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# SSW-2D: some open-source propagation software introducing split-step wavelet and wavelet-to-wavelet propagation techniques

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**Abstract**—In this paper, the open-source software SSW-2D for propagation simulation is introduced. The software includes the classical split-step Fourier, as well as the more recent split-step wavelet (SSW) and wavelet-to-wavelet propagation (WWP) techniques. SSW and WWP benefit from the properties of wavelet decompositions. Therefore, they are less memory-intensive and present a lower complexity than SSF. However, the coding is more demanding. It requires additional algorithmic steps – including the computation of wavelet propagators, which asks for a fine understanding of the decompositions.

This free and open-source Python software is proposed for encouraging the community to handle and improve SSW and WWP. The interaction with the computation core can be made through CSV files or with a dedicated GUI. Note that the final field is saved with respect to altitude, whereas only the wavelet decomposition of the total field is saved. This emphasizes the memory gain due to the sparse wavelet decompositions.

**Index Terms**—Atmospheric propagation, parabolic wave equation, wavelet transforms, open-source software.

## I. INTRODUCTION

The modelling of the propagation of radiowaves on large distances is a major issue for sizing many surveillance or telecommunication systems. When dealing with open spaces like suburban, rural, maritime, or even space environments, deterministic tools are often preferred. In this goal, the split-step Fourier (SSF) resolution of the parabolic equation is a widely used technique for the modelling of radio waves [1] in slowly-varying environments, as well as for acoustic waves [2].

SSF is based on a resolution of the propagation equation in two steps: a propagation step in the Fourier domain and an environment step – to account for atmosphere and relief – in the space domain. Here, we introduce a tool based on SSW [3] and WWP [4]. SSW mimics SSF, except that the propagation steps are performed in the wavelet domain. Basically, this brings two main advantages and one drawback. On one hand, the wavelet decomposition is faster than the Fourier transform and the wavelet representation of the signal allows for sparse representations. On the other hand, unlike in the Fourier domain, the propagator is not diagonal in the wavelet domain, which complicates this step.

In WWP, all the propagation is performed in the wavelet domain, which implies a faster computation and additional hypotheses.

The goal of this paper is to comprehensively introduce and illustrate the SSW-2D propagation tool and the techniques within. It is proposed for research and/or educational purposes first. Note that this software is available online [5].

In Section II, the SSW and WWP methods are introduced. The current capacities of the SSW-2D software are presented in Section III. Then, Section IV is dedicated to a discussion on SSW and WWP, including some research leads for the computational physics community.

## II. THE PROPAGATION ALGORITHMS

### A. Overview of the split-step techniques

We work on a 2D scenario in cylindrical coordinates  $(r, \theta, z)$ , assuming an invariance along  $\theta$ . In these conditions, one must solve the propagation equation in harmonic mode

$$\frac{\partial^2 u}{\partial r^2} - 2jk_0 \frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial z^2} + k_0^2(n^2 - 1)u = 0, \quad (1)$$

where  $u = Ee^{jk_0 r}$  is the reduced complex electric field,  $k_0$  is the wavenumber in the vacuum and  $n$  is the refractive index of the medium.

By neglecting the backward propagation, and using the approximation  $\sqrt{1+A+B} \approx \sqrt{1+A} + \sqrt{1+B} - 1$  for  $A, B \ll 1$ , this equation can be reduced to a one-order differential equation with  $r$ , denoted as the wide-angle parabolic equation [6]. Its iterative solution is

$$u(r + \Delta r, z) = e^{\left[-jk_0(n-1)-j\left(\sqrt{k_0^2 + \frac{\partial^2}{\partial z^2}} - k_0\right)\right]\Delta r} u(r, z). \quad (2)$$

The classical SSF algorithm numerically solves this equation by passing the second term in the Fourier domain, which yields

$$u(r + \Delta r, z) = e^{-jk_0(n-1)\Delta r} \mathcal{F}^{-1} \left[ e^{-j(k_r - k_0)\Delta r} \mathcal{F} [u(r, z)] \right], \quad (3)$$

where  $\mathcal{F}$  denotes for the Fourier transform and  $k_r$  is the horizontal wavenumber. The term  $e^{-j(k_r - k_0)\Delta r}$  is the free-space propagator in the sense that it transforms the plane wave spectrum at  $\Delta r$  into the plane wave spectrum at  $r + \Delta r$  considering free space between them.

### B. Split-step wavelet

Similarly to SSF, the numerical solution of (2) with SSW is [3]

$$u(r + \Delta r, z) = e^{-jk_0(n-1)\Delta r} \mathcal{W}^{-1} [\mathcal{P}\mathcal{W}[u(r, z)]], \quad (4)$$

where  $\mathcal{W}$  denotes for the wavelet decomposition (and compression),  $\mathcal{W}^{-1}$  is the wavelet recomposition, and  $\mathcal{P}$  is the free-space propagator in the wavelet domain detailed in Section II-D.

The wavelet decomposition leads to a multiscale representation of a discrete signal of size  $N_z$  given by

$$u[p_z] = \sum_{p=1}^{N_z/2^L} a_{L,p} \phi_{L,p}[p_z] + \sum_{l=1}^L \sum_{p=1}^{N_z/2^l} d_{l,p} \psi_{l,p}[p_z], \quad (5)$$

where  $\psi_{l,p}$  denotes a wavelet of level  $l$  and  $\phi_{L,p}$  is the scaling function. The translations  $p$  are necessary for the wavelets to cover the entire vertical domain. The fast wavelet transform (FWT) [7] is used to rapidly obtain the coefficients  $a_{L,p}$  and  $d_{l,p}$ , with a linear complexity with  $N_z$ . Most of the coefficients can be set to 0 afterwards with a minimal and controlled loss of information via a compression step [8].

The details and validations of the SSW algorithm are given in [3]. We have especially demonstrated that the algorithm has a linear time complexity whereas SSF complexity follows  $\mathcal{O}(N_z \log N_z)$ . Moreover, the compression steps in SSW imply an error that can be managed by choosing appropriate thresholds [9].

An example of a SSW result is plotted in Figure 1a. The propagation factor is displayed with respect to distance and altitude. It is calculated in the presence of a triangular relief (in black) and a trilinear atmospheric duct. Frequency is 500 MHz and the source is a complex source point placed 50 m before the first vertical of computation and at 50 m of altitude. Apodisation (absorbing layer) is performed above the dotted horizontal line with a Hanning window.

In Figure 1b, the wavelet decomposition of the vertical field at 35 km on  $L = 3$  levels is displayed. The white spaces correspond to null coefficients. One can see that most of the field is carried by the scaling functions  $\phi_L$ . The detail coefficients corresponding to wavelets of lower levels are nonzero only where the field is divergent, from 300 m to 500 m of altitude. That is consistent with the fact that the lower wavelets carry the higher part of the signal spectrum.

### C. Wavelet-to-wavelet propagation

In the WWP algorithm, the main idea is to perform all the propagation in the wavelet domain. Therefore, a step of propagation is given by

$$U(r + \Delta r) = \mathcal{N}\mathcal{P}U(r), \quad (6)$$

where  $U$  is the wavelet transform of  $u$ , and  $\mathcal{N}$  is the application of the refraction in the wavelet domain. This refraction is applied on the center position of each wavelet as detailed in [4]. The apodisation step is performed in the wavelet domain too. Therefore, to apply SSW, a first wavelet transform is necessary to initiate the algorithm, and an inverse wavelet transform is applied on the last vertical to obtain the final field. In other words, the actual field is not computed during the propagation, only its wavelet transform.

Since the propagation is performed in the wavelet domain, the ground composition and the relief cannot be accounted. The solution is to hybridize SSW with WWP. SSW is used to calculate the field at the lower altitudes whereas WWP is used for higher altitudes. This hybridization technique, denoted as WWP-H, is also detailed in [4].

The same simulation as in the previous section is performed with WWP-H. Only the relief is not present because it is not available yet in the software.

One can see in Fig. 2a that both refractivity and apodisation are efficiently accounted in the wavelet domain. However, the quantifying of this model error is still an open question. The wavelet coefficients at 35 km plotted in Fig. 2a show that in the absence of relief, all the energy is carried by the scaling functions. Therefore, the compression rate is high (83.7 % on this vertical).

In the next section, the generation and use of the free-space propagator in the wavelet domain are presented.

### D. The wavelet-domain propagator

The tricky part of the SSW and WWP algorithms is the generation and use of the wavelet-to-wavelet propagator.

From our knowledge, the propagation of a wavelet on a step  $\Delta r$  has no closed-form expression. Therefore, the propagator is numerically calculated beforehand using SSF on one free-space step then stored.

What we call the free-space propagator in the wavelet domain is actually a dictionary of local propagators for each wavelet. Since all the wavelets on one level of decomposition are identical, their propagated counterparts are also identical, and the final number of required propagators ( $2^L$ ) is kept small. The way to rapidly calculate these propagators, including the associated pseudo-code, is given by Bonnafont *et al.* [10]. Fig. 3 gives two examples of the local propagators used to generate Fig. 1a. They are associated to the scaling function and to a wavelet of level 2, respectively.

With these propagators, the propagation operator  $\mathcal{P}$  in the wavelet domain in (4) consists in applying the proper propagator to each wavelet coefficient of  $u(r, z)$  to obtain the wavelet decomposition of  $u(r + \Delta r, z)$  propagated in free space on one step  $\Delta r$ . The inverse wavelet transform is finally applied to obtain the very field.

Note that if geometrical parameters are modified during the propagation, the propagators must be updated. This operation is fast and can be performed on the fly [10].

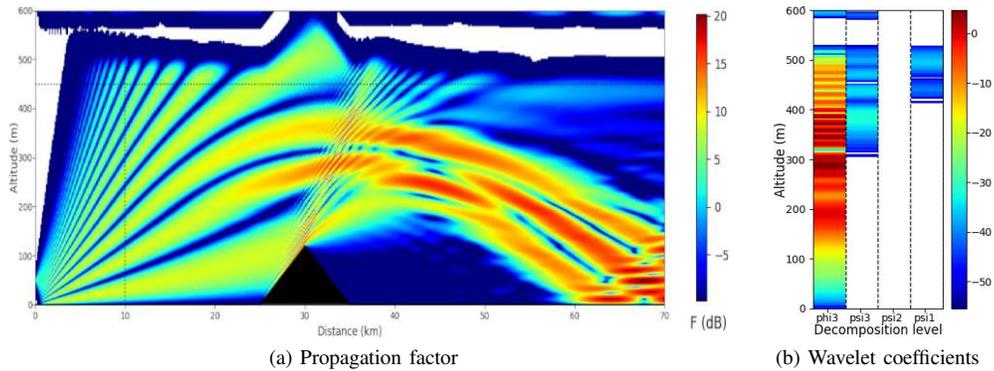


Fig. 1: Propagation factor (dB) obtained by SSW and the corresponding wavelet coefficients of the field in dB at 35 km.

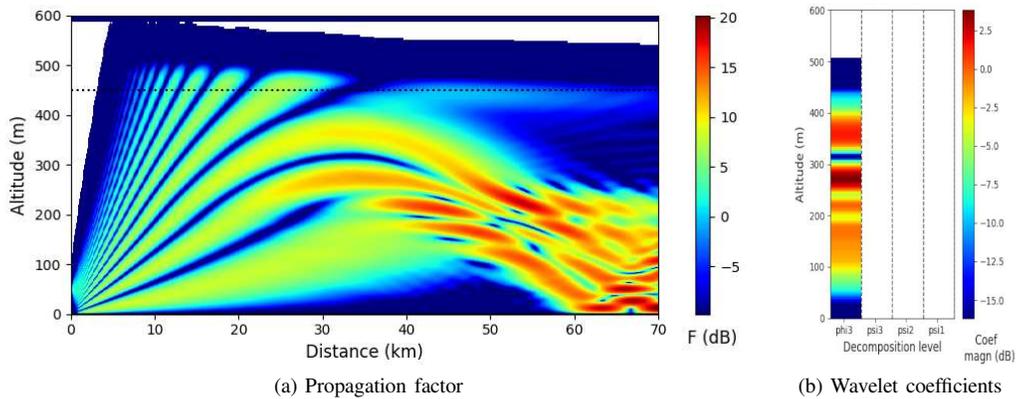


Fig. 2: Propagation factor (dB) obtained by WWP and the corresponding wavelet coefficients of the field in dB at 35 km.

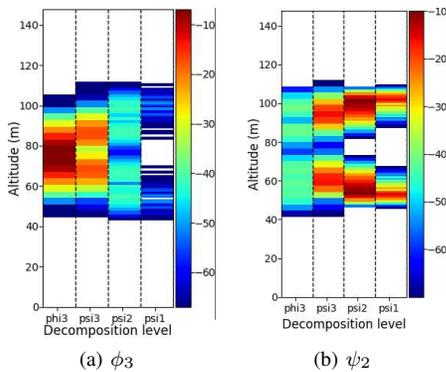


Fig. 3: Two propagators in the wavelet domain: for the scaling function and for one wavelet.

### III. PRESENTATION OF THE SOFTWARE SSW-2D

SSW-2D is a free and open source under the licence GNU GPL V3. It is organised in five Python modules. Each one comes with an html documentation generated by Doxify. In this section, these five modules are presented.

#### A. Source computation

The `source` module is dedicated to the initial field computation. For the time being, only the complex source point [11] is available. This module reads the source parameters,

calculates the associated field, and writes the corresponding electric field in a CSV file.

Any other source could be considered at minimum cost, including a field exported from an antenna simulation software.

#### B. Terrain generation

The `terrain` module is used to generate the relief. Triangular and realistic random reliefs are available. As for the initial field, this module writes this output in a dedicated file.

Similarly to the source input, other source generations can be considered as long as the output is in the proper format.

#### C. Propagation simulation

The `propagation` module is the core of the software. After the calculation of the dictionary of propagators, this function propagates the initial field with the selected technique. Up to now, available techniques are split-step Fourier (SSF), split-step wavelet (SSW), wavelet-to-wavelet propagation (WWP), and hybridized SSW-WWP (WWP-H).

Three variables are saved at the end of the calculation: the dictionary of propagators, the final field, and the wavelet coefficients all along the propagation (this requires an additional step for SSF). Note that the storage of the wavelet coefficients is much less demanding than the equivalent electric field.

#### D. Post processing

The `post_processing` module is mainly dedicated to the results display. At this time, it allows for plotting the dictionary of propagators, the total field from the saved wavelet coefficient or the corresponding propagation factor, and the wavelet decomposition of the field at a given distance from the initial plane. This last function also gives the compression rate obtained after the wavelet decomposition the field.

#### E. Graphic user interface

Finally, the module `GUI` is used to launch the graphic user interface (GUI). In this case, the inputs are directly filled in on the GUI, that automatically updates the CSV files read by the other modules. Press buttons launch the aforementioned modules `source`, `terrain`, and `propagation`, respectively. A display of the GUI is given in Fig. 4.

The use of the GUI is recommended to get familiar with SSW-2D and to test parameter values. Inputs through CSV files are more fitted for expert users willing to modify and/or improve the software.

On this GUI, the initial field and the environment (atmosphere and relief) are also plotted. After the propagation computation, the computed field and the final vertical cut are plotted. The type of output (electric field, propagation factor or Poynting vector) and the dynamic can be tuned as well. The values above the black dotted lines are in the absorbing layer.

Now that both the method and the software have been presented, the conclusion is dedicated to a discussion about the future of the research based on SSW and the usefulness of SSW-2D in this scope.

### IV. CONCLUSION AND FUTURE LEADS

First of all, SSW and WWP techniques are from now on mature techniques to model radiowave propagation. They are shown to be faster than SSF (specially WWP, having a much better complexity) and they allow for a sparse representation of the propagated field. The limitations are similar than for SSF, i.e., the paraxial cone of validity and the neglected backward propagation. The improved efficiency of SSW and SSW is at the cost of compressions of both the signal and the propagator that are acceptable.

In practice, SSW and WWP are particularly interesting on large domains and/or with very high frequencies for which the number of vertical points is large. For instance, it is obviously relevant in a radio occultation context [12].

As for WWP, the additional errors due to apodisation and atmosphere in the wavelet domain are not yet quantified. However, first tests show that these errors seems easily acceptable. This could not be the case in fast varying atmosphere, in the presence of turbulence notably.

As is, SSW-2D is a research code that does not claim to compete with the commercial SSF codes like AREPS or TEMPER. A significant amount of work would be necessary to optimise the algorithms and translate them in a compiled

language. That is also the reason why computation times are not mentioned in this work.

Also, many different features are planned during the next months of development: new sources, ground roughness, surface wave, new reliefs, other wavelet families. But the main objectives now are to track down bugs and propose a detailed documentation. The GIT deposit is the opportunity for the community to test and improve this software.

In addition to these simple features, some leads for future improvements still require research efforts. SSW-2D is promising for applications that demand lots of resources. The straightforward extension is 3D propagation. This has already been proposed [13] but relief and non-homogeneous impedance grounds are not properly accounted yet.

Apodisation may be improved using the wavelet decomposition. The residual field on top on the domain (see Fig. 1a) is due to the periodic boundary conditions for the wavelet decomposition that ensure the orthonormality of the basis. However, this could be improved as well.

Studies could also focus on the very propagators. Today, they are generated with SSF. A more appropriate technique could be derived. For example, a closed-form expression based on the wavelet connection coefficients could improve the overall method.

Last, these techniques can be straightforwardly adapted for acoustic or optic propagation. SSW-2D could be useful in these domains as acoustics deals with large 3D environments and optics with very small meshes.

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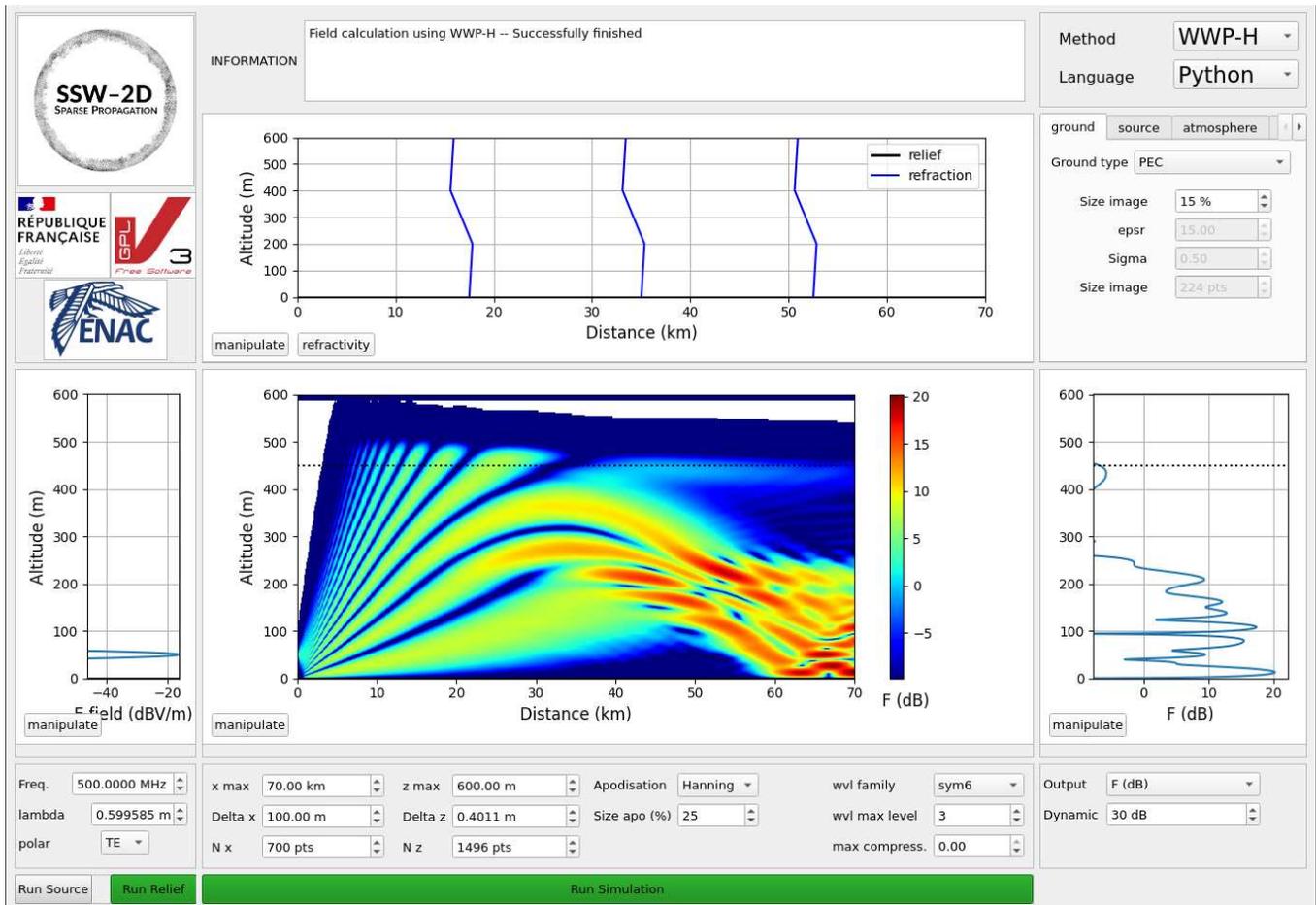


Fig. 4: User interface of the SSW-2D tool.

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