Investigation on Vertical Resolution of Atmospheric Simulations for the Propagation in Realistic Turbulent Media

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Abstract—In split-step methods, the contribution of tropospheric scintillation on the propagating electric field is taken into account by modifying the phase of the signal. The extended-LES method has been proposed to generate phase screens that precisely model atmospheric turbulence. It consists in extending the 3D refractive index fields obtained for the large scales of the medium resolved from atmospheric large eddy simulations (LES) to subrange scales following the K41 theory. In this article, we investigate the impact of the vertical LES resolution on an X-band 2D propagation case. To do so, the extended-LES method is tested in a BOMEX Marine Boundary Layer numerically generated for 3 different vertical meshes. Finally, a log-amplitude variance comparison tends to show that propagation results are dependent on the vertical LES resolution at 10 GHz.

Index Terms—parabolic equation, multiple phase screen, large-eddy simulation, marine atmospheric boundary layer, Kolmogorov spectrum

I. INTRODUCTION

The atmospheric boundary layer (ABL) is a dynamic medium where various atmospheric phenomena occur and can potentially degrade the performance of electromagnetic systems. In particular, tropospheric scintillation, that refers to the small scale fluctuations of the atmospheric refractive index, can cause variations in the amplitude and phase of the electromagnetic signal, inducing additional losses. An accurate modelling of turbulence is therefore crucial to quantify the impact of the atmosphere on the electromagnetic propagation.

The split-step wavelet (SSW) method is employed to model long-range propagation in complex media based on a 2D discrete “wide angle” parabolic wave equation (PWE) framework [1] [2]. In this iterative method, the contribution of the atmosphere is modelled through the implementation of a phase screen introducing a phase shift to the propagation signal. This 1D vertical phase screen is orthogonal to the propagation direction and is obtained by integrating the refractive index fields on all the propagation step.

When dealing with a turbulent atmosphere, the phase screens are usually randomly generated as a stochastic process following the classical K41 theory [3]–[5]. However, this approach has two main drawbacks. First, it is limited to the modelling of the inertial range of scales of the turbulent flow. Secondly, it assumes a vertical homogeneity of turbulence rarely observed in the ABL.

To better represent the phenomenon at large scales, it has been proposed to generate phase screens from an ABL Large-Eddy Simulation (LES). Due to computational time constraints, the LES grid cannot be as fine as the propagation grid. The subgrid scales are therefore modelled based on the K41 theory. This technique is denoted as extended-LES [6]. In this paper, we test this approach for different vertical LES resolutions on an X-band propagation case.

This article is organized as follows. Section II focuses on the generation of the turbulent atmospheric media. It introduces the studied Barbados Oceanographic and Meteorological Experiment (BOMEX) LES case and the extended-LES phase screens generation. Finally, Section III highlights the effects of the vertical LES resolution on an X-band 2D propagation case.

II. PHASE SCREENS GENERATION FROM A BOMEX LARGE-EDDY SIMULATION CASE

The objective of this section is to detail the 1D vertical phase screens generation method from LES outputs. The BOMEX LES case settings are initially outlined. The extended-LES generation method is then presented.

A. BOMEX case

The open-source atmospheric Large Eddy Simulation (LES) code MicroHH is used to simulate a BOMEX case of shallow cumulus [7] [8]. This classical case is initialized from data collected during the BOMEX measurement campaign conducted in the tropical Atlantic Ocean between May and July 1969. This simulation generates a convective cloud layer located between 500 and 1500 m of altitude, situated within a moist turbulent Marine Atmospheric Boundary Layer (MABL).

The time evolution of a $5 \times 5 \times 3$ km LES domain is studied with a $50 \times 50$ m horizontal resolution over 6 hours for 3 different vertical resolutions $d_z$: 50 m, 25 m and 10 m. The simulations become statistically steady after
3 hours. Starting from then the 3D LES fields are stored every \( \tau = l_x / \langle u \rangle = 10 \) minutes. \( l_x \) indicates the size of the LES domain and \( \langle u \rangle \) corresponds to the velocity of the flow in the \( x \)-direction averaged over the entire domain during the last 3 hours. Figure 1 represents the fluctuations of the refractive index field \( \Delta n(y, z) \) averaged over time and \( x \). It is given by

\[
\Delta n = n - \langle n \rangle_{x,y},
\]

where \( \langle \cdot \rangle_{x,y} \) indicates the mean operator over an horizontal plane. This figure highlights the vertical inhomogeneity of turbulence in the low troposphere. Details on boundary and initial conditions can be found in the comparative study led by Siebesma et al. [8]. The main LES simulation parameters used for this study are summarized in Table I.

### TABLE I

**BOMEX CASE : LES PARAMETERS**

<table>
<thead>
<tr>
<th>LES domain size : ( l_x \times l_y \times l_z )</th>
<th>( 5 \times 5 \times 3 ) km</th>
</tr>
</thead>
<tbody>
<tr>
<td>Horizontal resolution : ( d_x \times d_y )</td>
<td>50 \times 50 m</td>
</tr>
<tr>
<td>Vertical resolution : ( d_z )</td>
<td>[50; 25; 19] m</td>
</tr>
<tr>
<td>Simulated time</td>
<td>6 h</td>
</tr>
</tbody>
</table>

#### B. Phase screens computation

The purpose of this section is to generate 1D vertical phase screens from the 3D thermodynamic data obtained from the LES at each time step. To do so, the extended-LES phase screens generation method developed by Gilbert et al. [6] is used. It consists in considering the turbulent phase \( \Phi \) at an arbitrary \( y \) location of the LES domain as the combination of two contributions so that

\[
\Phi = \Phi_r + \Phi_s.
\]

\( \Phi_r \) indicates the turbulence induced by the resolved scales of the turbulent flow corresponding, for 1D vertical phase screens, to scales down to the LES resolution \( d_z \). It is computed by directly integrating the fluctuations of the radiofrequency refractive index fields obtained from LES outputs on a propagation step \( \Delta z \).

\( \Phi_s \) denotes the effect of the subgrid scales and enables to extend the phase spectrum to finer scales than the resolution imposed by LES. Its generation follows the same principle as the classical stochastic method [4]. It is here modelled through the following 1D phase spectrum [3]

\[
S_{\Phi_s}^{1D}(\kappa_z) = \begin{cases} 
2\pi k_0^2 \Delta z 0.055 \langle C_n^2 \rangle \kappa_z^{-8/3}, & \text{if } \kappa_z \in \left[ \frac{2\pi}{d_z}, \frac{2\pi}{\Delta z} \right], \\
0, & \text{otherwise.}
\end{cases}
\]

In this formula, \( k_0 \) is the electromagnetic wavenumber while \( \kappa_z \) indicates the vertical component of the wavenumber of the turbulent flow. \( C_n^2 \) corresponds to the structure constant of the refractive index that quantifies the scintillation intensity. Its value generally ranges from \( 10^{-18} \) m\(^{-2/3} \) in a case of weak turbulence to \( 10^{-12} \) m\(^{-2/3} \) under strong turbulence. It can be obtained at every point of the LES domain as proposed by Wilson and Fedorovich [9]. Here, \( \langle C_n^2 \rangle \) is fixed by computing the average value over the entire domain for the last 3 hours of simulation.

To illustrate the method, Figure 2 gives a schematic representation of the resulting total turbulent phase spectrum. \( \Delta z \) corresponds to the vertical resolution of the propagation grid. Figure 3 shows the impact of the modelled subgrid scales on the resulting phase screen. The extended-LES turbulent phase presents finer fluctuations.

The effect of turbulence is cumulative with range. Its impact becomes significant only from several tens of kilometers for X-band frequencies. Since periodic boundary conditions are applied in both the \( x \) and \( y \) homogeneous directions, ergodicity hypothesis allows to successively use LES results recorded at uncorrelated times to virtually extend the computational domain [6].
Phase screens from each domain are computed following the presented method and stored. The propagation is then performed by randomly selecting the screens to avoid any correlation phenomena.

### III. X-BAND PROPAGATION SIMULATION

The aim of this section is to test the presented extended-LES method for different vertical LES resolutions on an X-Band propagation case. The numerical results are analyzed through a log-amplitude variance comparison.

#### A. Propagation model and configuration

The propagation simulations are here performed with the SSW-2D software [10]. The propagation of a wave front at a 10 GHz frequency over a range of \( L_x = 95 \) km is investigated. The computational domain has a vertical size \( L_z \) of 3 km and a vertical discretization step \( \Delta z = 5\lambda/3 = 0.05 \) m. The value of \( \langle C_n^2 \rangle \) is computed from the atmospheric simulations as proposed in Section II giving the value \( 0.23 \times 10^{-12} \) m\(^{-2/3}\). The propagation step \( \Delta_x \) is fixed at 250 m which leads to the computation of \( N_s = 380 \) phase screens following the generation method presented in Section II. A complex source point of size \( 6\lambda \) is located at 10 m from the first phase screen at an altitude \( z_s = L_z/2 = 1500 \) m. The main propagation parameters are summarized in Table II.

**TABLE II**

<table>
<thead>
<tr>
<th>PROPAGATION CASE : SIMULATION PARAMETERS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency : ( f )</td>
</tr>
<tr>
<td>Propagation domain size : ( L_x \times L_z )</td>
</tr>
<tr>
<td>Propagation step : ( \Delta_x )</td>
</tr>
<tr>
<td>Number of screens : ( N_s )</td>
</tr>
<tr>
<td>Propagation vertical resolution : ( \Delta z )</td>
</tr>
<tr>
<td>( \langle C_n^2 \rangle )</td>
</tr>
</tbody>
</table>

#### B. Numerical results for various vertical LES resolutions

To highlight the impact of the vertical LES resolution on the electromagnetic propagation, the log-amplitude is computed at every point of the propagation domain as

\[
\chi(x, z) = \ln \left( \frac{|E_t(x, z)|}{|E_0(x, z)|} \right). \tag{4}
\]

\( E_t \) indicates the electric field propagated in the modelled turbulent medium while \( E_0 \) is the field obtained in a turbulent-free atmosphere. The log-amplitude variance \( \sigma^2_\chi \) quantifies the impact of turbulence on the electromagnetic propagation. It is numerically obtained at every range \( x \) by computing the variance of the vertical log-amplitude vector. \( \sigma^2_\chi \) profiles obtained for different \( d_z \) are plotted in Figure 4. The solid lines indicate a propagation through an atmosphere generated with the extended-LES method while the dashed lines denote the propagation through the non-extended atmospheric domain. These results are averaged over 500 Monte-Carlo runs.

Figure 4 first illustrates the cumulative effect of atmospheric turbulence with range. The log-amplitude variance computed using the extended-LES method reaches a value greater than 1 dB\(^2\) after 95 km of propagation, which indicates a strong scattering [11]. This figure also shows a significant difference between the propagation results in a non-extended LES atmosphere (dashed lines) according to the imposed resolution. These differences are due to the non-inclusion of scales that are not directly resolved by the atmospheric simulations. Moreover, these plots indicate that this gap is reduced by the use of the extended-LES method (solid lines). Indeed, simulations carried out for 3 different vertical resolutions seem to provide equivalent \( \sigma^2_\chi \) profiles up to 30 km of range. However, these results diverge from the one obtained for the finest resolution (\( d_z = 10 \) m) from 35 km for \( d_z = 50 \) m and from 60 km for \( d_z = 25 \) m.
Thus, Figure 4 confirms the need to take into account the subgrid scales of the flow to accurately model propagation in turbulent media. These plots also point out that the accuracy of the extended-LES method depends on the vertical resolution used for generating the atmosphere. When using the extended-LES method, it appears that insufficient LES resolution can lead to an underestimation of the impact of turbulence.

IV. CONCLUSION

In this paper, the extended-LES method has been tested on an X-Band 95 km range propagation case through a shallow cumulus MABL. The impact of the vertical LES resolution on the resulting electromagnetic propagation has been quantified by computing the log-amplitude variance for 3 different vertical meshes. The results appear to be dependent on this parameter at 10 GHz. These initial conclusions suggest the need for further investigations at finer LES resolutions as well as at higher signal frequencies.

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REFERENCES