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Slot allocation in a single airport aims to maximize the utilization of airport declared capacity, while slot allocation in a multi-airport system (MAS) has to take airspace capacity into account. Because the limited capacity of certain departure/arrival fixes in the terminal airspace can cause unnecessary flight delays. The uncertainty of flying time between airport and congested fixes makes it even more complicated for slot allocation in a MAS. Traffic flow may be over capacity when the flying times of flights change. In this paper, we propose a mixed integer-programming model for slot allocation in a MAS. The objective of the model is to minimize the total displacements of flights in the MAS while considering all the capacity constraints as well as the uncertainty of flying time. The constraints at departure/arrival fixes are transformed into chance constraints, and Lyapunov theorem is applied for the transformation. To test the proposed model, a case study of schedule optimization in the MAS of Guangdong-Hong Kong-Macao Greater Bay is presented. Specifically, the impact of the uncertainty of flying time from five airports to airspace fix YIN is investigated. Results show that the total displacements increased if the uncertainty of flying time was considered. The optimized schedule, however, is more robust which can satisfy capacity constraints in various scenarios.

Key Words: Slot Allocation, Multi-airport System, Uncertainty Model, Chance Constraint

1. Introduction

Slot allocation is an important means of air traffic flow management which can effectively balance air transport demand and airport capacity. At a strategic level, slot allocation for a single airport can improve the efficiency and effectiveness of the utilization of existing resources. Over the decades, the constraints and dependencies between different airports in a multi-airport system (MAS) have become more prominent, mainly due to the conflict in using shared departure/arrival fixes and routes. Traditionally, slot allocation has to consider airspace capacity. However, there may conflict between flights from different airports at a shared departure/arrival fix when allocating slots for each airport individually. Thus, it is necessary to consider all the airports at the same time for slot allocation to alleviate congestion and delay in the whole region.

The Estimated Time Over (ETO) fix of flight has to be calculated from the planned departure/arrival time of flights and the route flying time. The flying time varies due to traffic flow management, weather, airport congestion, etc., which makes the ETO uncertain. It leads to the fact that the flight flow through the fix within a certain time window varies.

In this paper, we propose a model that considers both airport declared capacity and airspace(fix) capacity, as well as the uncertainty of flying time for slot allocation in a MAS.

2. Literature review

The earliest work on MAS may be traced back to 2008 when the U.S. Joint Development and Planning Office (JDPO) first defined a multi-airport,[1)]as a cluster of airports served by multiple airports that are in close geographical proximity and interdependent on each other for inbound and outbound air routes, such that MAS forms a metroplex (a combination of metropolitan and complex).

Slot management problems can be classified into three categories: strategic, tactical, and pre-tactical. Most scholars studied the slot allocation problem at the tactical level to determine the best strategy for dynamically matching air traffic demand and capacity. In 2017, Zografos et al. conducted a review of the current research on the slot allocation problem at the strategic stage and the declared capacity modeling. They pointed out there is the need for further research to explore optimization objectives of the model and the need to enrich them with equity, resource utilization and environmental considerations. Future models can be incorporate airlines' tolerance and acceptability of flight allocation results, or to develop stable and feasible models that capture the complexity and dynamics of airport operations and weather uncertainty.[2] In 2018, Ribeiro et al. proposed a novel priority-based multi-objective slot allocation model (PSAM, priority -based multi-objective flight time allocation model), which is compliant with IATA regulations.[3)] In 2019, Z. Ye et al. developed a linear integer programming model to determine whether implementation difficulty can be used as a new mechanism to weaken grandfathering rights for the slot allocation. The results showed that by weight setting, the implementation difficulty can be significantly reduced without causing excessive shifts and disruptions in existing priorities.[4] In 2021, Zografos et al. considered the blocking problem of assigning flight moments at coordinated airports and proposed a new model for slot assignment that considers the regularity of schedules by limiting the range of assigned times. In 2022, Shui Xiaoyu et al. established a slot allocation model considering the fairness between airports, and used the Gini coefficient as a measure to compare different optimization plans based on peak demand and off-peak demand.[5]

Research work on uncertainty in air traffic flow management dates back to 2000s. In 2003, Wanke et al. developed a model for airspace demand forecasting analysis based on prototype TFM decision support system observations that quantifies NAS airspace demand forecasting. The statistical distribution of forecast errors allows the development of new techniques to display and exploit sector demand forecasts with known uncertainties.[6] In 2008, Hafner et al. tried to improve the predictability of airline schedules by enhancing the capability of flight schedules to resist Ground Delay Program (GDP) and inclement weather. They proposed strategic search heuristics to greatly improve the reliability of airline schedules by assigning airport takeoff and landing time slots to each flight in a schedule in a network of airports.[7] In 2011, Agogino et al. investigated linear programming models and nonlinear evolutionary algorithms for solving large-scale air traffic optimization problems. They argued that both methods suffer from perfection assumptions and that model robustness may be flawed when there is uncertainty in the problem.[8] In 2014, Luca Corolli et al. developed two stochastic programming models based on the certainty single-airport model proposed by Zografos et al [9] . Then the model is applied to European airports, where a set of test cases showed a total of 58% reduction in delays.[10] In 2020, Jacquillat addressed the problem of tactical operational procedures and strategic planning interventions being treated in isolation in air traffic management. An integrated model of airport network strategy and tactics is proposed that jointly optimizes planning interventions and ground waiting operations across a network of airports under operational uncertainty.[11]

In summary, many studies have addressed slot allocation problems for a single airport, or studied tactical air traffic flow management problems under uncertainty. However, no study has been done on slot allocation in a MAS that consider the uncertainty of flying time in the terminal airspace. In this paper, we developed a robust programming model to improve the overall adaptability and robustness of the optimization scheme.

3. Model

The objective of the model in this paper is to minimize the total displacements of flights given constraints from airport declared capacity, the capacity of key fixes in the airspace and the operating rules, and the uncertainty of flying times on the routes. The following reasonable assumptions are made before proceeding with the modeling.

Assumption 1. The slot in this paper does not refer to a point in time, but a time slice. Each segment has a start time and length. Airports can use these time periods to land and take off flights.

Assumption 2. The turnaround time of a flight must not be greater than the maximum turnaround time or less than the minimum turnaround time. The turnaround time is the time interval between the arrival of the preceding flight and the departure of the following flight, during which operations such as passenger loading and unloading, aircraft refueling, etc. need to be completed.

Assumption 3. We only consider flights from airports within the MAS. We do not consider the flight takeoff and landing times and turnaround times at airports outside the MAS.

Assumption 4: Fix capacity has to be calculated from historical data. We use traffic volume during periods with significant flight delays as the capacity.

Assumption 5. The frequency of different flying time observed in the historical data is equal to the probability of occurring in the future. Thus, the probability of flying time can be obtained by fitting the historical data.

3.1. Model Description

The inputs of the model in this paper are 1) the original flight schedule of the MAS, 2) the capacity of each airport and fix, and 3) the flying time from each airport to the fix.

The output is the optimized flight schedule. The decision variables of the model are set as follows

x_{it}^{arr} / x_{it}^{dep} :0-1 variable, $x_i = 1$ means that flight *i* is scheduled to arrive/depart no earlier than time slice *t* after optimization; $x_i = 0$ otherwise. $i \in F, t \in T$

Δ_i : The total displacements of flight *i*. The unit of displacement is 5 minutes. $i \in F$

The other model parameters are shown in Table 1.

Table 1. Model notation and description.

Notation	Description
T	Set of time slots throughout the day. $t = 1,2,3... \in T$
F	Set of flights in the MAS.
Q	Set of fixes in the MAS.
$\mathbb{F}$	Set connecting flight pairs in the MAS.
F_k^{arr} / F_k^{dep}	Set of arrival/departure flights of airport *k* in the MAS. $F_k^{dep}, F_k^{arr} \subset F$

F_q^{arr} / F_q^{dep}	Set of arrival/departure flights in the MAS passing fix *q*.
K	Set of airports in the MAS.
τ_{ij}^1	Minimum turnaround time between connecting flights. $(i,j)\in\mathbb{F}$
τ_{ij}^2	Maximum turnaround time between connecting flights. $(i,j)\in\mathbb{F}$
ω_{it}^{arr} / ω_{it}^{dep}	1 if arrival/departure time of flight *i* is not earlier than time slice *t* before optimization; 0 otherwise. $i\in F, t\in T$
S_i^{arr} / S_i^{dep}	Arrival/Departure time slot that flight *i* is scheduled before optimization. $i\in F$
k_i^{arr} / k_i^{dep}	The airport of arrival/departure flight *i*. $i\in F$
ft_{kq}^{arr} / ft_{kq}^{dep}	Fly time of arrival/departure route between airport *k* and fix *q*. $k\in K, q\in Q$
NK_{kt}^{arr} / NK_{kt}^{dep}	Arrival/Departure flight number of airport *k* in time period *t*. $k\in K, t\in T$
NQ_{qt}^{arr} / NQ_{qt}^{dep}	Arrival/Departure flight number of fix *q* in time period *t*. $q\in Q, t\in T$
C_{kt}^{arr} / C_{kt}^{dep} / C_{kt}^{all}	Arrival/Departure/Total capacity of airport *k* in time period *t*. $k\in K, t\in T$
D_{qt}	Total capacity of fix *q* in time period *t*. $q\in Q, t\in T$
μ_{iqt},σ_{iqt}^2	Mathematical expectation and variance of chance that flight *i* passes fix *q* in time period *t* . $i\in F, q\in Q, t\in T$
μ_{qt},σ_{qt}^2	Mathematical expectation and variance of traffic passing fix *q* in time period *t*. $q\in Q, t\in T$
eta_i / etd_i	Optimized scheduled arrival/departure slot of flight *i*.
t_m	The last time slot that flights must be adjusted to in slot allocation.

3.2. Objective

The objective of the model is to minimize the total displacements of all the flights. The calculation of the displacements for each flight is given by Eq. (2).

$$\min Z = \sum_{i\in F}\Delta_i \tag{1}$$

$$\Delta_i = \begin{cases} \sum_{t\in T}\left|x_{it}^{arr}-\omega_{it}^{arr}\right|, & i\in F^{arr} \\ \sum_{t\in T}\left|x_{it}^{dep}-\omega_{it}^{dep}\right|, & i\in F^{dep} \end{cases} \tag{2}$$

3.3. Certainty constraints

3.2.1. Existence of flights

Eq. (3)-(6) ensure that decision variables are set to an initial of 1 and ending of 0 throughout the day. Each flight is assigned at least one time slot.

$$x_{i,1}^{dep}=1 \quad \forall i\in F^{dep} \tag{3}$$

$$x_{i,t_m}^{dep}=0 \quad \forall i\in F^{dep} \tag{4}$$

$$x_{i,1}^{arr}=1 \quad \forall i\in F^{arr} \tag{5}$$

$$x_{i,tm}^{arr}=0, \forall i\in F^{arr} \tag{6}$$

3.2.2. Uniqueness of flights

Constraints (7) and (8) are flight uniqueness constraints. The decision variables must be monotonically decreasing with time *t*, ensuring that each flight is assigned at most 1 time slot.

$$x_{i,t}^{dep}\ge x_{i,t+1}^{dep} \quad \forall t\in T, \forall i\in F^{dep} \tag{7}$$

$$x_{i,t}^{arr}\ge x_{i,t+1}^{arr} \quad \forall t\in T, \forall i\in F^{arr} \tag{8}$$

3.2.2. Turnaround constraints for connecting flights

Constraints (9) and (10) are connecting flight turnaround constraints. For connecting flights, Eq. (9) limits the turnaround time to be greater than the minimum connecting time. Eq. (10) limits the turnaround time to be less than the maximum connecting time.

$$\sum_{t\in T}\left(x_{i,t}^{arr}-x_{j,t}^{dep}\right)\ge\tau_{ij}^1 \quad \forall (i,j)\in\mathbb{F} \tag{9}$$

$$\sum_{t\in T}\left(x_{i,t}^{arr}-x_{j,t}^{dep}\right)\le\tau_{ij}^2 \quad \forall (i,j)\in\mathbb{F} \tag{10}$$

3.2.3. Airport capacity constraints

Each airport has its arrival/departure declared capacity that set the limitation of slots. The number of arrival and departure flights at airport *i* in a time period cannot exceed the specified capacity. Eq. (11) and Eq. (12) give the calculation of the arrival and departure traffic for each time period of airport. Eq. (13) is the arrival capacity limit of airport. Eq. (14) is the departure capacity limit of airport, and Eq. (15) is the total capacity limit of the airport.

$$NK_{kt}^{arr}=\sum_{i\in F_k^{dep}}\left(x_{i,t}^{arr}-x_{i,t+1}^{arr}\right) \tag{11}$$

$$NK_{kt}^{dep}=\sum_{i\in F_k^{dep}}\left(x_{i,t}^{dep}-x_{i,t+1}^{dep}\right) \tag{12}$$

$$NK_{kt}^{arr}\le C_{kt}^{arr} \quad \forall t\in T, \forall k\in K \tag{13}$$

$$NK_{kt}^{dep}\le C_{kt}^{dep} \quad \forall t\in T, \forall k\in K \tag{14}$$

$$NK_{kt}^{arr}+NK_{kt}^{dep}\le C_{kt}^{all} \quad \forall t\in T, \forall k\in K \tag{15}$$

3.2.4. Fix capacity constraints

There are shared arrival and departure fixes in the MAS. The capacity of fix determines the number of flights that fix can serve per unit time. Eq. (16) and Eq. (17) give the calculation of the arrival and departure flight traffic passing fix q in time period t. In this case, the passing time of the arrival/departure flight is derived from the planned time of the flight and the flying time from the airport to the fix. Eq. (18) is the capacity constraint of fixes. Again, the arrival and departure traffic cannot exceed the capacity of the fix in any time period.

$$NQ_{qt}^{arr} = \sum_{i \in F_q^{arr}, k=k_i^{arr}} \left(x_{i,t+ft_{kq}^{arr}}^{arr} - x_{i,t+1+ft_{kq}^{arr}}^{arr} \right) \tag{16}$$

$$NQ_{qt}^{dep} = \sum_{i \in F_q^{dep}, k=k_i^{dep}} \left(x_{i,t-ft_{kq}^{dep}}^{dep} - x_{i,t+1-ft_{kq}^{dep}}^{dep} \right) \tag{17}$$

$$NQ_{qt}^{arr} + NQ_{qt}^{dep} \le D_{qt}^{all} \quad \forall t \in T, \forall q \in Q \tag{18}$$

3.3. Uncertainty constraints

3.3.1. Chance Constraint Establishment

In Eq. (16) and Eq. (17), the arrival and departure traffic passing a certain fix in a certain time period depend only on the decision variables. However, in practice, whether arrival or departure, the airport-to-fix flying time is not a certainty value, but a random variable following certain distributions. Therefore, for a specific set of decision variables, the arrival and departure traffic of a certain fix in a certain time window is also a random variable. Based on the above discussion, we will use the chance constraints in robust optimization method to solve the uncertainty problem. Where α is the violation probability of the chance constraint, showing the degree of violation that the decision maker may accept.

$$P\left(NQ_{qt}^{arr} + NQ_{qt}^{dep} \le D_{qt}^{all} \right) \ge 1-\alpha \quad \forall t \in T, \forall q \in Q \tag{19}$$

3.3.2. Chance constraint transformation

From equations (16), (17) and (18), we have

$$\begin{aligned} & P\left(NQ_{qt}^{arr} + NQ_{qt}^{dep} \le D_{qt}^{all} \right) \\ & = P\left(\begin{aligned} & \sum_{i \in F_q^{arr}, k=k_i^{arr}} \left(x_{i,t+ft_{kq}^{arr}}^{arr} - x_{i,t+1+ft_{kq}^{arr}}^{arr} \right) + \\ & \sum_{i \in F_q^{dep}, k=k_i^{dep}} \left(x_{i,t-ft_{kq}^{dep}}^{dep} - x_{i,t+1-ft_{kq}^{dep}}^{dep} \right) \le D_{qt} \end{aligned} \right) \end{aligned} \tag{20}$$

For $\left(x_{i,t+ft_{kq}^{arr}}^{arr} - x_{i,t+1+ft_{kq}^{arr}}^{arr} \right)$ and $\left(x_{i,t-ft_{kq}^{dep}}^{dep} - x_{i,t+1-ft_{kq}^{dep}}^{dep} \right)$:

They can be considered as random variables obeying some distribution. According to Lyapunov theorem, when n is large, the sum of n random variables approximately obeys a normal distribution, which can be expressed as Eq. (21).

$$\begin{aligned} & \sum_{i \in F_q^{arr}, k=k_i^{arr}} \left(x_{i,t+ft_{kq}^{arr}}^{arr} - x_{i,t+1+ft_{kq}^{arr}}^{arr} \right) \\ & + \sum_{i \in F_q^{dep}, k=k_i^{dep}} \left(x_{i,t-ft_{kq}^{dep}}^{dep} - x_{i,t+1-ft_{kq}^{dep}}^{dep} \right) \sim N\left(\mu_{qt}, \sigma_{qt}^2 \right) \end{aligned} \tag{21}$$

As a result, the constraint of Eq. (20) can be transformed as Eq. (22).

$$\begin{aligned} & P\left(\begin{aligned} & \sum_{i \in F_q^{arr}, k=k_i^{arr}} \left(x_{i,t+ft_{kq}^{arr}}^{arr} - x_{i,t+1+ft_{kq}^{arr}}^{arr} \right) \\ & + \sum_{i \in F_q^{dep}, k=k_i^{dep}} \left(x_{i,t-ft_{kq}^{dep}}^{dep} - x_{i,t+1-ft_{kq}^{dep}}^{dep} \right) \le D_{qt} \end{aligned} \right) \\ & = P\left(\frac{NQ_{qt}^{arr} + NQ_{qt}^{dep} - \mu_{qt}}{\sigma_{qt}^2} \le \frac{D_{qt} - \mu_{qt}}{\sigma_{qt}^2} \right) \\ & = \Phi\left(\frac{D_{qt} - \mu_{qt}}{\sigma_{qt}^2} \right) \\ & where \quad \mu_{qt} = \sum_{i \in F_q} \mu_{iqt}, \quad \sigma_{qt}^2 = \sum_{i \in F_q} \sigma_{iqt}^2 \end{aligned} \tag{22}$$

Where Φ denotes the distribution function of the standard normal distribution. Eq. (25) and Eq. (26) give the solutions for the expectation and variance of the traffic passing fix q in time period t.

$$let \quad x_{i,t+ft_{kq}^{arr}} - x_{i,t+1+ft_{kq}^{arr}} = {}_{\Delta}x_{i,t+ft_{kq}^{arr}}$$

$$and \quad eta_i = \sum_{t \in T} t \cdot {}_{\Delta}x_{i,t+ft_{kq}^{arr}}$$

$$\begin{aligned} \mu_{iqt} & = E\left({}_{\Delta}x_{i,t+ft_{kq}^{arr}} \right) \\ & = P\left(ft_{kq}^{arr} = eta_i - t \right) \times 1 + P\left(ft_{kq}^{arr} \ne eta_i - t \right) \times 0 \\ & = P\left(ft_{kq}^{arr} = eta_i - t \right) \quad \forall i \in F^{arr}, t \in T \end{aligned} \tag{23}$$

$$\begin{aligned} \sigma_{it}^2 & = E\left(\left({}_{\Delta}x_{i,t+ft_{kq}^{arr}} \right)^2 \right) - \left(E\left({}_{\Delta}x_{i,t+ft_{kq}^{arr}} \right) \right)^2 \\ & = \mu_{it} - \mu_{it}^2 \end{aligned} \tag{24}$$

Thus, Eq. (19) can be transformed into Eq. (25).

$$\Phi\left(\frac{D_{qt} - \mu_{qt}}{\sigma_{qt}^2} \right) \ge 1-\alpha \tag{25}$$

Eq. (26) can be obtained from the distribution function of the standard normal distribution.

$$\frac{D_{qt} - \mu_{qt}}{\sigma_{qt}^2} \ge \Phi^{-1}\left(1-\alpha \right) \tag{26}$$

3.5. Solution approach

The model is implemented using Python with Gurobi solver. The optimization problem in this paper is mixed-integer programming, which is one of the non-convex programming. The parameter "NonConvex" is set to 2. The model is run on Window 10 with 64-bit operating system with 16GB RAM and i7 processor.

4. Case study

4.1. Experiment setup

4.1.1. Experiment subject

In this paper, the Guangdong-Hong Kong-Macao Greater Bay Area of China is selected for the study. The MAS includes five airports, Guangzhou Baiyun International Airport (ZGGG), Shenzhen Baoan International Airport (ZGSZ), Zhuhai Jinwan Airport (ZGSD), Macau International Airport (VMMC) and Huizhou Pingtan Airport (ZGHZ). Since the flights of Hong Kong International Airport are basically isolated from other airports, they are not considered. Figure 1 shows the structure of airspace of the MAS. It can be seen that the airspace of the MAS is complex. Several airports have to share use of a single departure/arrival fix.

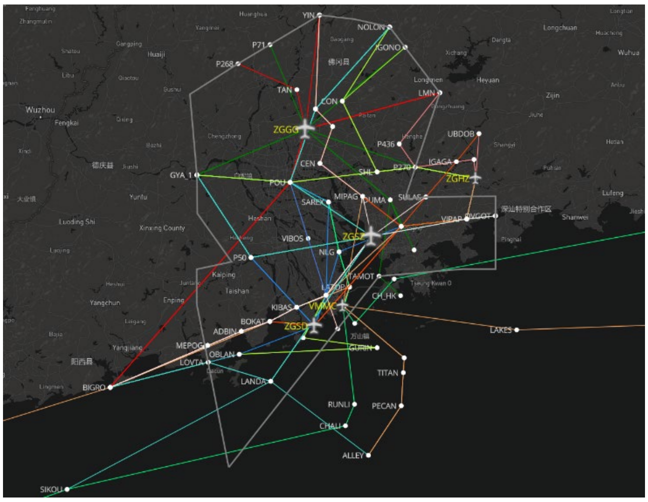

Fig. 1. MAS in the Guangdong-Hong Kong-Macao Greater Bay Area.

The flight schedule on December 21, 2019 is obtained as the model input. Statistically, a total of 2,969 flights were planned for the MAS including 1,494 arrival flights and 1,475 departure flights.

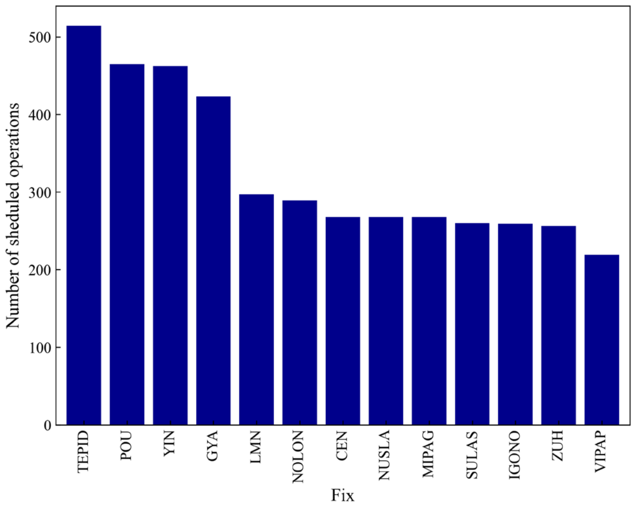

Fig. 2. Fix traffic without optimization (only part shown).

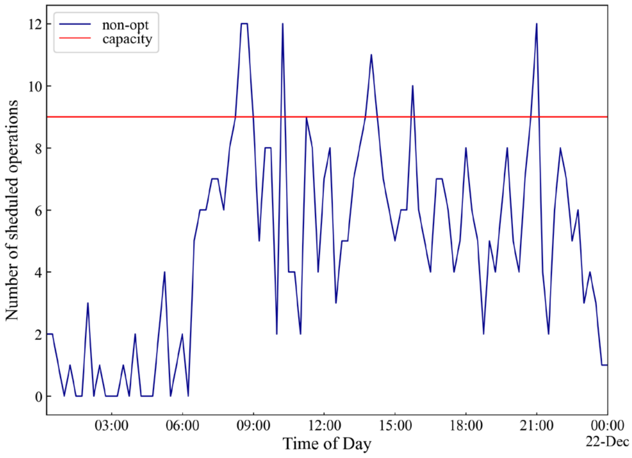

Fig. 3. Traffic change in YIN.

The departure fix, YIN, in the MAS, is selected for the study of flying time uncertainty because of the following considerations.

First, a total of 462 flights passed through YIN on December 21, 2019. It can be seen from Fig. 2 that the flight traffic passing through YIN point ranks the third and the traffic accounts for a relatively high percentage.

Second, YIN is a common departure point for five airports in the MAS. Thus it is representative to select this point for the optimization considering flying time uncertainty in the MAS.

Third, YIN has a high number of flights during peak hours and exceeds the capacity constraint frequently. Effectively solving the flying time uncertainty to this point will provide a reference for other fixes. Figure 3 shows the traffic of YIN before optimization, where the flying time from the airport to the fix is calculated using the median of historical data. From the figure, it is clear to see that the traffic of YIN fix is very high and full of ups and downs, and there are many points where the capacity limit is exceeded.

4.1.2. Parameter setup

In this paper, the length of each slot is set to 5 minutes. The airport capacity is set according to the declared capacity of the airport; the fix capacity is set with reference to China's *Technical Specification for Airport Moment Capacity Assessment*. The time window of capacity is divided into 15-minute capacity and 1-hour capacity. The minimum connecting time of flights is set to 30 minutes and the maximum connecting time is set to 180 minutes according to China's *Civil Aviation Normal Flight Statistics Measures*. The probability distribution of flying time from airports to fixes is obtained by fitting the actual historical flight operation data of December 2019. The frequency distribution of flying time from ZGSZ to YIN for departing flights is shown in Fig. 4. It can be seen that the flying time from ZGSZ to YIN presents an obvious form of multi-peak distribution, showing the uncertainty of flying time.

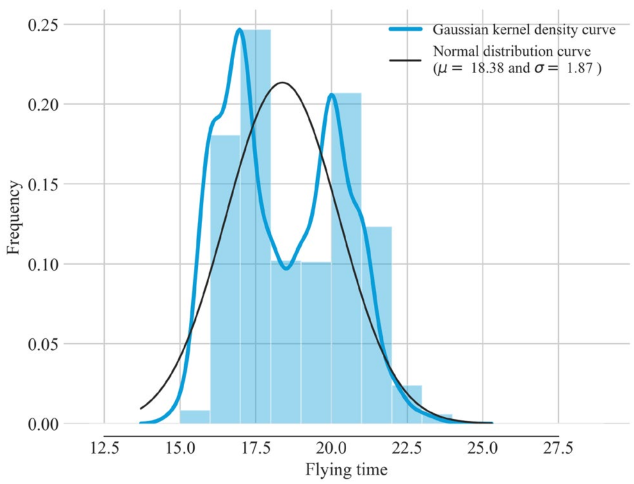

Fig. 4. Flying time distribution of departure flights from ZGSZ to YIN.

For the uncertainty constraint, we counted the flying time from each airport to YIN. According to the historical distribution, two cases with equal probability are divided. Each case corresponds to a value of flying time. For the certainty constraint, the flying time is taken as the median of the historical data. The range of flying time for the uncertainty constraint and the certainty condition are shown in Table 2.

Table 2. The flying time from different airports to YIN. (unit: 5 minutes)

Airport	uncertainty	certainty
VMMC	{4,5}	5
ZGGG	{2,3}	3
ZGHZ	{4,5}	5
ZGSD	{5,5}	5
ZGSZ	{4,4}	4

We adjust the violation probability to different values to examine the optimization results under different acceptance levels. We set the certainty model output as the control group, and then obtain a total of four different optimization schedules. The differences between the different optimization strategies are analyzed in the following section.

4.2. Results

4.1. Slot displacements

The total schedule displacements corresponding to the four different optimization schemes are shown in table 3. We can see that the total displacements are gradually increased with the decreasing of violation probability. This is mainly caused by the increasingly strict conditions of the chance constraint.

In the certainty scenario, the flying times from each airport to the fix are constant values. Thus, the ETO is completely determined by the flight's departure/arrival time. Therefore, a flight will only play an impact on the traffic within a specific time window of the fix.

Under uncertainty constraint, the flying time from each airport to the fix is uncertain, and the ETO is determined by both the departure/arrival time and the flying time. This leads to the situation that a single flight may affect more than one time window of the fix. Therefore, considering possible flying time in uncertainty optimization could make the model more conservative, while the total displacements increase as the probability of violation decreases.

In theory, the violation probability should be as small as possible. However, it is necessary to consider the economic cost of the total displacements due to different violation probabilities. In this paper, after verifying the optimization results of various values of violation probability, we found that α=0.4 can basically satisfy the capacity constraint in various scenarios, and α=0.3 can completely satisfy the capacity constraint in various scenarios. Therefore, a smaller value of α is no longer necessary, otherwise it will only increase the economic cost brought by unnecessary flying time displacements.

Table 3. Optimization results of the four schemes. (unit: 5 minutes)

Schemes	Displacements
Scheme 1(certainty)	570
Scheme 2(α=0.4)	647
Scheme 3(α=0.3)	756
Scheme 4(α=0.2)	975

4.2. Traffic at departure fix YIN

The purpose of considering flying time uncertainty is to enhance the robustness of the optimization results. The optimized flight schedule should satisfy the capacity constraint of fixes with different flying time for different operation scenarios. To examine this effect, a total of 8 experimental scenarios S1 to S8 are set up, and the flying time from the airport to the fix differ in the 8 experimental scenarios, as shown in Table 4.

Table 4. Flying time under different experimental scenarios. (unit: 5 minutes)

Airport	S1	S2	S3	S4	S5	S6	S7	S8
VMMC	5	4	5	4	5	4	5	4
ZGGG	3	3	2	2	3	3	2	2
ZGHZ	5	5	5	5	4	4	4	4
ZGSD	5	5	5	5	5	5	5	5
ZGSZ	4	4	4	4	4	4	4	4

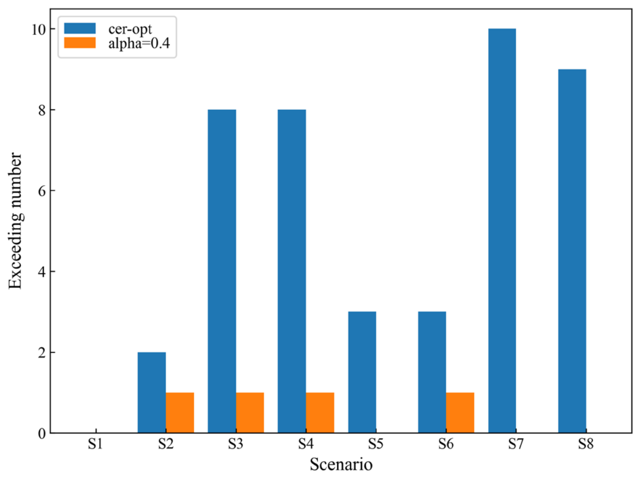

Fig. 5. Traffic exceeding the capacity of YIN.

We tested the four schemes in different experimental scenarios. We investigated whether different optimization plans in different experimental scenarios will cause the traffic to exceed the capacity. We found that the optimization results of scheme 3 and 4 can be perfectly cover all possible scenarios. Schemes 1 and 2 may have traffic exceeding the capacity. In different scenarios, the flights that exceed the capacity of scheme 1 and scheme 2 are shown in Fig. 5.

It can be seen from the figure that the traffic is over capacity in 7 scenarios from S2 to S8 in the basic model. While the optimized results of scheme 2 only slightly exceeds the capacity in a few cases. Even if the traffic exceeds the fix capacity, the number of flights is relatively small. In the following, we select S1, S6 and S8 for further analysis. The fix traffic changes corresponding to S1, S6 and S8 are shown in Figs. 6, 7 and 8.

In the figures, non-opt represents the original traffic calculated from schedule. Cer-opt represents the traffic obtained by basic model, while uncer-opt-alpha=0.4/0.3/0.2 represents the traffic with uncertainty optimization corresponding to different values of α. Observing the traffic change in YIN in different operation scenarios with different optimization schemes, we can have the following conclusions.

For scenario 1, the traffic with certainty optimization and uncertainty optimization does not exceed the capacity of the fix.

However, for S6 and S8, the traffic with certainty optimization exceeds the capacity of the fix, while the traffic with uncertainty optimization hardly exceeds the capacity. Only traffic in scheme 2 under S6 exceeds by one flight. From this it can be concluded that when the violation probability in the chance constraint reaches a certain threshold, even if there is still a violation probability, there will be no violation of the capacity constraint in actual operation.

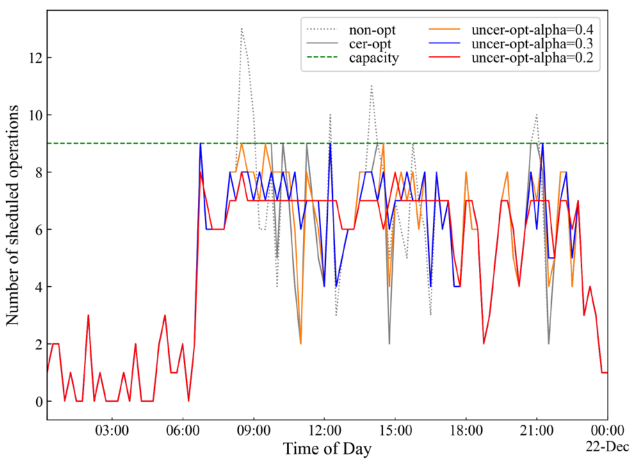

Fig. 6. Traffic in YIN under S1.

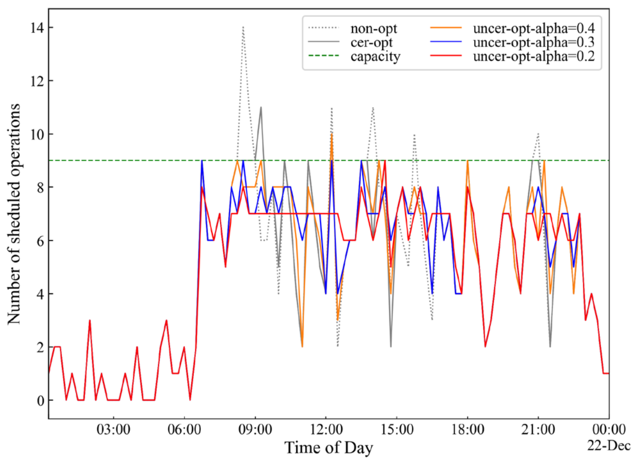

Fig. 7. Traffic in YIN under S6.

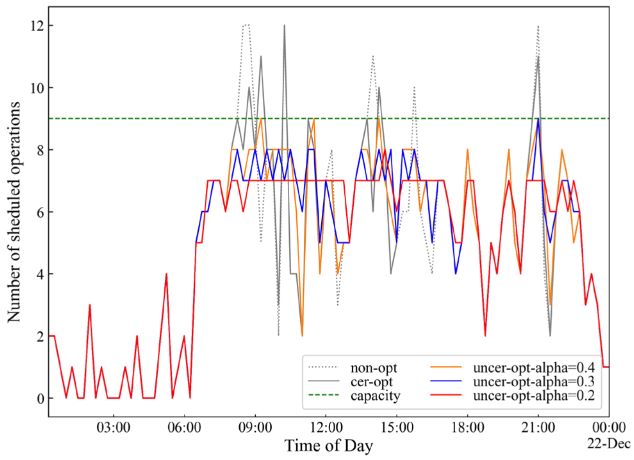

Fig. 8. Traffic in YIN under S8.

4.3. Exploration of violation probability value

We further explore the impact of different values of the violation probability on the results, and we obtain the relationships between the total displacements and the violation probability α as shown in Fig. 9. As can be seen from the figure, the objective function changes in a step-like manner with the probability of violation, because the number of flights within 15 minutes is not continuous variables, but discrete variables.

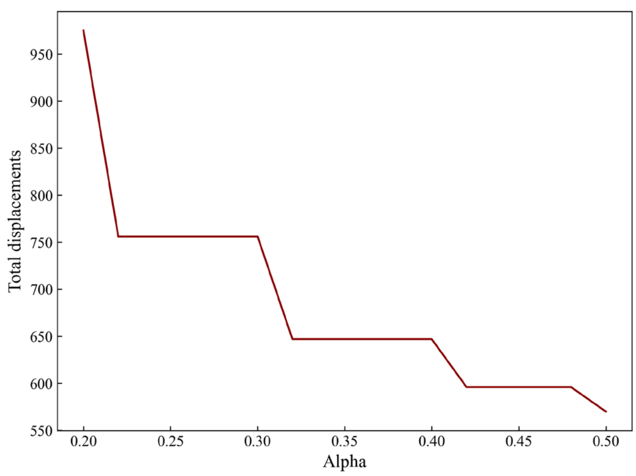

Fig. 9. The relationships between total displacements and α.

5. Conclusion

In this paper, we developed a chance-constraint model that takes the uncertainty of route flying time into account to improve the robustness of MAS schedules. The chance constraint is introduced for the capacity constraint of fixes within the MAS. Gurobi solver is applied to find the optimal solutions. A case study of Guangdong-Marco-Hong Kong MAS is presented.

1) The chance-constraint model is more conservative than the certainty optimization model. This is supported by the fact that the total slot displacements are higher in the former model than that in the later model.

2) The cost of uncertainty model increases as the violation probability decreases. There is a negative correlation between the value of the objective function value and the value of α.

3) The uncertainty model is more robust than the certainty optimization model. We found that the former optimized flight schedule can be well applied in various operation scenarios with different values of route flying time without exceeding the capacity of fixes.

4) The optimization results with α=0.3 can basically satisfy the capacity constraints under different experimental scenarios. Considering the slot displacements cost and operational delay cost, it is suggested that 0.3 is sufficient for the actual optimization.

There are still several limitation of the current work, and a few of areas that are deserved for future research.

1) We only considered the uncertainty constraint of one typical fix in the MAS. The flying times to other fixes are treated as constant values. The uncertainty constraints of all fixes can be considered in the future.

2) Lyapunov theorem is used to solve the chance constraint. Its applicability condition should be that the larger the sample size is, the better. The results for such a situation in this paper will have some errors. Thus, other solving methods should be done later.

3) The time windows in the fix capacity and airport capacity constraints are in a non-rolling form. A rolling form will be considered in the future to make the capacity constraints more stringent.

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