PRM or generated during the graph growth as in RRT. In both cases, these algorithms create a graph that avoids obstacles, and then finds a shortest path between two points. This paper proposes to use the FMT* algorithm to generate optimal paths in presence of contrails. Indeed, according

to the literature, this algorithm is the fastest sampling-based path-planning algorithm and seems particularly adaptable to aircraft trajectory design. The FMT* algorithm is described in Subsection 3.3.

3 PROBLEM MODELLING AND RESOLUTION METHOD

This section presents the problem model, contrail areas computation, and the resolution approach.

3.1 Problem model

The goal is to find an aircraft trajectory γ minimizing its total environmental impact taking into account both CO₂ and non-CO₂ effects (only contrails here). To do so, the fuel consumption is to be minimized and the passage in persistent contrails areas is penalized with a conversion factor to render it comparable to CO₂. The problem is written as follows:

$$\text{minimize}_{\gamma \in \Gamma} \int_{t_0}^{t_f} C_{CO_2} f + C_r r(x, y) dt,$$

where f is the fuel flow of the aircraft, C_{CO_2} is the conversion factor from the fuel quantity to the CO₂ quantity emitted (for weighting standard fuel, $C_{CO_2} = 3.16$ kg (CO₂) / kg (fuel)[17]), C_r is the factor used to balance the impact of CO₂ and that of contrails, typically here the GWP is used, and $r(x, y)$ is a penalty function to determine whether the aircraft is in a persistent-contrail area. It is equal to 1 when in contrail areas and 0 elsewhere. A constant fuel flow is assumed in the sequel. Indeed, the computation is done for the aircraft cruise phase with a constant altitude. Then, the aim is to minimize $\int_{t_0}^{t_f} 1 + g_H r(x, y) dt$, where g_H is the GWP subject to the time horizon H chosen.

3.2 Where are contrails?

Based on temperature, pressure and relative humidity data, contrails and persistent contrails areas are computed thanks to the following process, as usually done in the literature (see for instance [18]). First, at a given point, the *Schmidt-Appleman criterion* is applied: a contrail is formed if the relative humidity of the air in liquid water, RH_w is above a threshold r_{min} , where

$$r_{min} = \frac{G(T - T_c) + e_{sat}^{liq}(T_c)}{e_{sat}^{liq}(T)}, \quad (1)$$

$e_{sat}^{liq}(T)$ is the saturation vapor pressure over water, and T_c is the estimated threshold temperature (in Celsius degrees) for contrail formation at liquid saturation. The later is computed as follows:

$$T_c = -46.46 + 9.43 \log(G - 0.053) + 0.72 \log^2(G - 0.053), \quad (2)$$

where $G = \frac{EI_{H_2O} C_p P}{\varepsilon Q (1 - \eta)}$, $EI_{H_2O} = 1.25$ is the water vapor emission index, $C_p = 1004 \text{ J.kg}^{-1}.\text{K}^{-1}$ is the heat capacity of the air, P is the ambient pressure (in Pascals), $\varepsilon = 0.6222$ is the ratio of the molecular masses of water and dry air, $Q = 43 \cdot 10^6 \text{ J.kg}^{-1}$ is the specific heat of combustion, and $\eta = 0.3$ is the average propulsion efficiency of a commercial aircraft (using the same values as in [18]).

Then, if the point is in an ice-supersaturated area, it is considered in a persistent-contraill favorable area. In [18], these areas are determined thanks to the following criterion: $RH_i > 1$, where the relative humidity over the ice, noted RH_i is computed as follows:

$$RH_i = RH_w \frac{6.0612 \exp\left(\frac{18.102 T}{249.52 + T}\right)}{6.1162 \exp\left(\frac{22.577 T}{273.78 + T}\right)}, \quad (3)$$

where T is the ambient temperature in Celsius degrees.

3.3 Resolution algorithm

The resolution algorithm is an adaptation of the Fast Marching Tree algorithm to great circle distances, wind and soft obstacle considerations. The sequel describes the original algorithm before focusing on the adaptation done in this study.

The environment is sampled with N points (nodes). At each iteration, nodes are split into three sets:

1. $V_{unvisited}$: nodes never visited;
2. V_{open} : nodes already visited but for which the cost value is not yet definitively computed;
3. V_{closed} : nodes visited with a definitive cost value.

One iteration follows this process:

1. A lowest-cost node z is chosen in V_{open} .
2. For each neighbors node x of z which are in $V_{unvisited}$, an edge between x and an open neighbors of x is added to make the connection locally optimal without considering any constraint.
3. If this connection does not violate constraints (*e.g.* obstacles crossing), the edge is added to the tree and x is put in V_{open} .
4. Once all unvisited neighbors of z have been visited, z is put in V_{closed} .

All the nodes at a distance r_N of a node are considered as its neighbors. It is the *neighborhood radius* and is a function of the number N of samplings. The radius used in [14] is:

$$r_N = 2(1 + \eta) \left(\frac{1}{d}\right)^{1/d} \left(\frac{\mu(\mathcal{X}_{free})}{\zeta_d}\right)^{1/d} \left(\frac{\log(N)}{N}\right)^{1/d}, \quad (4)$$

where $\eta > 0$, d is the space dimension, \mathcal{X}_{free} is the free space, $\mu(\mathcal{X}_{free})$ is its Lebesgue measure, and ζ_d is the volume of the d -dimensional unitary ball.

First, based on the preliminary study [3], the problem where no contrails are taken into account is solved. In [3], the cost to be minimized between two nodes (Step 3) is the flight time, implying great circle distances and ground speeds. The first one is computed in a computationally non-expensive way as in [3]. The ground speed is computed taking into account spatially-dependent static wind.

In this study, the cost (used at Step 3) is further adapted to take into account contrails: the cost of edge (i, j) between node i and node j is:

$$c_{i,j} = t_{i,j} (1 + \lambda_{i,j} g_H), \quad (5)$$

where $t_{i,j}$ is the flight time between i and j (great circle distance divided by the ground speed), $\lambda_{i,j} \in [0, 1]$ is the proportion of the edge that is in a persistent contrail area, and g_H is the chosen GWP value, depending on the time horizon H . The cost function differs from the one used in [3], which only took flight time into account.

4 CASE STUDIES AND METHOD IMPROVEMENT

This section presents two study cases of our algorithm before focusing on an improvement on the sampling. All the presented results have been developed in Java programming on a computer with an Intel Core i5-10210U, 1.60 Hz, with 8 Go RAM and a Debian Linux OS. The weather data for wind consideration and data for contrail areas computation (see Subsection 3.2) have been extracted from ERA5 reanalysis data [19]. CO₂ emissions are estimated from OpenAP [20], considering an initial mass of 80% of the Maximum Take Off Mass of the aircraft. Unless otherwise indicated, the results are obtained with a uniform sampling of 10,000 points on a rectangle.

4.1 Berlin-Madrid case study

4.1.1 Instance setup

A typical short-range aircraft, the Airbus 320, is considered for the results presented in the sequel, and the airspeed considered is $V_a = 450$ kts [21].

The first study case is a flight from Berlin to Madrid at the Flight Level 300 on January 5th 2023, at 00:00 UTC. Figure 1 and Figure 2 show the wind and the persistent contrail areas encountered.

The computational experiments have been done with a GWP computed with two different time horizons (20 and 100 years).

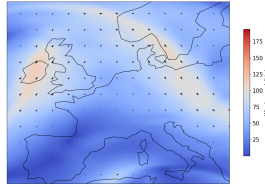


Figure 1: Wind setup, at FL300 on January 5th 2023, at 00:00 UTC.

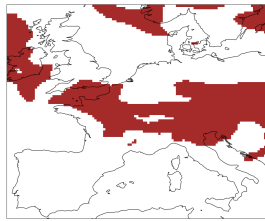


Figure 2: Contrails setup, at FL 300 on January 5th 2023, at 00:00 UTC.

4.1.2 Results

Figure 3 and Table 2 show the results obtained by our algorithm. Loss in flight time or CO₂ emissions and gain in flight time in contrail areas are computed in comparison with the case where no contrail are considered.

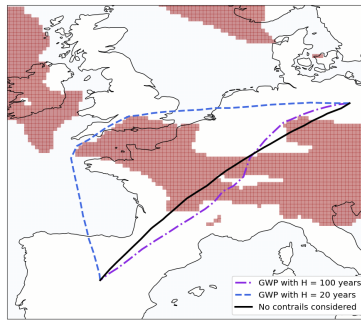


Figure 3: Results on Berlin-Madrid, at FL300 on January 5th 2023, at 00:00 UTC, with different time horizons for GWP.

This study case shows the efficiency of the algorithm, since the mean computation time is 6.5 seconds. It is then a fast algorithm allowing to avoid contrails while trading off CO₂ considerations which could be used at tactical level. Moreover, the impact of the chosen metric, in particular the time horizon H chosen for the GWP computation, is shown. In the case of a short time horizon, contrails have a strong impact (see Table 1) and the areas are thereby avoided, even at the expense of a big increase in CO₂. This kind of solution does not seem operationally acceptable. On the contrary, with a

Table 2: Results on Berlin-Madrid, at FL300 on January 5th 2023, at 00:00 UTC, with different time horizons for GWP.

	Flight time in minutes (loss)	CO ₂ emissions in kg (loss)	Time in contrail areas in minutes (gain)
No contrail considerations	134	13,595	55
GWP with H=100 years	141 (+5%)	14,294 (+5.1%)	27 (-51%)
GWP with H=20 years	165 (+23%)	16,674 (+22.6%)	0 (-100%)

longer time horizon, the computed solution is more balanced between total CO₂ and time spent in contrails.

4.2 Long haul study case

A long-range flight is considered: Paris to Los Angeles with a typical long-range aircraft, the Boeing 777-300ER and the airspeed $V_a = 490$ kts [22]. The flight level chosen is FL 340 (250 hPa).

Figure 4 shows the wind encountered for the transatlantic area presented in this subsection and Figure 5 shows the considered persistent-contrail areas.

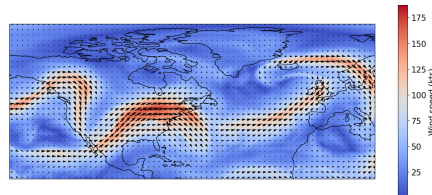


Figure 4: Wind setup, at FL340 on January 01th 2022, at 11:00 UTC.

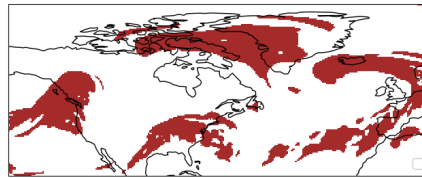


Figure 5: Contrails setup, at FL340 on January 01th 2022, at 11:00 UTC.

Table 3 shows the quantitative results obtained with a mean computation time of 6.8 seconds.

Table 3: Results on study from Paris to Los Angeles, at FL340 on January 01th 2022, at 11:00 UTC.

	Flight time in minutes (loss)	CO ₂ emissions in kg (loss)	Time in contrail area in minutes (gain)
No contrail considerations	597	256,649	95
GWP with H=100 years	622 (+4.2%)	266,735 (+3.9%)	11 (-88.4%)

These results shows that, even though transatlantic flights are constrained by wind, contrails can be avoided at the expense of a small increase in CO₂ emissions. This is obtained within a small computation, allowing the user to perform several tests if ensemble forecasts [23] are considered for instance or to update the trajectory all along the flight, based on the updates of the weather forecasts.

4.3 Sampling improvement

Some areas, when sampling a rectangle, are never visited by the algorithm. Indeed in the case of the transatlantic flights, some areas of the sample rectangle are not visited or will not be with certainty in the final solution, as shown in Figure 6.

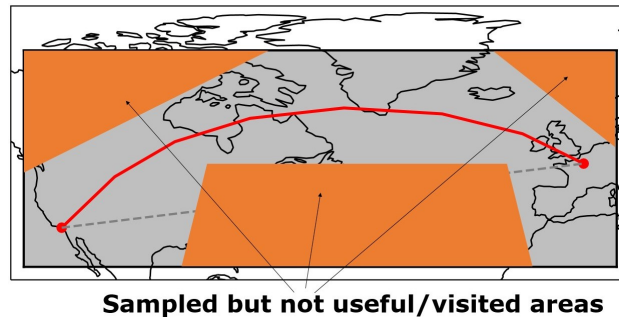


Figure 6: Some areas are sampled but are not useful in the algorithm.

By avoiding sampling these areas, with a same total number of sampling points, the area of interest could be more finely sampled. Similarly, the number of points can be reduced by the same factor as the area is reduced, since the sampling is uniform, reducing thereby computation time and memory-space use. Moreover, this allows one to constrain the trajectory in a specific part of the space, avoiding areas that are too far for instance.

To reproduce a desirable search space for trajectories between Paris and Los Angeles, it is possible to process historical flight data (shown on Figure 7) and determine a tube that the computed trajectory should not leave. A safe margin is taken with re-

spect to the flown extremes, arbitrary fixed to 10% here. The construction principle is described on Figure 8 and such a sampling tube is shown on Figure 9

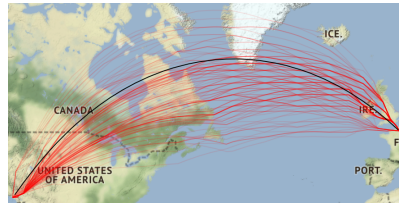


Figure 7: Past flights from Paris to Los Angeles in September 2019, from Eurocontrol R&D data [24] (in black: the great circle trajectory).

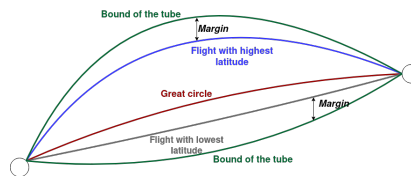


Figure 8: Tube construction principle.

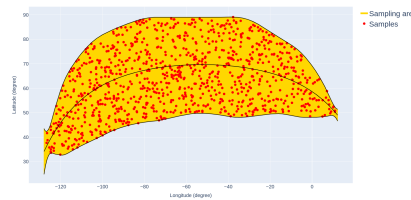


Figure 9: Example with 1000 samples in a tube between Paris and Los Angeles and bounded by extremal latitudes.

Then, two experiments are conducted:

1. 10,000 samples, as before but within the tube;
2. 7,350 samples in a reduced sampling area, *i.e.*, using the same distribution as before with a sampled area reduced by a factor 1.36. A reduced computing time with roughly the same performance is expected.

The setup of the previous studies is used. Figure 10 and Table 4 show the results with 10,000 samples (obtained in 6.7 seconds on average) and with 7,350 samples (in 5.7 seconds on average). Costs and trajectories are likely the same with a lowest computation time.

To conclude, this improvement reduce the number of samples or increase the concentration of the sampling. If the number of samples can be reduced, then the algorithm

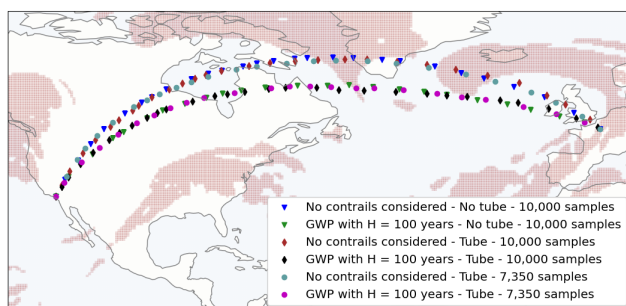


Figure 10: Results on Paris-Los Angeles, at FL340 on January 01th 2022, at 11:00 UTC, with tube sampling.

Table 4: Results on study from Paris to Los Angeles with tube sampling. Comparison are computed regarding cases with the same sampling.

	Flight time in minutes (loss)	Time in contrail areas in minutes (gain)
No contrail considerations - No tube - 10,000 samples	597	95
GWP with H=100 years - No tube - 10,000 samples	622 (+4.2%)	11 (- 88.4 %)
No contrail considerations - Tube - 10,000 samples	596	89
GWP with H=100 years - Tube - 10,000 samples	623 (+4.5%)	9 (- 89.9 %)
No contrail considerations - Tube - 7,350 samples	596	89
GWP with H=100 years - Tube - 7,350 samples	623 (+4.5%)	8 (- 91 %)

is faster and uses less memory. Thus, it would allow the user to implement it on board or to repeat more easily computations to obtain updated solutions throughout the flight which is also useful considering uncertainties. By increasing the concentration of the sampling, better results are expected with a same number of samples.

5 CONCLUSION

This paper presents a new method for contrail mitigation in a free-flight context. To do this, an adaptation of a sampling-based method, the Fast Marching Tree (FMT*), is implemented. FMT* is known to be efficient for obstacle avoidance [14], and has been used in a first study for wind optimal trajectory computation [3]. It is adapted here to address soft obstacles, such as contrail areas.

Based on experiments on short and long-range flights, it is shown that trajectories

avoiding contrails balancing CO₂ according to a given metric within small computational time. For instance, on a long range, with a time horizon of 100 years for GWP computation, the CO₂ emissions increases by less than 5% when the time spent in the contrails decreases by around 90%. Moreover, an improvement is proposed, allowing the user to constrain the trajectory to a desirable space and reducing computation time.

These encouraging results pave the way to possible extensions. In particular, in the dynamic case, the trajectory could be updated continuously, as weather information is received. The proposed sampling strategy in a tube could avoid the algorithm to modify too much the trajectory from one step to another, while taking advantage of the new information. The margin considered can also be adapted to the flight envelope, improving sampling area to meet flying constraints. In addition, uncertainties could be considered using ensemble forecasts. Finally, an extension to three dimensions could be envisaged by imposing operational constraints.

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