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Dual-Horizon Reciprocal Collision Avoidance for Aircraft and Unmanned Aerial Systems

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Abstract

The aircraft conflict detection and resolution problem has been addressed with a wide range of centralised methods in the past few decades, e.g. constraint programming, mathematical programming or metaheuristics. In the context of autonomous, decentralized collision avoidance without explicit coordination, geometric methods provide an elegant, cost-effective approach to avoid collisions between mobile agents, provided they all share a same logic and a same view of the traffic. The Optimal Reciprocal Collision Avoidance (ORCA) algorithm is a state-of-the art geometric method for robot collision avoidance, which can be used as a Detect & Avoid logic on-board aircraft or Unmanned Aerial Vehicles. However, ORCA does not handle well some degenerate situations where agents operate at constant or near-constant speeds, which is a widespread feature of commercial aircraft or fixed-winged Unmanned Airborne Systems. In such degenerate situations, pairs of aircraft could end up flying parallel tracks without ever crossing paths to reach their respective destination. The Constant Speed ORCA (CS-ORCA) was proposed in 2018 to better handle these situations. In this paper, we discuss the limitations of both ORCA and CS-ORCA, and introduce the Dual-Horizon ORCA (DH-ORCA) algorithm, where two time horizons are used respectively for short-term collision avoidance and medium-term path-crossing. We show that this new approach mitigates the main issues of ORCA and CS-ORCA and yields better performances with dense traffic scenarios.

Keywords: Collision Avoidance, aircraft conflict resolution, self-separation, Optimal Reciprocal Collision Avoidance, Air Traffic Control, Unmanned Airborne Systems

1 Introduction

Automated conflict detection and resolution is a topic of major interest in the field of air traffic management ([Allignol et al \(2012\)](#)). The problem has been largely addressed from a centralized or decentralized point of view, using deterministic or stochastic algorithms and considering the problem from a strategical or tactical point of view.

Current research and advances around Urban Air Mobility (UAM), Unmanned Air Vehicles (UAV) and more generally Unmanned Airborne Systems (UAS) flying in the lower airspace lead researchers to study collision avoidance algorithms in order to separate UAS. The expected emergence in a near future of a large-scale, dense UAS traffic brings the focus on short-horizon self-separation methods in a multi-agent context.

052 Collision avoidance has been widely studied in
 053 the context of robots trajectory planning. Among
 054 the most efficient algorithms for multi-agent colli-
 055 sion avoidance, the Optimal Reciprocal Collision
 056 Avoidance (ORCA) introduced ([van den Berg et al](#)
 057 [\(2011\)](#)) relies on a geometric approach where each
 058 agent modifies its velocity vector so that the rela-
 059 tive velocity with any other agent falls outside the
 060 region where collision would occur in a given time
 061 horizon. This geometric approach is designed to
 062 guarantee conflict resolution for the next minutes
 063 in an autonomous manner, without explicit coord-
 064 ination among the agents. Provided all agents
 065 apply the same logic for the geometric algorithm
 066 and share the same view of the traffic situation
 067 (positions and velocities of the other agents), the
 068 maneuvers computed independently by each agent
 069 are implicitly coordinated.

070 Note that the term "optimal" in ORCA relates
 071 to the fact that the respective sets of collision-
 072 avoiding velocities from which two conflicting
 073 robots choose their optimal respective velocities
 074 are reciprocally-avoiding and maximal (see def-
 075 inition 1 in [van den Berg et al \(2011\)](#)): one
 076 cannot find a larger couple of velocity sets that
 077 is reciprocally-avoiding. With ORCA, the small-
 078 est change in the relative velocity is chosen so as
 079 to prevent collision in a given time horizon, and
 080 this relative velocity change is fairly shared among
 081 the two agents. However, ORCA does not provide
 082 optimal solutions in terms of overall trajectory de-
 083 viations. In fact, it can be considered as a greedy
 084 heuristic: The relative velocity of each conflicting
 085 pair is chosen optimally at each time step in the
 086 set of conflict-free velocities, but without trying to
 087 minimize trajectory deviations over a whole time
 088 interval.

089 Although very efficient for robots which can
 090 modify their direction and speed at will, [Du-](#)
 091 [rand \(2018\)](#) showed that the ORCA algorithm has
 092 limitations for agents such as fixed-wing UAVs
 093 or aircraft that can only move at constant or
 094 nearly-constant speeds. The Constant-Speed Op-
 095 timal Reciprocal Collision Avoidance (CS-ORCA)
 096 proposed by Durand showed improvements over
 097 the ORCA logic, with fewer pairs of flights end-
 098 ing on parallel paths without ever reaching their
 099 destination.

100 In this paper, we discuss the limitations of
 101 CS-ORCA and propose a further improvement
 102 on the ORCA and CS-ORCA algorithms. We

introduce the Dual-Horizon Reciprocal Collision
 Avoidance (DH-ORCA) algorithm where a short-
 horizon collision avoidance logic is complemented
 by a longer-horizon logic that helps agents to cross
 path with surrounding agents. We show that this
 dual-horizon logic provides improvements over the
 ORCA and CS-ORCA algorithms and has a sig-
 nificant impact on the residual separation losses
 and other safety and efficiency metrics.

We may expect that algorithms such as DH-
 ORCA could help tackle the problem of UAV
 Traffic Management in a safe, decentralized, au-
 tonomous way, even with the very large amounts
 of traffic expected in the near future. Such geo-
 metric algorithms are good candidates for possible
 airborne self-separation systems on-board com-
 mercial aircraft, or as the "Detect & Avoid"
 (D&A) logic envisioned for future Unmanned
 Aerial Systems.

After a short literature review on conflict de-
 tection and resolution methods in Section 2, we
 briefly discuss the context and the objectives of
 our study in Section 3. We then describe the
 ORCA algorithm in Section 4. The CS-ORCA
 variant is presented in Section 5, where its lim-
 itations are also discussed. Section 6 introduces
 the new dual-horizon method DH-ORCA. The ex-
 periment setup and traffic scenarios are presented
 in Section 7. Finally, the results are presented in
 Section 8, before the conclusion.

2 Literature Review

En-route conflict detection and resolution is cur-
 rently managed by air traffic controllers who have
 a global view of traffic. They use a horizontal 2D
 representation of the traffic (a radar screen) and
 give different types of maneuvers (e.g. heading
 or flight level changes, small speed modifications)
 to maintain a minimum standard separation be-
 tween aircraft. It has been known since the 90s
 that this task is highly combinatorial (see [Granger](#)
[and Durand \(2003\)](#) for a discussion on the sub-
 ject). When more than two aircraft are involved,
 it is more efficient to consider the global situa-
 tions than to solve conflicts by pairs. However,
 when the number of aircraft increases, a global ap-
 proach may become too complex and autonomous
 algorithms can become a good alternative. Au-
 tomatic conflict resolution methods can thus be
 divided in two categories: on one hand centralized

approaches consider the whole situation and give instructions to every aircraft, on the other, autonomous approaches delegate the separation task to the different aircraft.

2.1 Centralized Approaches

Two types of centralized approaches can be found in the literature. In the first one, greedy methods (Krella et al (1989); Chiang et al (1997); Hu et al (2002)) use sequential algorithms to optimize trajectories one by one after ranking aircraft. The main challenge is then to find an appropriate ordering (Archambault and Durand (2004)). The second type of centralized approach considers the traffic situation as a global problem and tries to find a solution without prioritizing aircraft, using global optimization techniques such as evolutionary algorithms, mathematical programming, constraint programming, and making various assumptions on the traffic and/or the aircraft performances.

Durand et al (1996) showed that a genetic algorithm could efficiently solve multiple aircraft conflicts with simple maneuvers on real traffic data. The algorithm is all the more efficient when uncertainties on trajectory prediction can be reduced (Alliot et al (2001)). Other approaches used mathematical methods (semi-definite programming) (Oh et al (1997); Frazzoli et al (2001)), but solutions were only locally optimal, and the model required perfect trajectory prediction which is unrealistic. A perfect trajectory prediction was also used by Pallottino et al (2001, 2002) in the early 2000s in a mathematical model using Mixed Integer Linear Programming, which could be solved by CPLEX and ensure the global optimality of the solution. Christidoulou extended the model in 3D, but still required constant speed during climbing phases (Christidoulou and Kontogeorgou (2008)). An improved version adding uncertainties to the trajectory headings was introduced by Gariel and Feron (2009) in 2009 but all maneuvers needed to be executed at the same time. Omer and Farges (2013) introduced in the early 2010s a Mixed-Integer Linear Programming approach taking into account the trajectory recovery. Alonso-Ayuso et al (2016) used a Mixed Integer Non Linear Optimization model that also took the trajectory recovery into account. Peyronne et al

(2015) introduced a trajectory model using unrealistic B-Splines trajectories and a semi-infinite programming formulation of the constraints. Rey and Hijazi (2017) recently proposed a new complex number formulation and convex relaxations for the centralized problem.

Allignol et al (2013) proposed to separate the model from the resolution algorithm. For each aircraft, a number of alternative trajectories and a matrix of pairwise conflicts were pre-calculated, taking various uncertainties into account, before the optimization process was performed. A similar approach was proposed by Lehouillier et al (2017). In Wang et al (2020) different resolution methods were compared on the benchmark proposed in Allignol et al (2013).

2.2 Autonomous Approaches

Autonomous approaches appeared in the 90s with the Free Flight debate motivated by Air Traffic Control cost reduction. The development of Unmanned Airborne Systems in the lower airspace recently gave a new interest to these approaches because the high traffic demand in the lower airspace (Bulusu et al (2016)) might create dense situations that centralized approaches might not be able to handle, or at a greater cost. Sliding forces to coordinate maneuvers between aircraft (Zeghal (1998)) were one of the first autonomous approaches proposed in the late 90s. Potential or vortex fields (Košecká et al (1998)) as well as a model based on an analogy with electrical particle repulsion (Eby and Kelly (1999)) were also used. These algorithms took into account the aircraft speed constraints¹. For situations involving many aircraft, these approaches relied on the principle that forces or potential fields virtually generated by aircraft on each other would add up. There is no guarantee that this principle always leads to a conflict free solution. Eby and Kelly (1999) inspired the Airborne Separation Assurance System (ASAS) approach developed by Hoekstra et al (2002). ASAS used a Modified Voltage Potential (MVP) close to Eby's definition. It was tested in different contexts, such as the Mediterranean Free-Flight (Ruigrok and Hoekstra (2007)). More recently, it was used to model airspace stability

¹When cruising, an aircraft can easily change its direction while respecting a maximum turning rate, but it can only slightly modify its speed, within a narrow speed range.

154 and capacity in a decentralized separation context
 155 (Sunil et al (2017)) and in a Capacity Assessment
 156 Model (Emmanuel Sunil and Hoekstra (2018)).
 157 A geometric optimization approach solving com-
 158 plex situations by using an iterative process was
 159 proposed by Bilimoria (2000) and used in the
 160 Future Air traffic management Concepts Evalu-
 161 ation Tool (FACET) project (D. Bilimoria et al
 162 (2001)). The Free-flight Autonomous and Coordi-
 163 nated Embarked Solver (Granger et al (2001b,a))
 164 used a token allocation strategy to coordinate se-
 165 quential maneuvers. Le Ny and Pappas (2010) also
 166 used a geometric approach for scheduling crossing
 167 times of aircraft through a metering fix. Pallottino
 168 et al (2007) proposed a model where coordination
 169 is ensured by protected stacking areas in which
 170 each aircraft can move without conflicting with
 171 other aircraft. The areas are headed to the flight
 172 destination. Schouwenaars and Feron (2004) used
 173 a similar approach to plan safe trajectories in a
 174 decentralized way.

175 Some approaches use a set of rules to coordi-
 176 nate the conflict resolution of multiple aircraft. For
 177 instance, D’Amato et al (2020) use the *Right of*
 178 *Way rules*, as prescribed by the International Civil
 179 Aviation Organization (ICAO) for human piloted
 180 flights. This *Right of Way rules* creates constraints
 181 for each UAV. Then, considering these constraints,
 182 each UAV optimizes its own trajectory using
 183 constrained Model Predictive Control.

184 Geometric approaches using an implicit au-
 185 tomated coordination were first introduced by
 186 van den Berg et al (2008) with the Reciprocal
 187 Velocity Obstacles and then with the Optimal Re-
 188 ciprocal Collision Avoidance (ORCA) (van den
 189 Berg et al (2011)) algorithm used in the present
 190 paper. ORCA was initially built to simulate robots
 191 using self-separation logic in a 2D environment.
 192 In air traffic control, Snape and Manocha (2010)
 193 have extended the model to the vertical di-
 194 mension without segregating the horizontal and
 195 vertical maneuvers. More recently, geometric ap-
 196 proaches have led to the definition of Solution
 197 Space Diagrams (SSD) that were used to ana-
 198 lyze the dynamic of air traffic controller workload
 199 (d’Engelbronner et al (2015)) and were recently
 200 compared to the Modified Voltage Potential ap-
 201 proach (Balasooriyan (2017)) showing that the
 202 latter seemed to be more efficient in many sit-
 203 uations. As opposed to ORCA which provides
 204 coordinated maneuvers for agents actively trying

to avoid each other, SSD – which is actually a
 Velocity Obstacle (VO) algorithm with speed and
 turning rate constraints – does not provide coordi-
 nated maneuvers. In ORCA, a modification of
 the relative speed is computed for the considered
 pair of aircraft using a Reciprocal Velocity Obsta-
 cle (RVO). This relative speed modification is then
 split equally between the involved aircraft. In the
 Variable Responsibility ORCA (VR-ORCA) (Guo
 et al (2021)) the required relative speed modifi-
 cation is not shared equally, the relative speed
 modification is split between the two aircraft in
 order to minimize a cost function that takes into
 account all the aircraft visible by both aircraft.

ORCA maximizes the number of feasible ve-
 locities near the current speed vectors, whereas
 RORCA (Wang et al (2021)) maximizes the num-
 ber of feasible velocities near the zero speed vector.
 It can be proved that the resulting speed con-
 straints always contains the zero speed vector,
 ensuring a non-empty solution set for robots that
 can stop. Despite being useful for ground vehicles
 and multi-rotor UAVs, it does not provide useful
 guarantees for commercial air traffic or fixed-wing
 UAVs.

Durand (2018) highlighted an issue with the
 original ORCA algorithm when applied to com-
 mercial air traffic or fixed-wing UAVs flying at
 similar constant or nearly-constant speeds: air-
 craft could end-up flying parallel tracks without
 ever crossing paths to reach their respective des-
 tinations. In the same publication, they proposed
 a Constant-Speed ORCA algorithm (CS-ORCA)
 that can better handle these situations.

In this paper, we will discuss the limitations
 of CS-ORCA, then propose a new logic with two
 time horizons DH-ORCA that improves on ORCA
 and CS-ORCA.

3 Context and objectives

3.1 Potential context of application

Geometric algorithms such as ORCA, CS-ORCA,
 or the DH-ORCA algorithm introduced in this
 paper can be used in a variety of contexts, with
 different system architectures.

In a centralized architecture, the geometric
 algorithm could be implemented in a central com-
 puter with the positions and velocities of all traffic
 as input. The resulting modified velocity vectors

would then be transmitted via data-link to the aircraft or UAVs. Using geometric algorithms in a centralized approach would be fast in terms of computation time, but less optimal in terms of overall trajectory deviations than global optimization methods such as the ones presented in the literature review.

Geometric algorithms are actually intended for a decentralized, autonomous approach. They could be of use either in the context of autonomous airborne separation assurance systems for commercial aircraft or in the *detect & avoid* logic that could be deployed in future Unmanned Airborne Systems. In such a context, each aircraft or UAV computes its own maneuvers on-board, and no explicit coordination among the agents is required. If the same ORCA logic is implemented aboard all agents, and if they all have the same view of the positions and velocities of the traffic, the velocity changes computed aboard each UAV or aircraft are implicitly coordinated (following the implemented logic) with the maneuvers of the other traffic. In other words, the resulting maneuvers are coordinated among all agents without passing or broadcasting any message.

This implies that all agents should apply the same logic, with the same look-ahead time (or time horizon) for the conflict detection. The time horizon parameter(s)² should be pre-set before deployment. In section 7.5, we propose a method to tune these parameters, by choosing the values providing the best results on a set of random scenarios.

3.2 Handling the altitude

The original ORCA algorithm from [van den Berg et al \(2011\)](#) was initially designed for a 2D-environment, to simulate robot collision avoidance. As will be detailed in section 4, it works by moving the 2D-vector of relative velocity outside a forbidden domain consisting of a blunt 2D-cone (see Fig. 1).

Aircraft and UAVs fly in a 3D-environment, and several ways have been proposed to adapt the 2D-ORCA algorithm to the 3D-space. The first one in [Snape and Manocha \(2010\)](#) considers a 3D-vector for the relative velocity and a blunt 3D-cone

for the velocity obstacle – the head of the 3D-cone being a sphere around the other aircraft.

The second one in [Alligier et al \(2018\)](#) uses vertical cylinders instead of spheres around the aircraft. Each cylinder has a circular basis of radius, the horizontal separation standard, and its height is equal to the vertical separation standard. This model is more consistent with the current operational notion of separation loss in aviation: two flights are not separated when their horizontal distance is closer than the horizontal separation standard, and when their vertical separation is less than the vertical separation standard.

In this paper, we will focus on 2D-separation in the horizontal plane exclusively, and introduce a dual-horizon algorithm that improves on ORCA and CS-ORCA when applied to constant-speed aircraft flying in a horizontal plane. The extension of this dual-horizon algorithm to 3D-space could be made as in [Snape and Manocha \(2010\)](#) or [Alligier et al \(2018\)](#), without difficulty.

3.3 Objective of this paper

The limitations of ORCA in the context of airborne separation come from the fact that the speed range and acceleration of commercial aircraft are small. Whereas robots can reduce their velocity at will and can even stop completely, fixed-winged aircraft and UAVs cannot reduce their speed below the stall speed otherwise they fall to the ground, and their upper speed is limited by their airframe structure and motorization. Actually, fuel-efficiency dictates to operate within much tighter bounds than that. In the ERASMUS speed control project ([Bonini et al \(2009\)](#)), the speed modification range for the cruising speed was set to $[-6\%; +3\%]$ around the nominal speed.

When applying the original ORCA algorithm with a limited speed range, conflicting aircraft flying on converging tracks at similar speeds may end up on parallel tracks, postponing the crossing of their paths for a very long time and potentially never reaching their destination.

The objective of the research presented in this paper is to propose a new algorithm for self-separation in the horizontal plane that is less subject to the parallel-track issue than ORCA or its constant-speed variant CS-ORCA and that helps converging flights to cross paths.

²There is only one time horizon for ORCA and CS-ORCA, but the new logic we propose DH-ORCA uses two different time horizons.

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256 As the final intended application is a decen-
 257 tralized autonomous system where each aircraft
 258 computes its maneuver without explicit coordina-
 259 tion with the others, the algorithm must use only
 260 the information available to the own aircraft: typi-
 261 cally, the direction toward its own destination and
 262 the positions and velocities of the other aircraft
 263 obtained through the on-board radar. In particu-
 264 lar, we assume that the intended destination of the
 265 other aircraft is not known by the own aircraft.

266 In our traffic scenarios, we will focus on patho-
 267 logical cases where all aircraft or UAVs fly at
 268 constant speeds that might differ from one an-
 269 other, but only in a limited range. These scenarios
 270 are more prone to exhibit the parallel tracks' be-
 271 haviour. We here describe these cases as pathologi-
 272 cal, in the sense that the original ORCA algorithm
 273 was not intended to handle them, but such situ-
 274 ations may actually occur very often in real life,
 275 typically for commercial aircraft flying at similar
 276 speeds in the upper airspace. This is not neces-
 277 sarily the case for UAVs flying in lower airspace,
 278 as there are a variety of UAVs (fixed-wing, rotor,
 279 etc) having different speed and acceleration char-
 280 acteristics. Nonetheless, the self-separation logic
 281 of these UAVs should be able to handle conflicts
 282 involving UAVs with similar speeds, for safety and
 283 efficiency reasons, as such situations may actually
 284 occur in the dense traffic situations envisioned in
 285 the near future.

287 4 Optimal Reciprocal 288 Collision Avoidance 289 (ORCA) 290

291 This section describes the Optimal Reciprocal
 292 Collision Avoidance (ORCA) algorithm developed
 293 in [van den Berg et al \(2011\)](#) in the specific case
 294 where speeds are constrained. First, the maneu-
 295 ver model is detailed for an aircraft pair, then,
 296 the maneuver calculation is explained when more
 297 than two aircraft are involved in a conflict.

298 As explained in section 3, we focus on self-
 299 separation in the horizontal plane for aircraft
 300 flying at a same altitude, so ORCA and its variants
 301 are applied in 2D and provide lateral maneuvers
 302 only, in the form of velocity changes. We will
 303 assume in this paper that only the direction of
 304 the velocity can be changed: the velocity domain
 305 is a circular arc (see Fig. 2). All the algorithms
 306

presented hereafter can easily be extended to non-
 constant speeds by considering a velocity domain
 that is a portion of a disc limited by the minimum
 and maximum speed and by the turning rate.

4.1 Separation Constraint Model for Two Aircraft

Let us consider two aircraft A and B, as illustrated
 in Figure 1.a. Let d be the required minimum
 separation and τ be a time horizon (also called
 look-ahead time, or anticipation in the literature).
 Let us denote A and B the respective positions of
 aircraft A and B at the current time (here $t = 0$
 by convention), and \vec{v}_A and \vec{v}_B their respective
 velocity vectors.

Assuming both flights continue on their cur-
 rent course, the future position of aircraft A
 relative to aircraft B at a future time $t \geq 0$ is
 given by the vector $\overrightarrow{BA}(t) = \overrightarrow{BA} + (\vec{v}_A - \vec{v}_B)t =$
 $\overrightarrow{BA} + \vec{v}_r t$, where \vec{v}_r is the velocity of A
 relative to B (*i.e.* considering B fixed).

Given a time horizon τ , and assuming $t = 0$
 at the current aircraft positions, there is a conflict
 between aircraft A and B if and only if we antic-
 ipate a separation loss in the future, within the
 time horizon τ : $\exists t, 0 \leq t \leq \tau \mid \|\overrightarrow{BA}(t)\| < d$

If we mentally try to picture, in Figure 1.b,
 where the extremity of vector $\overrightarrow{BA} + \vec{v}_r t$ lies, we
 see that it is on a line $\mathcal{L}(A, \vec{v}_r)$ passing through
 A and directed by \vec{v}_r . If this line $\mathcal{L}(A, \vec{v}_r)$ inter-
 sects the circle $\mathcal{C}(B, d)$ of radius d centered on B,
 then this intersection defines two times t_1 and t_2
 at which a future separation loss begins and end:
 $\forall t \in [t_1, t_2] \mid \|\overrightarrow{BA}(t)\| < d$. This can only happen
 if \vec{v}_r lies inside the cone which extremity is A and
 which sides are the tangents to the circle $\mathcal{C}(B, d)$
 passing through A (in grey on the figure). In the
 case where \vec{v}_r lies outside this cone of extremity A
 – which is not the case in Figure 1.b – there will
 never be a separation loss. To summarize, the fact
 that \vec{v}_r lies inside or outside the full cone of ex-
 tremity A allows us to detect if there is a conflict
 within an infinite time horizon $\tau = +\infty$, or not.

In order to detect conflicts occurring within
 a finite time horizon τ , let us now consider in
 Figure 1.b the smaller circle $\mathcal{C}(B', \frac{d}{\tau})$ of radius
 $\frac{d}{\tau}$ centered at B' such that $\overrightarrow{AB'} = \frac{\overrightarrow{AB}}{\tau}$. This
 small circle and the tangents to the big circle

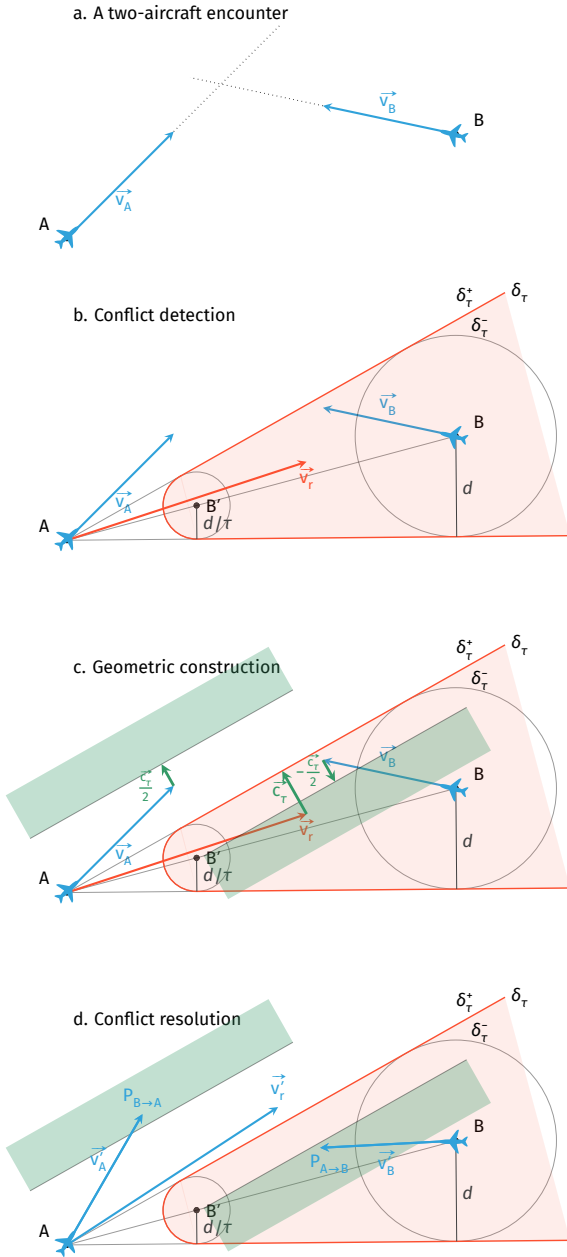


Figure 1: Conflicting aircraft model: a loss of separation will occur within time τ if and only if the relative velocity \vec{v}_r lies in the red forbidden zone.

$\mathcal{C}(B, d)$ define a zone in light red and denoted δ_τ^- in Figure 1.b, bounded by the boundary δ_τ (the bold red curve on the figure). This red zone has the form of a blunt cone (with the small circle at the head) and is called a *reciprocal velocity obstacle*. The relative velocity \vec{v}_r intersects with this zone

δ_τ^- if and only if it intersects the disc of size $\frac{d}{\tau}$ centered in B' , i.e. if and only if there exists $\lambda \in [0; 1]$ such that $\|\vec{B'A} + \lambda\vec{v}_r\| < \frac{d}{\tau}$ or $\|\vec{BA} + \lambda\tau\vec{v}_r\| < d$ which means that A and B are in conflict at time $\lambda\tau \leq \tau$. Consequently, a conflict will occur within time τ if and only if \vec{v}_r lies in this zone δ_τ^- .

Note that the reciprocal velocity obstacle δ_τ^- lies in the space of relative velocities. When a conflict occurs, it can be avoided simply by modifying the velocities \vec{v}_A and \vec{v}_B such that the relative velocity is moved outside the reciprocal velocity obstacle. The ORCA (Optimal Reciprocal Collision Avoidance) algorithm is based on the principle that the effort to keep \vec{v}_r outside the reciprocal velocity obstacle should be minimal and shared by the two agents A and B. In its original version, the necessary minimal relative velocity change, here denoted \vec{c}_τ , is split equally between the two agents, as shown in Figure 1.c. The vector \vec{c}_τ is simply obtained by connecting the tip of \vec{v}_r with its orthogonal projection on the closest boundary of the reciprocal velocity obstacle δ_τ . The half-vector $\frac{\vec{c}_\tau}{2}$ is added to \vec{v}_A in order to define a semi-plane $P_{B \rightarrow A}^\tau$ perpendicular to \vec{c}_τ , and this same half-vector is subtracted from \vec{v}_B to define a semi-plane $P_{A \rightarrow B}^\tau$. Both semi-planes are shown in light green on Figure 1.c.

If the new velocities \vec{v}'_A and \vec{v}'_B are chosen in the semi-planes $P_{B \rightarrow A}^\tau$ and $P_{A \rightarrow B}^\tau$ respectively, as illustrated in Figure 1.d, the new relative velocity \vec{v}'_r falls outside δ_τ^- and the conflict is solved, at least for a time horizon τ and assuming both aircraft stay on their new course.

The two semi-planes $P_{B \rightarrow A}^\tau$ and $P_{A \rightarrow B}^\tau$ constraining the velocities of A and B respectively can be formally defined as follows:

$$P_{B \rightarrow A}^\tau := \left\{ \vec{v}'_A \in \mathbb{R}^2 \mid \left(\vec{v}'_A - \left(\vec{v}_A + \frac{\vec{c}_\tau}{2} \right) \right) \cdot \vec{\eta}_\tau \geq 0 \right\}$$

$$P_{A \rightarrow B}^\tau := \left\{ \vec{v}'_B \in \mathbb{R}^2 \mid \left(\vec{v}'_B - \left(\vec{v}_B - \frac{\vec{c}_\tau}{2} \right) \right) \cdot \vec{\eta}_\tau \leq 0 \right\}$$

In the above definitions, $\vec{\eta}_\tau$ denotes a unit vector perpendicular to the curve δ_τ at the point $\vec{v}_r + \vec{c}_\tau$, and pointing towards the outside of δ_τ^- . Using these equations, we can prove that:

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$$\begin{aligned} 358 \quad & \forall v'_A \in P_{B \rightarrow A}^\tau, \forall v'_B \in P_{A \rightarrow B}^\tau, \\ 359 \quad & \left(\vec{v}'_A - \vec{v}'_B - (\vec{v}_r + \vec{c}_\tau) \right) \cdot \vec{\eta}_\tau \geq 0 \\ 360 \end{aligned}$$

361 This equation guarantees that $\vec{v}'_r = \vec{v}'_A - \vec{v}'_B$ is outside the red zone δ_τ^- (Figure 1.d). Note that the
362 constraints $P_{B \rightarrow A}^\tau$ and $P_{A \rightarrow B}^\tau$ are always enforced, whether a conflict is detected or not between A
363 and B. When no conflict is detected at the current time step, these constraints guarantee that
364 the next velocities chosen for A and B remain conflict-free.
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367 In our description of ORCA, we have chosen to consider the reciprocal velocity obstacle for the
368 relative velocity $\vec{v}_r = \vec{v}_A - \vec{v}_B$, considering B fixed, but we could have reversed the roles of A and B
369 and considered the velocity obstacle for $\vec{v}_B - \vec{v}_A$, considering A fixed. This would have given exactly
370 the same semi-planes and the same result. This computation can be made on a central computer
371 deciding the maneuvers for both aircraft, but more interestingly it can also be made separately on-
372 board the two aircraft, with each aircraft deciding of its own maneuver within its conflict-free semi-
373 plane. If both aircraft share the same view of the situation and apply the same logic, their maneu-
374 vers will be implicitly coordinated, without having to exchange messages.
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387 4.2 Speed and Turn Rate Constraints

388 Track changes are the only possible maneuvers for the aircraft or fixed-wing UAVs considered in
389 this study. The aircraft speed remains constant throughout the conflict resolution process. This
390 can be expressed as the following constraints on the velocities of aircraft A and B:
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$$396 \quad \left\| \vec{v}'_A \right\| = \left\| \vec{v}_A \right\|$$

$$397 \quad \left\| \vec{v}'_B \right\| = \left\| \vec{v}_B \right\|$$

398 In addition, the turn rate of every aircraft is limited to a maximum value, which can be spe-
399 cific to each aircraft. In the current study, it was limited to 3 degrees per second for all aircraft.
400

401 With the above constraints, the new velocity assigned by ORCA to an aircraft must be chosen
402 so that the head of the velocity vector is on an arc of limited range, centered on the current aircraft
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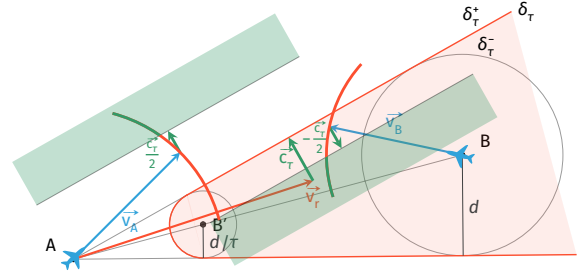


Figure 2: Conflicting aircraft model: the effort is shared by the two aircraft. The new velocities must be chosen on the green portion of the arc.

position (see Figure 2). The arc range is limited by the maximum turning rate. If the time step is set to 5 seconds, for example, the arc range is ± 15 degrees for the standard turning rate (3 degrees per second).

Considering the separation constraints $P_{B \rightarrow A}^\tau$ and $P_{A \rightarrow B}^\tau$ presented in the previous section 4.1, we see in Figure 2 that the velocity vectors must be chosen in the green portions of the arcs, inside these semi-planes, in order to move the relative velocity vector outside the red blunt cone δ_τ^- while satisfying the speed constraints.

4.3 Multiple-Aircraft Separation Constraints

When an aircraft i is in conflict with more than two aircraft, the intersection of the semi-planar separation constraints $P_{k \rightarrow i}^\tau$ with $k \neq i$ creates a 2d polyhedral set C_i . This 2d polyhedral set C_i might be unbounded. This is the case for example in Figure 3 where A is conflicting with B and D. The conflict-free convex set C_A for aircraft A is the intersection of the two semi-planes $P_{B \rightarrow A}^\tau$ and $P_{D \rightarrow A}^\tau$.

The intersection of C_i and the arc A_i of possible turning angles for aircraft i (Section 4.2) defines an arc S_i of admissible conflict-free velocities for aircraft i . If every aircraft selects its new velocity in this conflict-free arc, then no loss of separation will occur in the next τ seconds.

If, for a given aircraft i , the intersection of the semi-planes is empty, then every semi-plane is equally slightly moved until a non-empty intersection appears. The resulting convex C_i does not guarantee a conflict-free maneuver anymore, but it remains close to the conflict-free domain. This

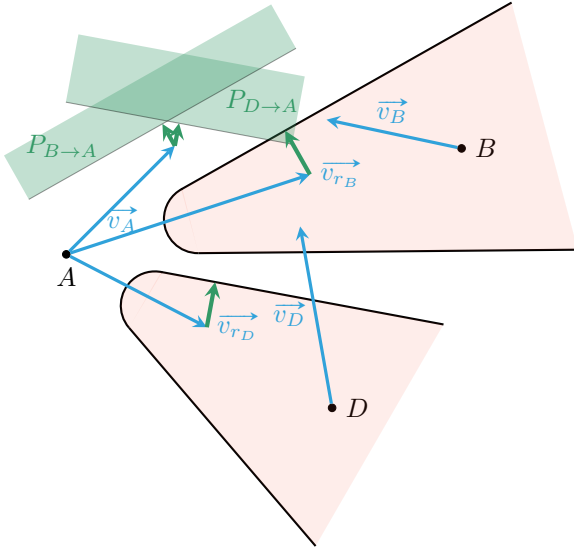


Figure 3: Multi-Conflicting aircraft model: for aircraft A, a conflict will occur within time τ with aircraft B (resp. D) if and only if the relative velocity \vec{v}_{r_B} (resp. \vec{v}_{r_D}) lies in the forbidden zone in red associated with aircraft B (resp. D).

process is described by Van den Berg [van den Berg et al \(2011\)](#).

4.4 Summary of the ORCA algorithm applied to constant-speed aircraft

Algorithm 1 summarizes the ORCA algorithm applied to constant-speed aircraft (or UAVs). Time is discretized using a time step δt (see Table 1 in section 7.3 for the parameter values chosen in the experiments). As long as every aircraft has not reached its destination, every aircraft pair (i, j) is checked to calculate the semi-planes $P_{j \rightarrow i} = P_{j \rightarrow i}^\tau$ and $P_{i \rightarrow j} = P_{i \rightarrow j}^\tau$. For every aircraft i , the convex C_i is computed as the intersection of the semi-planes $P_{k \rightarrow i}$, with $k \neq i$. The admissible domain S_i for the new velocity is then the intersection of C_i and the arc A_i of possible turning angles for aircraft i .

The new velocity \vec{v}_i is chosen as the closest vector to \vec{v}_i^{pref} in the admissible domain S_i , where \vec{v}_i^{pref} is the preferred velocity vector for aircraft i , directed from the aircraft current position towards its destination.

Note that the admissible velocity domain S_i can be empty for two reasons: either the convex C_i is not empty but its intersection with A_i is empty, or the convex C_i is empty. In both cases, we select the new velocity \vec{v}_i that violates the constraints as little as possible. This is done by incrementally relaxing the constraints – as explained at the end of the previous subsection 4.3 – until S_i is not empty.

Algorithm 1 ORCA algorithm, applied to constant-speed aircraft

Input: simulation time step δt , time horizon τ , and a scenario with the planned origin, destination, cruising speed and scheduled entry time of each aircraft

Output: aircraft trajectories

- 1: **while** every aircraft has not reached destination
 - 2: **do**
 - 3: Define the semi-plane constraints $P_{j \rightarrow i}$ and $P_{i \rightarrow j}$
 - 4: **end for**
 - 5: **for** every aircraft i **do**
 - 6: Calculate the convex intersection C_i of every semi-planes $C_i = \bigcap_{k \neq i} P_{k \rightarrow i}$
 - 7: Calculate the arc A_i of possible aircraft i turning range.
 - 8: Calculate $S_i = C_i \cap A_i$
 - 9: Calculate the ideal velocity \vec{v}_i^{pref}
 - 10: Choose \vec{v}_i the velocity closest to \vec{v}_i^{pref} inside S_i
 - 11: Change current velocity: $\vec{v}_i = \vec{v}_i$
 - 12: Move aircraft i with current velocity \vec{v}_i
 - 13: **end for**
 - 14: increase current time $t = t + \delta t$
 - 15: **end while**
-

5 Constant-Speed ORCA (CS-ORCA)

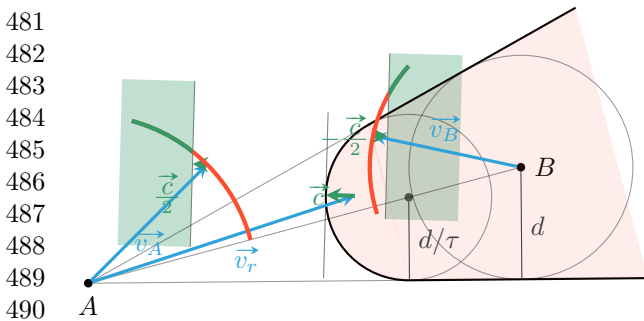
This section describes the Constant-Speed ORCA algorithm (CS-ORCA) introduced by Durand in [Durand \(2018\)](#). This variant aims at correcting the pathological behaviour exhibited by the original ORCA algorithm when constant-speed aircraft converge at a small angle [Durand and Barnier \(2015\)](#). In such cases, ORCA tends to postpone the conflict outside the time horizon τ , indefinitely, by setting the flights on parallel tracks.

460 We will first explain this behaviour in more de-
 461 tail in Section 5.1, then give a brief description
 462 of CS-ORCA (Section 5.2), before explaining in
 463 Section 5.3 why CS-ORCA does not completely
 464 solve the pathological cases it was designed to
 465 solve.

467 5.1 Pathological solutions where 468 flights cannot cross paths

470 In Figure 1, we presented a conflict situation
 471 where \vec{v}_r lied deep within the forbidden zone
 472 δ_τ^- (the blunt cone in red), and the vector \vec{c}_τ
 473 computed by ORCA to move \vec{v}_r outside δ_τ^- was
 474 perpendicular to one of the cone sides.

475 Let us now consider Figure 4 showing a conflict
 476 in which the relative velocity \vec{v}_r is close to the
 477 circular part of the blunt cone δ_τ^- (in red). Such
 478 situations are more likely to occur at the beginning
 479 of a conflict.



491 **Figure 4:** Example where the tip of the relative
 492 velocity \vec{v}_r is projected onto the circular part of
 493 the reciprocal velocity obstacle (in red), leading to
 494 ORCA velocity constraints (semi-planes in green)
 495 that move the velocities of A and B apart, making
 496 them more parallel.

499 In such cases, the original ORCA algorithm
 500 computes a vector \vec{c}_τ joining the tip of \vec{v}_r to the
 501 closest point on the circular part of the frontier
 502 δ_τ^- of the velocity obstacle. The direction of \vec{c}_τ
 503 is not perpendicular to either one of the cone sides,
 504 and the semi-planes defining the constraints for
 505 \vec{v}_A and \vec{v}_B – which are perpendicular to \vec{c}_τ – are
 506 not parallel to either one of the cone sides. They
 507 actually tend to move the velocities of A and B
 508 slightly apart.

510

This move ensures that there will be no loss
 of separation within the next τ seconds. How-
 ever, the conflict may reappear in the next time
 steps—and be solved in the same way—if the air-
 craft are still converging. This may typically occur
 when the aircraft must cross paths to reach their
 respective destination.

As a result, depending on the initial condi-
 tions, the two aircraft velocities tend to become
 parallel. Instead of solving the conflict by cross-
 ing the paths of the two aircraft, thus allowing
 them to reach their destination, the ORCA algo-
 rithm tends to postpone indefinitely the conflict
 outside the time horizon by setting them on paral-
 lel tracks. This is called the horizon effect in game
 theory.

5.2 The CS-ORCA algorithm

To deal with the problem described in the pre-
 vious section, the semi-planes in CS-ORCA are
 computed differently than with ORCA, but the
 principle of the algorithm is the same. The finite
 time horizon τ is used to detect conflicts (i.e. fu-
 ture separation losses), exactly as in ORCA, by
 checking if the relative velocity \vec{v}_r lies inside the
 reciprocal velocity obstacle (the blunt red cone, on
 Fig.4, with a circle of radius $\frac{d}{\tau}$ at the head, and
 denoted δ_τ^- in Section 4.1).

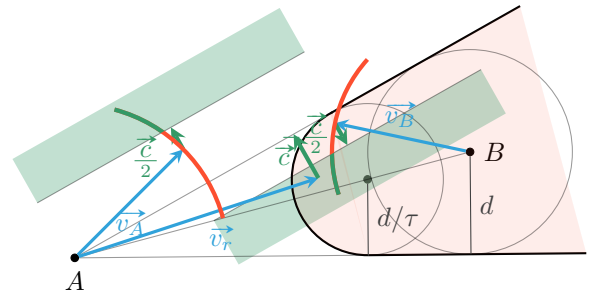


Figure 5: CS-ORCA velocity constraint model:
 the relative speed \vec{v}_r is now projected on the clos-
 est side of the cone instead of the circular part,
 only when \vec{v}_r is inside the forbidden zone (in red).

When no conflict is detected, *i.e.* when \vec{v}_r lies
 outside δ_τ^- , the semi-planes $P_{A \rightarrow B}^\tau$ and $P_{B \rightarrow A}^\tau$ are
 defined as in ORCA, by considering the orthogo-
 nal projection of the tip of \vec{v}_r onto the closest
 boundary of δ_τ^- . In this case, these semi-planes

prevent from selecting new velocities that would lead to a conflict at the next time step. Depending on how \vec{v}_r is positioned outside δ_τ^- , the semi-planes $P_{A \rightarrow B}^\tau$ and $P_{B \rightarrow A}^\tau$ might be parallel to one of the straight sides of δ_τ^- , or parallel to a tangent to the round head of δ_τ^- .

The difference between ORCA and CS-ORCA lies in how the semi-planes implementing the velocity constraints are computed when a conflict is detected, *i.e.* when \vec{v}_r lies inside δ_τ^- . This difference is illustrated in Figures 4 and 5. Figure 4 shows how ORCA builds the semi-planes by projecting the tip of \vec{v}_r onto the closest boundary of δ_τ^- – which can be the round part at the head of the cone δ_τ^- , as in Fig. 4. Figure 5 shows how CS-ORCA builds the relative velocity deviation \vec{c} by projecting the tip of \vec{v}_r onto the closest side of the full cone of extremity *A*, instead of the closest boundary of the blunt cone. Remembering from Section 4.1 that this full cone is in fact the reciprocal velocity obstacle corresponding to an infinite time horizon $\tau = +\infty^3$, we can denote $P_{B \rightarrow A}^{\tau=+\infty}$ and $P_{A \rightarrow B}^{\tau=+\infty}$ the semi-planes defined using this full cone $\delta_{\tau=+\infty}^-$.

With these notations, the semi-plane implementing the constraint induced by aircraft *B* on the velocity of aircraft *A* can be mathematically summarized as follows for CS-ORCA:

$$P_{B \rightarrow A}^{\text{CSORCA}_\tau} := \begin{cases} P_{B \rightarrow A}^{\tau=+\infty} & \text{if } \vec{v}_r \in \delta_\tau^- \\ P_{B \rightarrow A}^{\tau=\tau} & \text{otherwise} \end{cases}$$

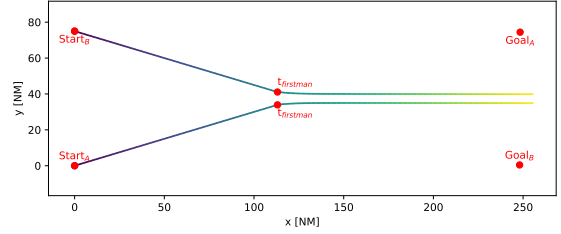
CS-ORCA is just Algorithm 1 with $P_{i \rightarrow j} = P_{i \rightarrow j}^{\text{CSORCA}_\tau}$ for all aircraft pairs (i, j) with $i \neq j$.

As shown in Figure 5, with CS-ORCA the two aircraft turn left, as opposed to the choice made with ORCA in Figure 4. This allows both aircraft to reach their destination, without postponing the conflict beyond the time horizon.

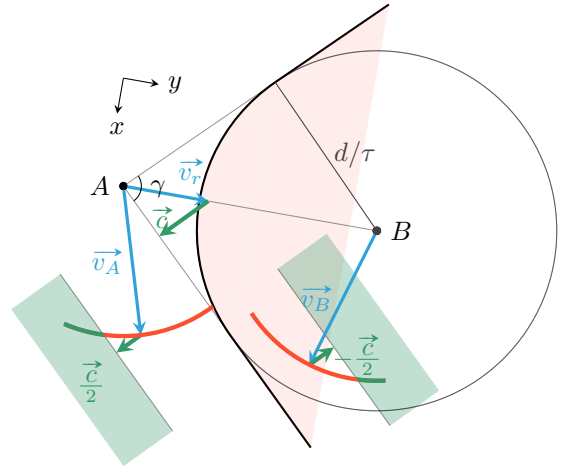
5.3 Issue with CS-ORCA

Let us now explain why CS-ORCA does not always solve the pathological situations, exposed in section 5.1, that it was intended to solve. When aircraft have to cross one another, τ must be chosen large enough in order to avoid the parallel

³One can also realize that by considering how the shape of δ_τ^- is modified when increasing τ to $+\infty$: B' moves to the position of *A*, and the radius of the round head of δ_τ^- tends to 0.



(a) With $\tau = 1$ min, CS-ORCA sets the two aircraft on parallel courses. The conflict is avoided, but the aircraft do not reach their destinations. Figure 6b depicts the first maneuver at $t_{firstman}$ that led to this situation.



(b) First maneuver performed at $t_{firstman}$, when \vec{v}_r first enters the forbidden cone. Here, this happens when the aircraft are close. As a consequence, the cone angle is large. The resulting constraints (green semi-planes and arcs) tend to move the velocities toward parallel directions. The x and y directions are shown so that it is easier to relate this figure with Fig. 6a.

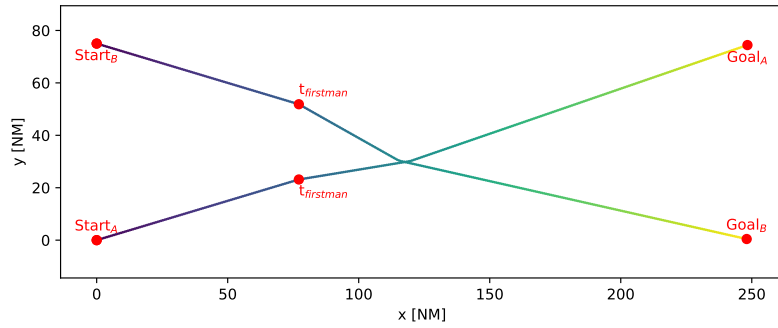
Figure 6: Example with $\tau = 1$ min where CS-ORCA sets two aircraft on parallel tracks (Fig. 6a), by moving the velocities apart (Fig. 6b). Figure 6b is scaled to improve readability by choosing the minute as unit of time. This explains why *B* is here the center of both the circles of radius d and of radius $\frac{d}{\tau}$.

behaviour. To understand this, we can look at a situation with only two aircraft with the exact same speed converging at a small angle, as illustrated in Figure 6.

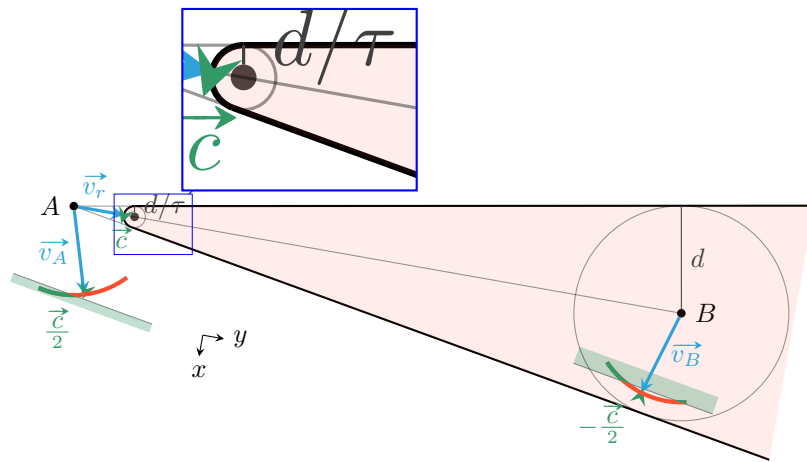
If τ is too small, CS-ORCA chooses parallel tracks, the conflict is avoided, but aircraft do not

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(a) With $\tau = 10$ min, CS-ORCA makes both aircraft turn right, the conflict is avoided, and the aircraft reach their destinations. Figure 7b depicts the situation at $t_{firstman}$ that explains this choice.



(b) First maneuver performed at $t_{firstman}$. Here, the aircraft are far from each other when \vec{v}_r first enters the forbidden cone. As a consequence, the cone angle is small. The resulting constraints (semi-planes and arcs in green) make both aircraft turn right, here increasing their relative closing speed, which helps them to cross paths. The x and y directions are shown so that it is easier to relate this figure with Figure 7a.

Figure 7: Example with $\tau = 10$ min where CS-ORCA makes both aircraft turn right and reduce their relative closing speed, allowing them to cross paths and reach their destination.

reach their destinations as depicted in Figure 6a. As can be seen in Figure 6b, the first maneuver begins when \vec{v}_r enters the forbidden zone δ_r^- . With a small τ , aircraft are close when this happens.

As a consequence, the angle $\gamma = 2 \arcsin \frac{AB}{d}$ made by the cone is large. Thus, the angle between the chosen semi-planes and \vec{AB} , equal to $\frac{\gamma}{2}$, is large too. This results in constraints forcing a reduction of the velocity vectors' component

along the vector \vec{AB} : the aircraft reduce their closing speed by making their velocities parallel, as plotted in Figure 6b.

Conversely, if τ is large enough, CS-ORCA makes them both turn right (or left), and both aircraft reach their destination as illustrated in Figure 7a showing the same example as in Fig. 6b, with a larger time horizon $\tau = 10$ min. The maneuver begins when \vec{v}_r enters the forbidden cone as depicted in Figure 7b. With a large τ , the aircraft are far from each other when this happens.

As a consequence, the angle between the chosen semi-planes and \overrightarrow{AB} is small, thus favouring maneuvers with a non-null closing speed and allowing aircraft to cross paths.

Ideally, one would want a large τ when aircraft have to cross one another, in order to avoid the parallel behaviour. When aircraft do not want to cross one another, a smaller τ would result in less restrictive constraints and be sufficient to avoid conflict efficiently.

6 Dual-Horizon ORCA (DH-ORCA)

We have seen in the previous section that CS-ORCA was initially meant to avoid the parallel behaviour when aircraft have to cross one another, but in order to work a large enough value of τ must be chosen. The drawback of having a large value of τ is that it will be used in all situations, even ones that do not require aircraft to cross paths.

With a large τ , we have a loss of efficiency – as the constraints imposed on the velocity vectors are more restrictive than required – and with a small τ , the CS-ORCA algorithm exhibits the pathological behaviour we are trying to avoid.

With this in mind, we propose a new logic, called Dual-Horizon ORCA (DH-ORCA), with two time horizons. A small value τ_{conflict} is used to enforce systematically the standard ORCA constraints. A larger value $\tau_{\text{cross}} > \tau_{\text{conflict}}$ is used to enforce optional CS-ORCA-like constraints.

The optional constraint is enforced by an aircraft only when its preferred course towards its destination crosses the current trajectory of another aircraft, and is in conflict with it in the τ_{cross} time horizon. Note that this logic does not require the aircraft to broadcast their preferred course to others: Each aircraft only needs its own intent to decide whether to enforce the optional constraint.

6.1 The DH-ORCA Constraints and Algorithm

The two constraints created by τ_{conflict} , one for each aircraft, are always enforced. This guarantees that there will be no loss of separation within a time horizon τ_{conflict} . These constraints are denoted $P_{\rightarrow}^{\tau=\tau_{\text{conflict}}}$. They are less restrictive than the $P_{\rightarrow}^{\tau=+\infty}$ constraints of CS-ORCA.

The two constraints created by τ_{cross} , one for each aircraft, are here to avoid the parallel behaviour and will be created using the cone side closest to $\overrightarrow{v_r}$, as in CS-ORCA. Let us denote $P_{\rightarrow}^{\tau=+\infty}$ these crossing constraints. Because the separation constraints $P_{\rightarrow}^{\tau=\tau_{\text{conflict}}}$ created with the shortest time horizon τ_{conflict} are sufficient to guarantee conflict-free trajectories—at least in the τ_{conflict} time horizon—the constraints created by τ_{cross} are somewhat optional. In fact, each aircraft decides if it enforces or ignores the crossing constraint generated by the other.

Making the $P_{\rightarrow}^{\tau=+\infty}$ constraint optional avoids to restrict the velocity choices when not necessary. Here, the constraint will be enforced by an aircraft only when its preferred course towards its destination crosses the current trajectory of the other aircraft and if a loss of separation is predicted to occur in the τ_{cross} time horizon when following this preferred course.

Let us denote $\text{Cross}(A, B)$ this condition, allowing A to decide if it should enforce a $P_{B \rightarrow A}^{\tau=+\infty}$ constraint because of aircraft B . This condition is expressed in the following equation, where d is the required minimum separation and $\overrightarrow{v_A}^{\text{pref}}$ is the preferred velocity vector of aircraft A , directed toward its destination:

$$\text{Cross}(A, B) := \exists t \in [0, \tau_{\text{cross}}] \text{ s.t. } \left\| \overrightarrow{BA} + (\overrightarrow{v_A}^{\text{pref}} - \overrightarrow{v_B})t \right\| < d$$

Note that aircraft A assumes that B keeps following its current velocity vector $\overrightarrow{v_B}$. Using $\overrightarrow{v_B}^{\text{pref}}$ instead of $\overrightarrow{v_B}$ would more accurately model a conflict between the intended paths of both aircraft, but would require aircraft B to share its intended velocity with A . Here $\overrightarrow{v_B}$ is used as a proxy of $\overrightarrow{v_B}^{\text{pref}}$. If $\overrightarrow{v_B}$ is not close to $\overrightarrow{v_B}^{\text{pref}}$ then $\text{Cross}(A, B)$ could be false despite aircraft A and B having to cross one another to go to their destination. Note also that the $\text{Cross}(\cdot, \cdot)$ criteria is not symmetric: $\text{Cross}(A, B)$ might have a value different from $\text{Cross}(B, A)$.

The constraint $P_{B \rightarrow A}^{\tau=+\infty}$ generated by B is enforced by A if and only if $\text{Cross}(A, B)$ is true.

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664 The DH-ORCA constraints can be mathemati-
665 cally summarized as follows:

$$666 \quad P_{B \rightarrow A}^{\text{DH-ORCA}_{\tau_{\text{conflict}}}^{\text{cross}}} :=$$

$$667 \quad \begin{cases} P_{B \rightarrow A}^{\tau = \tau_{\text{conflict}}} \cap P_{B \rightarrow A}^{\tau = +\infty} & \text{if Cross}(A, B) \\ P_{B \rightarrow A}^{\tau = \tau_{\text{conflict}}} & \text{otherwise} \end{cases}$$

672 DH-ORCA is just Algorithm 1 with $P_{i \rightarrow j} =$
673 $P_{i \rightarrow j}^{\text{DH-ORCA}_{\tau_{\text{conflict}}}^{\text{cross}}}$ for all aircraft pairs (i, j) with
674 $i \neq j$.

676 6.2 Geometrical Representation of 677 the DH-ORCA constraints 678

679 A geometrical representation of the $\text{Cross}(\cdot, \cdot)$ cri-
680 teria is also possible: $\text{Cross}(A, B)$ is true if and
681 only if $\vec{v}_A^{\text{pref}} - \vec{v}_B$ is inside the τ_{cross} forbidden
682 zone: $\vec{v}_A^{\text{pref}} - \vec{v}_B \in \delta_{\tau_{\text{cross}}}^-$. Using the geomet-
683 rical interpretation of this criterion, Figure 8
684 depicts the three possible cases for the $P_{\rightarrow}^{\tau = +\infty}$
685 constraints:

- 687 1. Both aircraft ignore their $P_{\rightarrow}^{\tau = +\infty}$ constraint
688 as depicted in Figure 8a: only the classic con-
689 straints from ORCA with τ_{conflict} are consid-
690 ered. This gives more freedom to the aircraft
691 to modify their velocity. The new velocities
692 are chosen as if only ORCA was used with
693 $\tau = \tau_{\text{conflict}}$. This case is typically encountered
694 when the aircraft are far from each other, or
695 when the direction of the preferred velocity of
696 each aircraft diverges from the other aircraft's
697 current velocity.
- 698 2. Only one of the two aircraft enforces the
699 crossing constraint $P_{\rightarrow}^{\tau = +\infty}$, as depicted in
700 Figure 8b: this crossing constraint is considered
701 along with the classic ORCA-style separation
702 constraints. See subsection 6.3 for a discussion
703 on the effect of the asymmetrical enforcement
704 of the DH-ORCA constraints.
- 705 3. Both $P_{\rightarrow}^{\tau = +\infty}$ constraints are enforced as de-
706 picted in Figure 8c: Both aircraft must choose
707 a velocity satisfying $P_{\rightarrow}^{\tau = +\infty}$. As a consequence,
708 the new relative velocity \vec{v}_r shall fall outside
709 the cone $\delta_{\tau = +\infty}^-$, allowing the aircraft to safely
710 cross one another.

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6.3 Discussion on the Asymmetry of DH-ORCA Constraints

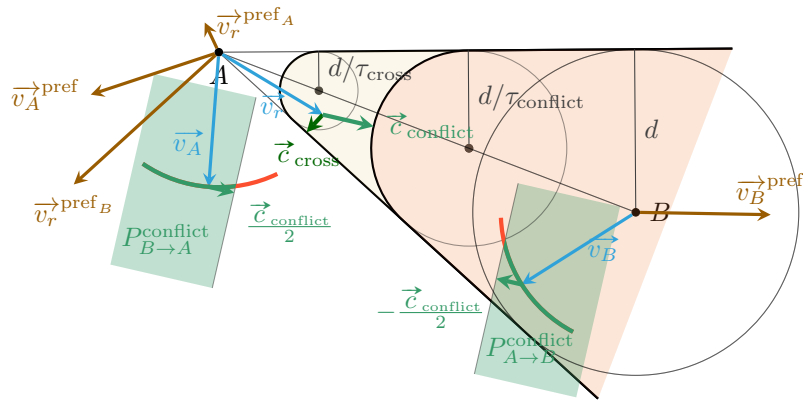
When both aircraft enforce their $P_{\rightarrow}^{\tau = +\infty}$ con-
straint, as on Figure 8c, both aircraft change their
velocity vector to move \vec{v}_r outside the $\delta_{\tau = +\infty}^-$ cone.
The maneuver is equally split among the two air-
craft and each aircraft do its part to avoid the
other. If both aircraft do this, \vec{v}_r is guaranteed to
be outside the cone $\delta_{\tau = +\infty}^-$ and consequently no
conflict will ever happen assuming all aircraft stay
on their course.

In Figure 8b, where only one aircraft enforces
its $P_{\rightarrow}^{\tau = +\infty}$ constraint, we do not have such a guar-
antee: only one aircraft is actively maneuvering to
avoid the other. However, it was experimentally
observed that having only one aircraft maneu-
vering is sometimes enough to avoid the parallel
behaviour. Roughly speaking, at each time step,
the constrained aircraft takes half of the required
speed change. It might not be enough to actually
move \vec{v}_r outside the $\delta_{\tau = +\infty}^-$ cone, but it tries to
move \vec{v}_r toward the edge of the cone: $\delta_{\tau = +\infty}$. As
aircraft are still far from each other, the enforced
 $P_{\rightarrow}^{\tau = +\infty}$ constraint do not drastically reduce the
aircraft's speed component along \overline{AB} , thus avoid-
ing the parallel behaviour. Furthermore, having
 \vec{v}_r close to $\delta_{\tau = +\infty}$ somewhat pre-solves the cross-
ing problem: as time goes by, \vec{v}_r will eventually
get inside $\delta_{\tau = \tau_{\text{conflict}}}^-$ but as \vec{v}_r is close to $\delta_{\tau = +\infty}$,
chances are that the minimal maneuvers to get \vec{v}_r
outside $\delta_{\tau = \tau_{\text{conflict}}}^-$ will be also maneuvers that get
 \vec{v}_r outside $\delta_{\tau = +\infty}^-$, allowing aircraft to cross one
another.

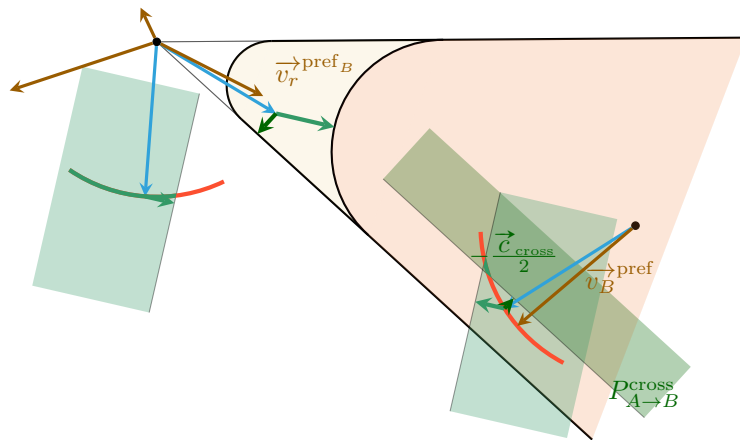
6.4 Illustration of the Behavior of ORCA, CS-ORCA and DH-ORCA on a simple example.

Let us consider the simple example shown in
Figure 9a where 5 flights cruising at 230 kts at the
same altitude must cross paths to reach their re-
spective destinations. The separation standard d
is 5 NM. The departure points (labeled "Start")
are evenly spaced apart by a distance of 25 NM,
as well as the arrival points (labeled "Goal").

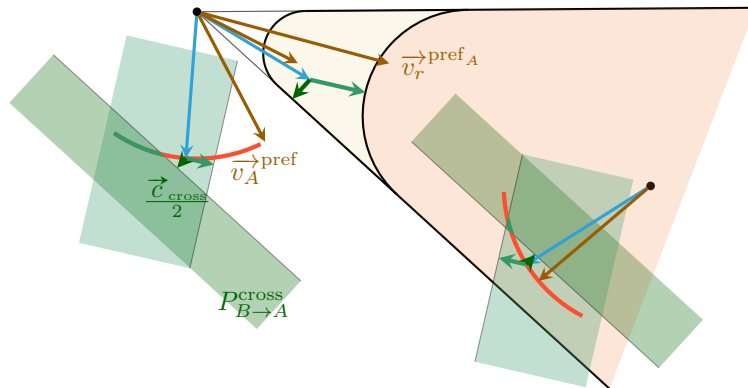
When no *Detect & Avoid* algorithm is applied
(Fig. 9a), all 5 aircraft fly directly to their desti-
nations, but separation losses occur, with a total
duration of 605 s.



(a) In this example, the \vec{v}_r^{pref} are not inside the τ_{cross} forbidden zone, no P^{cross} constraints are added.



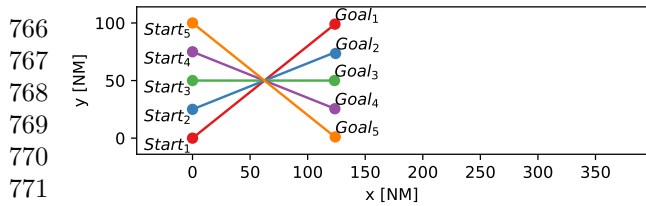
(b) In this example, \vec{v}_r^{pref} is inside the τ_{cross} forbidden zone, $P_{A \rightarrow B}^{\tau=+\infty}$ is added.



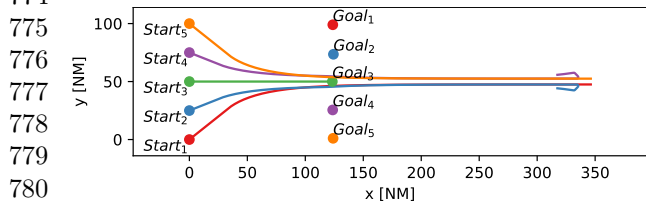
(c) In this example, both $\vec{v}_r^{\text{pref}_A}$ and $\vec{v}_r^{\text{pref}_B}$ are inside the τ_{cross} forbidden zone, both $P_{A \rightarrow B}^{\tau=+\infty}$ and $P_{B \rightarrow A}^{\tau=+\infty}$ are added.

Figure 8: Three examples with different preferred velocities are considered. They illustrate the three possible cases concerning the P^{cross} constraints. Let us note $\vec{v}_r^{\text{pref}_A} = \vec{v}_A^{\text{pref}} - \vec{v}_B^{\text{pref}}$ and $\vec{v}_r^{\text{pref}_B} = \vec{v}_B^{\text{pref}} - \vec{v}_A^{\text{pref}}$. Reading from Figure 8a to Figure 8c, only the modified elements are labelled in order to improve readability.

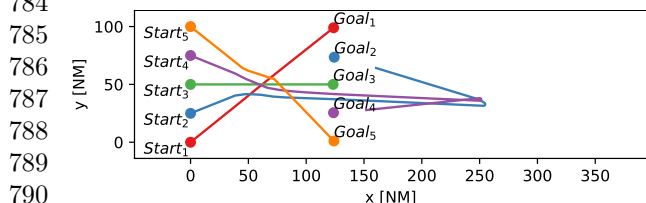
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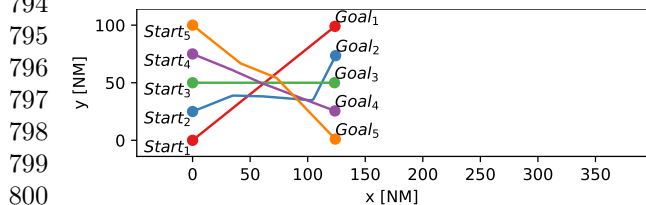
(a) No resolution is performed here, resulting in a 605 s separation loss duration.



(b) ORCA algorithm is used, no loss of separation and two parallel behavior are observed on this simple scenario.



(c) CS-ORCA algorithm is used, no loss of separation and one parallel behavior are observed on this simple scenario.



(d) DH-ORCA algorithm is used, no loss of separation and no parallel behavior are observed on this simple scenario.

Figure 9: An example in which the parallel behavior is observed with ORCA and CS-ORCA but not with DH-ORCA.

When applying a *Detect & Avoid* algorithm (Figures 9b to 9d) with the τ -values tuned in section 7.5 and found in Table 4, no separation loss remains, whatever the chosen algorithm (ORCA, CS-ORCA, or DH-ORCA). However, we see that both algorithms ORCA (Fig. 9b) and CS-ORCA (Fig. 9c) exhibit the pathological parallel-tracks

behavior described in section 5.1, whereas DH-ORCA does not (Fig. 9d)

Of course, we cannot draw general conclusions from this simple illustration and DH-ORCA does not guarantee that the pathological solutions will never occur. In the following, we will show experimentally in Section 8 that DH-ORCA drastically reduces the occurrences of pathological solutions when compared with ORCA and CS-ORCA on hours of random scenarios, described in next Section 7, especially for aircraft flying at the same speed.

7 Traffic Scenarios, Simulation, and Experiment Setup

7.1 Traffic Scenarios (Flight Plans)

In our experiments, we compare the different methods described in the previous sections on random traffic scenarios.

We consider a circular area of chosen radius R . A new flight enters the area at a random location on the boundary, taken in the admissible domain determined by exclusion zones around the positions of the preceding flights, so as to avoid conflicting situations on entry. More precisely, as illustrated in Figure 10, we extrapolate the positions of the flights already in the area at the time t when the new flight is scheduled to enter the area, assuming each flight follows its initial straight route toward its destination. We then define a circular exclusion zone of chosen radius (greater than the separation standard d) around each extrapolated position. The admissible entry domain (in green on Figure 10) is obtained by removing the intersections between the outer boundary and the exclusion zones.

Let us denote Δ_{entry} the separation that we wish to enforce between an entering aircraft and the aircraft already in the area. To account for the fact that the traffic is simulated with a user-chosen time step δt , the exclusion zones defined around the positions of these other aircraft must have a radius $\Delta_{entry} + V \delta t$, where V is the speed of the entering flight. The additional margin $V \delta t$ accounts for the fact that the entering aircraft actually enters the area at a time t_{start} between t

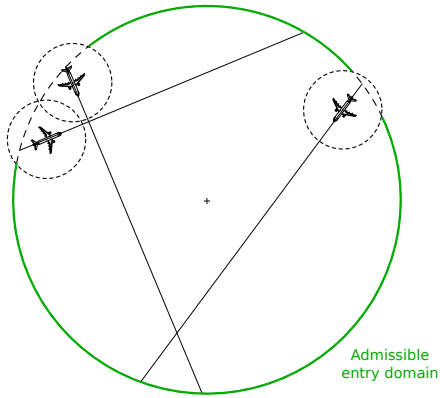


Figure 10: Admissible domain for an entry point (in green), and exclusion zones around flights already in the area.

and $t + \delta t$ – where t is the time of the current simulation step – and it flies over a certain distance between t_{start} and $t + \delta t$, that could put it within a circle of radius Δ_{entry} around another flight already in the area. In our experiments, we chose four times the separation standard d for Δ_{entry}

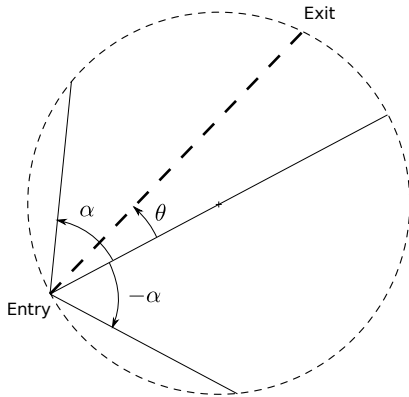


Figure 11: Route selection, at a random angle θ in $\mathcal{U}([- \alpha, \alpha])$ from the direction towards the center of the area. The exit point is determined by the selected route.

Once the entry point is chosen, the flight will follow a straight route at a random angle θ from the direction towards the center of the area (see Fig. 11). This random angle θ is drawn from a uniform distribution $\mathcal{U}([- \alpha, \alpha])$. The destination of each flight is the intersection of this straight route and the circular boundary opposite to the entry point. It can be easily shown that the average

crossing distance of the circular area is $\frac{2R \sin \alpha}{\alpha}$ when $\alpha > 0$ and $2R$ when $\alpha = 0$.

The aircraft speed V is randomly chosen between a minimum value V_{min} and a maximum value V_{max} . We assume no wind.

The average crossing time is then $\frac{2R \sin \alpha}{\alpha} / \left(\frac{V_{min} + V_{max}}{2} \right)$.

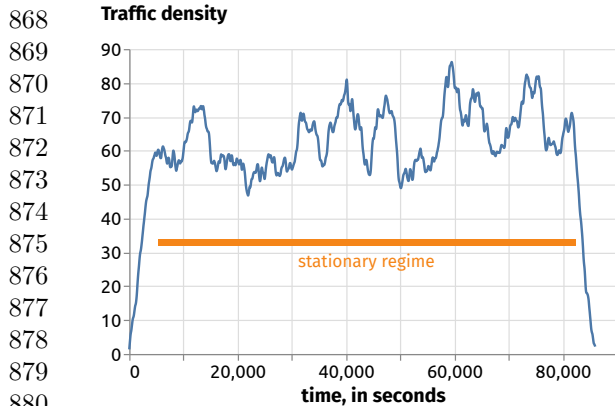
In all random scenarios, the flight entry times follow a Poisson distribution of parameter λ (the desired average number of flight entries per unit of time). By definition of the Poisson distribution, the average number of flights entering the area over a given period of time T is λT and the variance of this number of entering flights is also λT .

We are interested in measuring some performance metrics at a given average traffic density. The traffic density is the number of aircraft or drones within the area at a given moment. This number depends on the inbound and outbound flows. Assuming the circular area is supplied in flights between t_{start} and t_{end} following the Poisson distribution, there will be a transition period at the beginning of the scenario, when the traffic density increases while the outbound flow remains at zero. After this transition period, the traffic density remains approximately stable – with some variations mainly due to the Poisson distribution of the inbound flow and also to the distribution of the crossing times – before decreasing after t_{end} until it reaches zero at time t_f , the end of the simulation. Figure 12 illustrates the typical evolution of the traffic density when the incoming flow follows a Poisson distribution, over a period of 24 hours.

To summarize, a scenario is characterized by several parameters: radius of circular area considered in the experiment, average number of flights entering the circular area per unit of time, duration of the scenario, upper and lower bounds for the ground speed, angle α defining the angular bounds for the route, radius Δ_{entry} of the exclusion zones for the entry conflicts.

Note that the scenarios are designed so that all flights reach their exit point before the end of the simulation t_f when the flights follow their initial straight route as initially planned (when no D&A maneuver is performed).

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881 **Figure 12:** Typical evolution of the traffic density in a random scenario where flight entry times are sampled from a Poisson distribution. The stationary regime is shown by the horizontal orange segment.
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886 887 888 7.2 Traffic Simulation with a Detect 889 and Avoid (D&A) module

890 We have developed a small generic fast-time traffic simulator to execute the traffic scenarios and evaluate several detect and avoid algorithms such as the ones described in previous sections. Any D&A algorithm or conflict resolution algorithm can be easily plugged in the generic simulator to provide maneuvering instructions to the flights every δt seconds, where δt is the user-chosen time step of the simulation.
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900 A D&A algorithm may modify the trajectory of a flight by changing the norm and/or the direction of its velocity at any time step of the simulation. However, in the current experiment, the norm is kept constant, and we only change the direction of the velocity vector. Note that whatever the trajectory modifications, the objective of a flight still remains to reach the exit point that was initially planned, so the D&A algorithm should at some point allow it to resume its navigation towards this exit point, unless it is unable to because of other traffic.
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911 The random scenarios described in section 7.1 are designed such that no flight should conflict with others when entering the airspace, if all flights follow their initial route. When a D&A algorithm is used, some trajectories may be deviated to avoid conflicts. These modified trajectories may conflict with entering flights, with not enough
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anticipation for the D&A algorithm to find solutions to these conflicts. Such conflicts on entry are solved by the simulator by delaying the entering flight until it is separated from the other traffic by a distance greater than Δ_{entry} . These delays are measured during the simulations. A large amount of delay, as well as large trajectory deviations, will indirectly result in less flights reaching their destination before the end of the simulation. This will in turn result in a lower traffic throughput than with no D&A. The traffic throughput is measured by the throughput metric presented in Section 7.4.

7.3 Experiment Setup and Scenario Parameters

In this paper, some parameters will be the same for all scenarios. The values of these common parameters can be found in Table 1. The aircraft speed is constant during the simulation; only course changes are possible. However, each aircraft might have a different speed. Each aircraft speed is drawn in a uniform distribution $\mathcal{U}([V_{min}; V_{max}])$. Different speed ranges will be tested as the speed diversity has a great impact on the observed performance metrics. However, the speed ranges $[V_{min}; V_{max}]$ will be chosen so that $(V_{min} + V_{max})/2$ is set to a constant value for all scenarios, so that the traffic density is not impacted by the choice of the speed range. This way, the traffic density is only controlled through the average number of flights entering the circular area per unit of time λ . We will use random scenarios with different values of this incoming flow λ in our experiments, as we want to test the algorithms on various traffic densities.

Table 2 gives the values of λ tested in this paper as well as the measures of the traffic density and overall duration of separation loss for each of the three scenarios (low, medium, and high density). These metrics are measured without applying any D&A algorithm in order to characterize the difficulty of each scenario. The cumulated duration of separation loss is simply the sum over all flights of the time during which an aircraft was separated by less than the required minimum separation d from another aircraft.

Parameter	Description	Value
d	Separation standard	5 NM
Δ_{entry}	Radius of exclusion zones preventing conflicts on entry	$4d$
R	Radius of the domain entry	150 NM
α	Random angle range	$[-70^\circ; 70^\circ]$
δt	Simulation timestep	5 s
$\frac{V_{min}+V_{max}}{2}$	Average speed	230 kts
	Simulation duration inside the stationary regime	6 h

Table 1: Common parameters for all scenarios

Parameter	Description	Low	Medium	High
λ	Incoming flow [flight/h]	60	120	160
	Aircraft density [flight/10 000 NM ²]	8.5	17	22.7
	LoS duration without D&A [s]	60 783.1	244 710.1	432 360.4

Table 2: Low, medium, and high density scenarios. The traffic density is controlled by the value of the incoming flow parameter λ . LoS here means Loss of Separation.

7.4 Performance Metrics

Executing avoidance maneuvers during the simulation of a scenario has an impact on the traffic that can be measured through a number of metrics: number and/or duration of remaining separation losses, delays at entry, trajectory lengthening, number of flights not terminated—i.e. that could not reach their exit point before the end of the simulation because they were maneuvered—and distance remaining to be flown to destination for these flights⁴.

In this paper, we propose to assess the performance of D&A algorithms through five metrics only: Two safety metrics measuring the duration and severity of separation losses respectively; Three efficiency metrics, the first measuring the total duration of the degenerate situations with parallel tracks not reaching their destinations, the second measuring the trajectory lengthening due to the D&A maneuvers, and the third the traffic throughput, which is directly related to the amount of delay assigned to entering flights.

Because we want to assess the D&A algorithms performances at a given average traffic density, these safety and efficiency metrics are measured

only for the flights that are scheduled to enter the simulation in the time interval corresponding to the stationary regime of the scenario (see Figure 12). So we consider only the flights departing between $t_i^{\text{statio}} = t_i + T_{\text{transition}}$ and $t_f^{\text{statio}} = t_f - T_{\text{transition}}$ where t_i and t_f are the beginning and end times of the scenario with no detect and avoid, and $T_{\text{transition}}$ is the transition time at the beginning or at the end of the scenario. The other flights can be of use when computing some of the metrics, however. For example, a separation loss involving a flight in the selected list and another one that is not in the list will still be counted.

In our experiments, we have taken a transition time $T_{\text{transition}} = T_{\text{max}} + 2\sigma$ where $T_{\text{max}} = 2RV_{\text{min}}$ and $\sigma = \sqrt{\lambda T_{\text{max}}}$, in order to be safely within the stationary regime interval.

The performance metrics used to assess the three methods (ORCA, CS-ORCA and DH-ORCA) on scenarios of low, medium and high traffic densities are listed below. The separation loss metrics and the one measuring the parallel tracks duration consider only the trajectory points in the stationary regime of each scenario. The trajectory lengthening metric considers only the trajectory parts starting after the beginning of the stationary regime. Most metrics are normalized so that we can compare their values on scenarios of various traffic densities.

- **normalized_LoS:** This metric quantifies the duration of the separation losses. In order to

⁴Remember that a full simulation with the D&A off will show no delay and have all flights reaching their destination, although it will certainly have many separation losses, whereas with the D&A on we may expect much less separation losses, but some flights might be delayed at entry because of maneuvering aircraft, and some other flights might not reach their destination before the end of the simulation.

970 avoid scenario-related variations, we normal-
 971 ize this total duration. The **normalized_LoS**
 972 metric is the total duration of the separation
 973 losses observed in the simulation when using the
 974 evaluated D&A algorithm divided by the total
 975 duration of separation losses when no D&A al-
 976 gorithm is used. This can be summarized by
 977 the following formula, where LoS is the total
 978 duration of separation losses:

$$\text{normalized_LoS}(\text{scenario}, \text{algo}) = \frac{\text{LoS}(\text{simu}(\text{scenario}, \text{algo}))}{\text{LoS}(\text{simu}(\text{scenario}, \text{without D\&A}))}$$

984 • **normalized_LoS_Severity**: This metric quan-
 985 tifies the severity of the separation losses by
 986 considering the minimum distance min_LoS_dist
 987 between aircraft over all separation losses, and
 988 normalizing using the separation standard d
 989 (*i.e.* the minimum required separation). This
 990 metric is 0 when no separation loss occurs,
 991 and is otherwise computed using the following
 992 formula:

$$\text{normalized_LoS_Severity}(\text{scenario}, \text{algo}) = 1 - \frac{\text{min_LoS_dist}(\text{simu}(\text{scenario}, \text{algo}))}{d}$$

998 The closer to 1 this metric is, the worst the
 999 closest separation loss is.

1000 • **normalized_length**: With this metric we quan-
 1001 tify the trajectory deviations due to the D&A
 1002 maneuvers. A simple way to do this could be
 1003 to compute the ratio of the length of the ac-
 1004 tual trajectories over the length of the initial
 1005 planned routes. However, for the flights that do
 1006 not reach their destination before the end of
 1007 the simulation, it makes no sense to measure
 1008 the ratio between the length of the incomplete
 1009 trajectory and the length of the initial planned
 1010 route. Thus, we consider the distance between
 1011 the last point of the flown trajectory and the
 1012 destination, and add it to the length of the
 1013 flown trajectory, before computing the ratio be-
 1014 tween this total length and the length of the
 1015 planned route. This can be summarized by these
 1016 formulas:

$$\text{length}(\text{scenario}, \text{algo}) =$$

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$$\sum_{\text{traj} \in \text{simu}(\text{scenario}, \text{algo})} \text{length}(\text{traj}) + \|\text{last}(\text{traj}) - \text{dest}(\text{traj})\|$$

$$\text{normalized_length}(\text{scenario}, \text{algo}) = \frac{\text{length}(\text{scenario}, \text{algo})}{\sum_{\text{traj} \in \text{simu}(\text{scenario}, \text{algo})} \|\text{first}(\text{traj}) - \text{dest}(\text{traj})\|}$$

With all aircraft flying straight routes to their destination, this metric would be equal to 1. When aircraft deviate from the straight course, this metric takes values greater than 1. It gives the ratio of the overall trajectory lengths with the D&A active over the overall shortest travel distances from entry to exit that would be flown with the D&A off.

• **throughput**: With this metric we quantify the ability of the D&A to accommodate the incoming flow of aircraft. To measure this, inside the stationary regime, the number of aircraft reaching their destination is compared with the expected theoretical number of aircraft reaching their destination. As we consider a stationary regime, this expected number of aircraft reaching their destination is equal to the expected number of incoming aircraft. This can be summarized by this formula:

$$\text{throughput}(\text{scenario}, \text{algo}) = \frac{\text{count_arrived}(\text{simu}(\text{scenario}, \text{algo}))}{\text{incoming_flow} \times \text{simulation duration}}$$

If the D&A is not able to accommodate the demand, the amount of delay and the total trajectory lengthening will be high, resulting in a low value of **throughput** (greatly inferior to 1). Please note that this metric can be superior to 1 because the actual number of aircraft reaching their destination can be superior to the expected one. This can be due to the randomness of the Poisson process determining the desired entry times, or it can be due to a large number of aircraft already in the simulation at the beginning of the considered time window in the stationary regime over which the metric is measured.

• **parallel_total_duration**: With this metric we quantify the parallel behavior discussed in Sub-Section 5.3. Two flights at respective positions A and B with velocities \vec{v}_A and \vec{v}_B are

considered as locked on parallel tracks when the following conditions are met: $\vec{v}_A \cdot \vec{v}_B > 0$ (same direction); the angle between \vec{v}_A and \vec{v}_B is less than 5 degrees (parallel tracks); the distance between A and B is less than $1.5d$ (proximity) where d is the separation standard; and the intent of each aircraft is to cross path with the other. Denoting \vec{v}_A^{pref} and \vec{v}_B^{pref} the preferred velocities of the two flights toward their respective destinations, the last criterion (path-crossing) can be expressed as the fact that \vec{v}_A^{pref} and B are on the same side of \vec{v}_A , and \vec{v}_B^{pref} and A are on the same side of \vec{v}_B .

The `parallel_total_duration` metric is the total duration of the parallel behavior observed in the simulation when using the evaluated D&A algorithm.

7.5 D&A Parameter Selection

Each D&A algorithm has parameters that we must calibrate before comparing the methods: the time horizon τ for ORCA and CS-ORCA and $(\tau_{\text{conflict}}, \tau_{\text{cross}})$ for DH-ORCA. To select the best parameter values for each method, we perform a grid search: The best parameter value (or combination of values) is selected from a grid of values evaluated on a set of scenarios. For each D&A algorithm, each parameter is tested on 3 traffic densities with 20 random scenarios for each traffic density.

Table 3 gives the grid of values tested for each method. For DH-ORCA, note that we try all couples of parameter values such that $\tau_{\text{conflict}} < \tau_{\text{cross}}$.

D&A algorithm	Grid of parameters to be tested
ORCA	$\tau \in \{212 \text{ s}, 236 \text{ s}, \dots, 740 \text{ s}\}$
CS-ORCA	$\tau \in \{212 \text{ s}, 236 \text{ s}, \dots, 740 \text{ s}\}$
DH-ORCA	$\tau_{\text{conflict}} \in \{20 \text{ s}, 52 \text{ s}, \dots, 212 \text{ s}\}$ $\tau_{\text{cross}} \in \{212 \text{ s}, 244 \text{ s}, \dots, 532 \text{ s}\}$ such that: $\tau_{\text{conflict}} < \tau_{\text{cross}}$

Table 3: Time horizon (τ) values tested for each D&A algorithm.

When choosing the best parameter value(s), one wants to minimize both the separation losses `normalizedLoS` and the trajectory length `normalized.length`, with a higher priority on the safety metric. Considering Figure 13, we see that

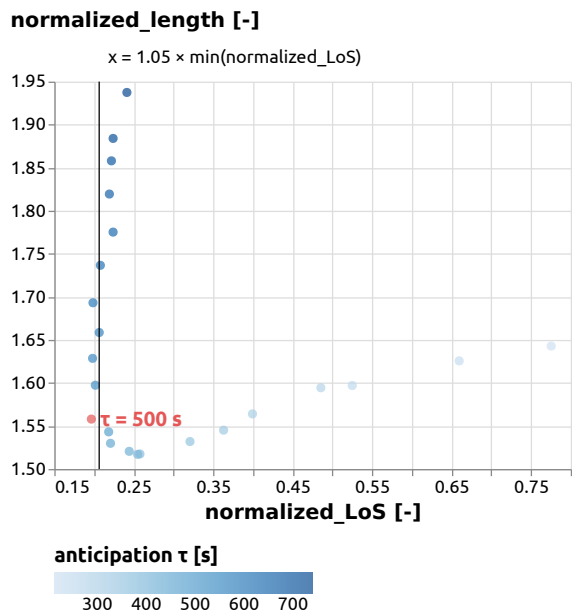


Figure 13: For the ORCA method, `normalized.length` is plotted as a function of the `normalizedLoS`. Each point is the result obtained by using ORCA with a specific τ value. Each point is obtained by computing the metrics average over the 20×3 calibration scenarios. The red point is the selected parameter value $\tau = 500$ s for the ORCA method.

there is a trade-off to be found between the two conflicting objectives (safety and efficiency), when selecting the value of the time horizon parameter τ . This figure plots the normalized trajectory length as a function of the normalized separation loss duration, averaged over the 3×20 calibration scenarios, here for the ORCA method. Each point on this figure represents a parameter value from the selection grid.

As the primary objective is to ensure safety, we could simply select the value of τ minimizing the `normalizedLoS` metric. However, we can see on Figure 13 that there is a range of τ values – near the y -axis – that perform very well in terms of safety, but show different performances in terms of efficiency.

In order to take also the efficiency criteria into account, we will select the τ parameter value having the minimum value of `normalized.length` among all the τ -values for which the `normalizedLoS` measure is less

1072 than 5% above the minimum `normalized_LoS`.
 1073 This 5% safety threshold is materialized, for
 1074 ORCA, on Figure 13 by the vertical line close to
 1075 the y -axis. This selection process is applied for all
 1076 the time horizon parameters of the three D&A
 1077 methods tested in this paper (ORCA, CS-ORCA
 1078 and DH-ORCA).

1079 As stated in section 5.3, when aircraft have to
 1080 cross one another, a large τ is required to avoid
 1081 the parallel behavior. In DH-ORCA, we have two
 1082 horizons, one to avoid the parallel behavior (τ_{cross})
 1083 and one to avoid conflict (τ_{conflict}). As a conse-
 1084 quence this allows τ_{conflict} to be much smaller than
 1085 τ_{cross} and the τ values CS-ORCA. This interpreta-
 1086 tion seems to be confirmed by the best parameter
 1087 values obtained with the grid search, shown in
 1088 Table 4.

D&A algorithm	Best performing parameters
ORCA	$\tau = 500$ s
CS-ORCA	$\tau = 308$ s
DH-ORCA	$\tau_{\text{conflict}} = 52$ s $\tau_{\text{cross}} = 372$ s

1094 **Table 4:** Best values of the time horizon param-
 1095 eter(s) for each D&A algorithm. The τ -values are
 1096 selected from the parameter grid shown in Table 3,
 1097 considering the average performance over 3×20
 1098 calibration scenarios.

1100
 1101 The time horizon parameter values of Table 4
 1102 are the ones used in section 8 when comparing
 1103 the three D&A methods (ORCA, CS-ORCA, DH-
 1104 ORCA). Please note that the 3×20 scenarios
 1105 used to select the time horizon parameter values
 1106 are not used in section 8, where the performances
 1107 of each method are assessed on new, unseen scen-
 1108 arios, using the metrics presented in section 7.4.
 1109 This avoids any overly optimistic bias. To draw
 1110 a parallel with Machine Learning, one can see
 1111 these 3×20 scenarios as a training set to learn
 1112 the time horizon parameters. Likewise, the scenar-
 1113 ios used in Section 8 can be seen as the test set
 1114 used to evaluate the performance with new traffic
 1115 instances.

1116 Note also that the time horizon parameter
 1117 values found in this section are specific to the
 1118 context of the current study, and that more re-
 1119 alistic traffic conditions – with a different mix of
 1120 aircraft speeds – might require another selection

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of these parameters before deploying the chosen
 D&A algorithm.

8 Results

Let us now compare ORCA, CS-ORCA and DH-
 ORCA on traffic scenarios of low, medium and
 high densities (see section 7.3 for a description
 of the scenarios). In subsection 8.1, we compare
 the three methods using a same aircraft speed
 distribution for all the scenarios. Then, in subsec-
 tion 8.2, we examine the influence of the aircraft
 speed distribution on the results, considering sev-
 eral speed distributions. As we will see, these two
 scenario parameters (traffic density, speed distri-
 bution) have an impact on the results, although
 the performance ranking of the three methods
 remains the same.

For each tested traffic density and speed dis-
 tribution, 100 random scenarios are simulated.
 For each simulation, we measure the performance
 metrics described in section 7.4. The 100 values
 obtained for each metric are summarized in a box-
 plot or by their mean value, depending on the
 considered figure.

The results were computed using only one core
 of an AMD Ryzen Threadripper 1920X. Computa-
 tion time is not an issue for geometric D&A
 algorithms: For instance, it takes 355 s of CPU
 user time, on average, to simulate 6 hours of traffic
 using DH-ORCA with the largest incoming flow of
 160 flights/hour. Out of these 355 s, the time spent
 computing the new velocities (*i.e.* the execution
 time of DH-ORCA) is 262 s.

8.1 Comparison on low, medium, and high traffic density scenarios

In this section, the aircraft speeds in all scenar-
 ios are drawn from a same uniform distribution
 $\mathcal{U}([200 \text{ kts}; 260 \text{ kts}])$.

Let us first see how the ORCA, CS-ORCA and
 DH-ORCA methods fare in terms of safety, consid-
 ering the normalized duration of separation losses
 on Figure 14, and the severity of separation losses
 on Figure 15. We see that ORCA is the less safe
 among the three methods, showing the longest and
 most severe separation losses in all types of scenar-
 ios, whereas DH-ORCA outperforms both ORCA
 and CS-ORCA on these two safety metrics.

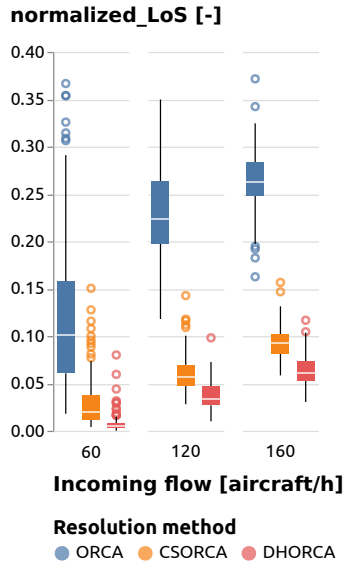


Figure 14: Normalized duration of the separation losses with aircraft speed randomly drawn in $\mathcal{U}([200 \text{ kts}; 260 \text{ kts}])$ for different incoming flows.

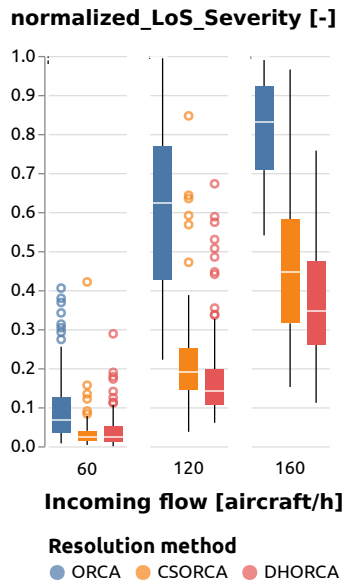


Figure 15: Normalized severity of the separation losses with aircraft speed randomly drawn in $\mathcal{U}([200 \text{ kts}; 260 \text{ kts}])$ for different incoming flows.

Now considering the efficiency metrics – parallel total duration on Figure 16, normalized trajectory lengthening on Figure 17, and normalized throughput on Figure 18 – we observe similar

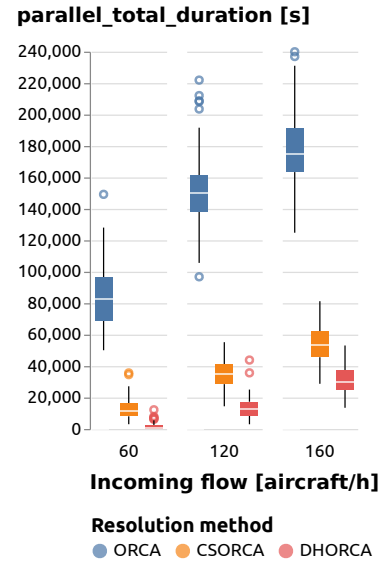


Figure 16: Parallel behaviour total duration with aircraft speed randomly drawn in $\mathcal{U}([200 \text{ kts}; 260 \text{ kts}])$ for different incoming flows.

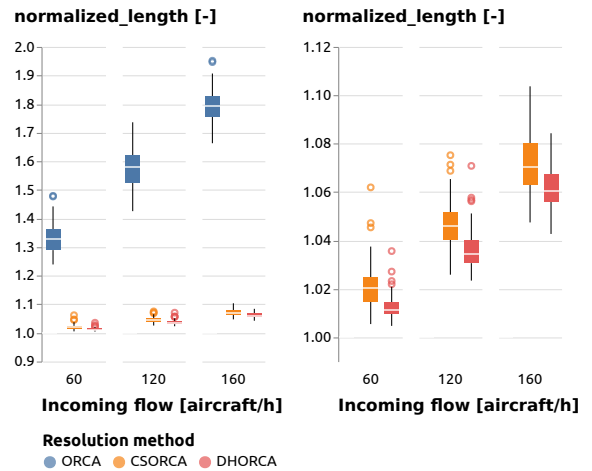
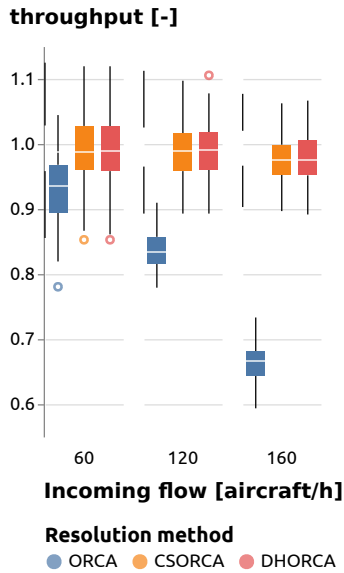


Figure 17: Normalized length with aircraft speed randomly drawn in $\mathcal{U}([200 \text{ kts}; 260 \text{ kts}])$ for different incoming flows.

results, with ORCA performing worst and DHORCA performing best among the three methods.

Figure 16 shows the total duration of the parallel tracks occurrences for the three methods. ORCA performs worst, by far. We also see that CS-ORCA does actually improve very significantly over ORCA on the parallel-tracks issue. DH-ORCA performs best, significantly improving

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1192 **Figure 18:** throughput with aircraft speed ran-
1193 domly drawn in $\mathcal{U}([200 \text{ kts}; 260 \text{ kts}])$ for different
1194 incoming flows.

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1197 over both ORCA and DH-ORCA. We will see in
1198 the next sub-section that the improvement is even
1199 more significant when the speed range is tighter.

1200 The left part of Figure 17 shows that the tra-
1201 jectory lengthening due to the D&A algorithm is
1202 really large for ORCA, and much smaller for CS-
1203 ORCA and DH-ORCA. This was expected, con-
1204 sidering the issues that were raised in section 5.1
1205 concerning ORCA when applied to aircraft fly-
1206 ing at constant or near-constant speeds. The right
1207 part of Figure 17 zooms on the results of the two
1208 best methods, and shows a clear advantage for
1209 DH-ORCA.

1210 The better performance of DH-ORCA on both
1211 the parallel tracks duration (Fig. 16) and the tra-
1212 jectory lengthening metric (Fig. 17) confirm that
1213 the dual horizon actually performs as expected,
1214 helping aircraft to cross paths more efficiently
1215 than with the two single-horizon methods.

1216 The normalized throughput shown on
1217 Figure 18 is closely related both to the additional
1218 time spent by aircraft in the area due to maneu-
1219 vering, and to the delays assigned to entering
1220 aircraft, due to other traffic maneuvering close
1221 to their entry point. Remind that the scenarios
1222 are designed such that no conflict occurs at entry
1223 when no D&A algorithm is used, but that delays
1224 may be assigned when a D&A algorithm modifies

the trajectories. When the cumulated delays are important, this reduces the throughput of the system. We see on Figure 18 that both CS-ORCA and DH-ORCA are able to maintain a nearly constant throughput ratio, whatever the traffic density, whereas the throughput drastically decreases with the traffic density (which is controlled through the incoming flow) when ORCA is used.

8.2 Influence of the aircraft speed distribution

In all the scenarios in this paper, every aircraft flies at a constant speed: only course changes are possible. The individual aircraft speeds are drawn in a uniform distribution centered on a same average speed (see Table 1 in section 7.3). In the current section, we investigate the influence of the speed range of the uniform distribution on the results.

As seen in Section 5.3, it may happen that the D&A algorithm solves a conflict by postponing it, setting the two involved aircraft on parallel courses. If the aircraft fly at different speeds, one will eventually overtake the other, and both aircraft will resume their navigation towards their respective destination. However, if they fly at the same speed, the relative position of the aircraft will stay the same: they are interlocked. This situation is more likely to occur when the speed diversity is low.

Let us now confirm that DH-ORCA actually mitigates this issue. When comparing DH-ORCA with CS-ORCA on a variety of speed ranges, DH-ORCA should perform better than CS-ORCA when the speed diversity is low. This is actually the case, as can be seen on Figures 19, 20, 21, 22, and 23 showing the evolution of the metrics with the speed range. In these figures, the mean values of the metrics are plotted along with 95% confidence intervals for the estimated means.

Figures 19 and 20 show that the trajectory lengthening and the parallel behavior total duration remain approximately constant with DH-ORCA, whatever the speed range, whereas with CS-ORCA they both dramatically increases when the speed diversity is low. Likewise, the throughput in Figure 21 is stable for DH-ORCA whereas it slightly drops for CS-ORCA as speed diversity decreases.

Incoming flow [aircraft/h], parallel_total_duration [s] (right axis)

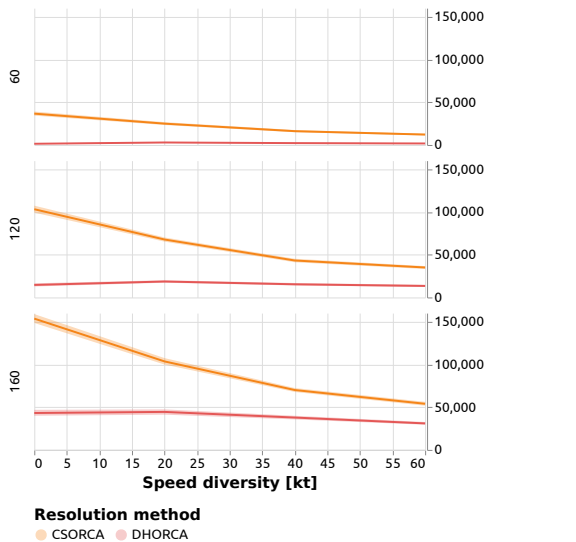


Figure 19: Evolution of the parallel behaviour total duration with the speed range.

Incoming flow [aircraft/h], normalized_length [-] (right axis)

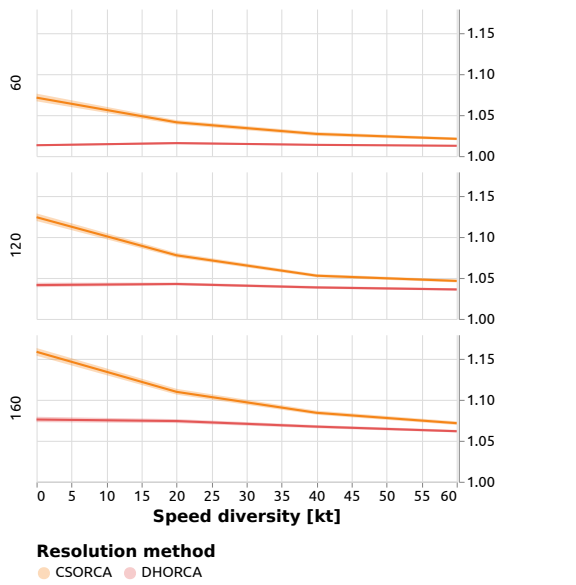


Figure 20: Evolution of the normalized trajectory lengthening with the speed range.

We can conclude that the pathological behavior for aircraft flying at similar constant speeds—*i.e.* aircraft being set on parallel tracks—that occurred when using ORCA is still observed with CS-ORCA and the more so with low speed diversity. Although it still exists with DH-ORCA, this

Incoming flow [aircraft/h], throughput [-] (right axis)

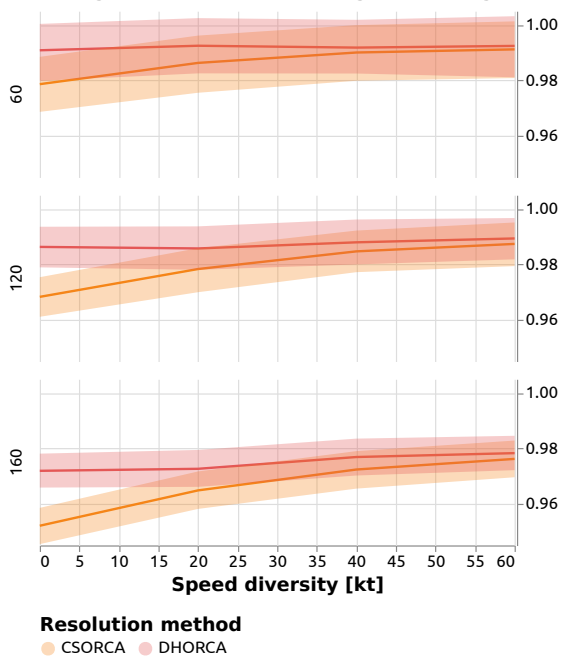


Figure 21: Evolution of the normalized throughput with the speed range.

Incoming flow [aircraft/h], normalized_LoS [s] (right axis)

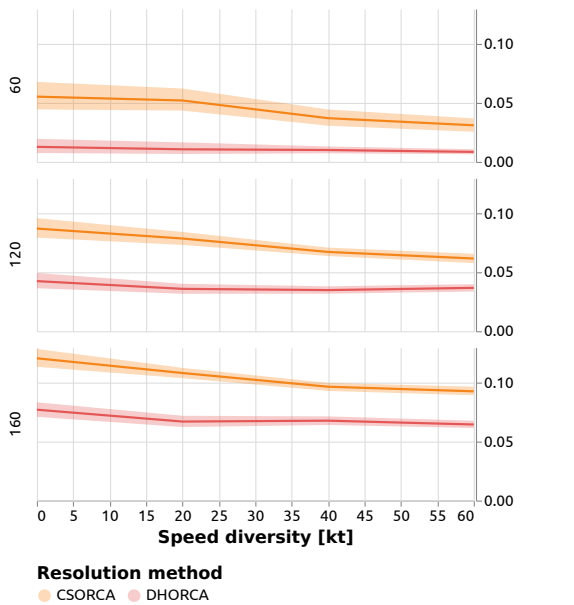
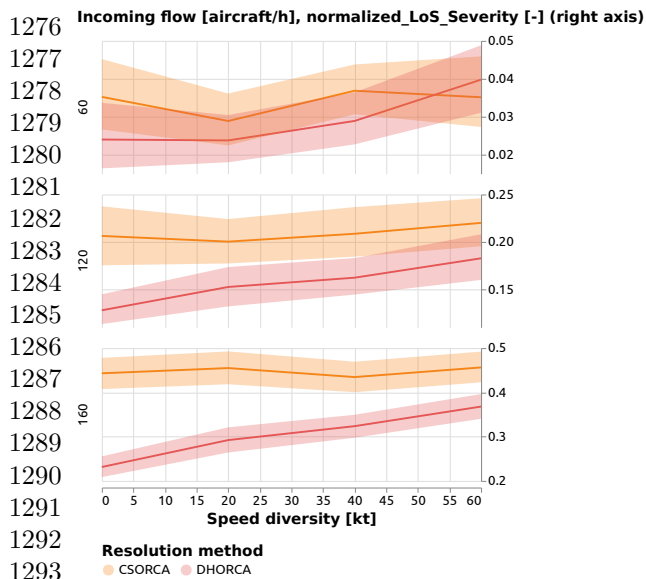


Figure 22: Evolution of the normalized duration of separation losses with the speed range.

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1294 **Figure 23:** Evolution of the normalized severity
1295 of the separation losses with the speed range.

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1297 pathological behavior is significantly mitigated by
1298 the use of a dual horizon.

1300 Concerning the safety metrics (duration of separation losses and separation loss severity), we see on Figures 22 and 23 that DH-ORCA consistently fares better than CS-ORCA on all traffic densities and speed ranges. On Figure 22, we can also observe that DH-ORCA is somewhat less sensitive to the speed diversity than CS-ORCA, in terms of conflict duration.

1308 Looking at Figure 23, we can see that the remaining separation losses are less severe with DH-ORCA than with CS-ORCA for the `normalizedLoSSeverity` metric. Considering the evolution of the severity with the speed range, we see that the advantage of using DH-ORCA is even more apparent when aircraft fly at very similar speeds.

1317 9 Conclusion

1319 To conclude, we have seen that the original Optimal Reciprocal Collision Avoidance (ORCA)
1320 algorithm, initially proposed in van den Berg et al
1321 (2011) for robot collision avoidance, exhibits a
1322 pathological behavior when applied for Detect &
1323 Avoid purposes to aircraft or UAS flying at similar
1324 constant speeds. The Constant-Speed ORCA
1325 (CS-ORCA) variant of the algorithm introduced

in Durand (2018) was designed as a way to solve this issue.

In the current paper, we have exposed why CS-ORCA might still exhibit the undesired behavior where two flights converging at a small angle might be set on parallel tracks by the algorithm, preventing them to cross paths to reach their respective destinations. We have shown that CS-ORCA is less subject to this issue when using a large value of the time horizon parameter τ . However, our results in section 8.2 show that this issue still remains when using CS-ORCA, even though we selected the τ value in section 7.5 to get the best possible results on the kind of scenarios considered in this study.

We have proposed a new algorithm Dual-Horizon ORCA (DH-ORCA) combining a short-term ORCA-like logic with a longer-term CS-ORCA-like logic with optional constraints. In DH-ORCA, a relatively small time horizon τ_{conflict} is used to enforce separation, and a larger time horizon τ_{cross} is used to help aircraft (and/or UAVs) to cross paths. The optional constraints computed with the larger time horizon τ_{cross} are designed so that each aircraft only requires its own intent to decide to enforce the constraint or not: There is no need for any aircraft to broadcast its destination to the others.

Our results show that DH-ORCA outperforms both ORCA and CS-ORCA on scenarios of various traffic densities. They also show that the gain in safety and efficiency of DH-ORCA is most important when the speed diversity is low, as expected.

The results of our study were obtained on traffic scenarios where all aircraft fly at similar, constant speeds, in order to make the undesired behavior described above more likely to occur. Aircraft flying at similar speeds within a very limited speed range is actually a common feature of the commercial traffic in the upper airspace, although these aircraft usually fly at greater speeds than in our study. In the lower airspace, the air traffic that we may envision in the future will probably also include a number of UAVs and/or aircraft operating at similar speeds—closer to the speed ranges of our scenarios—and for which a speed modification is either of very limited range (e.g., large fixed-wing UAVs) or costly in terms of energy budget (e.g., multi-rotor or hybrid UAVs).

So the Detect & Avoid logic that could be deployed in the future on these aircraft or UAVs should be safe and efficient in all situations, including when aircraft and/or UAVs flying at similar speeds converge at a small angle. We have shown that DH-ORCA can fulfill this objective with greater safety and efficiency than ORCA or CS-ORCA.

However, none of the geometric algorithms studied in this paper provides a formal guarantee to avoid the parallel tracks degenerate situations. Although they are much less likely to occur with DH-ORCA than with ORCA or CS-ORCA, these situations still happen in dense traffic and we cannot be sure they will never occur even with low traffic.

Further research could focus on finding an algorithm that would provide such formal guarantees. One could also examine the robustness of geometric algorithms such as ORCA or its variants CS-ORCA and DH-ORCA, and possibly other D&A logics, in more realistic operational conditions where all aircraft and/or UAVs might not share the exact same representation of the traffic, and might have slightly different implementations of the Detect & Avoid logic.

Declarations

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- **Conflict of Interest/Competing Interests** The authors have no financial or proprietary interests in any material discussed in this article.
- **Data Availability** Not applicable, as no real data have been used to realize this work.
- **Code availability** The custom code used during the current study is available from the corresponding author on reasonable request.
- **Authors' Contributions** Richard Alligier: Conceptualization, Data curation, Formal analysis, Investigation, Methodology, Software, Validation, Visualization, Writing – original draft, Writing – review & editing. David Gianazza: Conceptualization, Methodology, Software, Writing – Original Draft, Writing – Review & Editing, Supervision. Nicolas Durand: Conceptualization, Methodology, Supervision, Writing – original draft, Writing – review &

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- **Consent to Participate** Not applicable.
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