Analysis of Antenna Radiation Patterns by Means of Spherical Wavelets

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***Abstract*—A new tool for analysing antenna radiation patterns is proposed in this article. This tool consists in applying a spherical wavelet transform to the radiation which is assumed to be known on a regular spherical-coordinates grid. This is an alternative solution to usual plane-waves or spherical harmonic expansions. The interest of using wavelets is to obtain a better analysis in terms of localization in position and/or direction. The direct link between spherical wavelets and spherical harmonics is also highlighted. The properties of this tool are illustrated by means of numerical experiments performed on two typical antennas, a canonical aperture and a pyramidal horn.**

***Index Terms*—wavelets, antennas, radiation pattern, multiresolution analysis.**

I. Introduction

In the literature, expansions of antenna radiation are generally based on plane-waves or spherical harmonics. Such solutions are notably used in the context of antennna measurements for performing near-field to far-field transforms. Nevertheless, they are not optimal in case of a radiation that is localized in terms of position and/or direction. Indeed plane waves and spherical harmonics do not have this characteristic of localization in position and direction of propagation. A more efficient solution would be to use expansions in terms of elementary waves which possess this double localization property. Such expansions already exist, *e.g.* Gabor frames and wavelets. They come from the time-frequency analysis of signal processing and have been applied in other domains of electromagnetics [1], [2], [3], [4].

A very promising expansion method is the multiresolution analysis in wavelets. The first definition of the wavelet transform was achieved by Jean Morlet and Alex Grossman [5] in the 80's. Then, Yves Meyers [6], Ingrid Daubechies [7] and Stéphane Mallat [8], [9] gave a good formalization of it. Today it is quite popular and used in a lot of domains. Wavelets are used for denoising [10] and data compression [11], [12], [13], in signal theory and image processing. In electromagnetics, wavelets are often used as basis-tests functions in finite elements [14], integral equations [15], [2], [16], and in FDTD because of their compression capacity [17]. A model of atmospheric propagation have recently been computed on wavelets [4],[18].

Recent research works have been led to develop wavelet expansions specifically for spherical geometry [19], [20]. These methods have notably been used to analyze the cosmic microwave background [21] but never for antenna radiation.

A wavelet is a zero-mean oscillating function able to dilate and multiply itself to generate a mathematical basis. The original wavelet is called "mother", and the dilated and translated ones "daughters". Wavelets have very interesting properties as they are localized both in space and spectrum allowing them to take into account both local and global properties of the signal. Morever, by means of the wavelet transform, any signal can be decomposed over this basis. On a short term, the wavelet transform can be compared with the Fourier transform. No simple analytical formulation exists for the propagation of wavelets but this expansion is characterized by a reduced computation time and by its ability to compress the information.

A key-issue to apply wavelets to antenna radiation is the consistency of the expansion with a spherical geometry, and thus spherical coordinates. The field could be analysed by means of 2D separable wavelets using a projection of the sphere onto a 2D grid. However, any type of such projections inherently introduces deformations of angles or areas. This would jeopardize the locality of the expansion. Another method is the recently proposed spherical wavelet analysis on the sphere [22] which results from spherical harmonics and sphere sampling theorems. Due to its spherical definition, this is a suitable solution to analyse spherical radiation pattern.

The objective of this paper is to introduce the spherical wavelet multiresolution transform as a new tool to analyse antenna radiation and to apply it to typical antennas via numerical experiments. This transform is here applied to a scalar pattern, such as the co and cross-components of the radiated fields.

This paper is organised as follows. In Section II a definition of 1D wavelets is reminded. Then in Section III the sampling theorem over the sphere is presented. In Section IV spherical wavelets are described and the relation between spherical harmonics and spherical wavelets is shown. Finally in Section V simulations are performed and far-field radiations are analysed thanks to spherical wavelets.

II. 1D wavelets

In this section, basics about 1D wavelets are explained [23]. First, a mother wavelet is defined as a function ψ of zero mean,

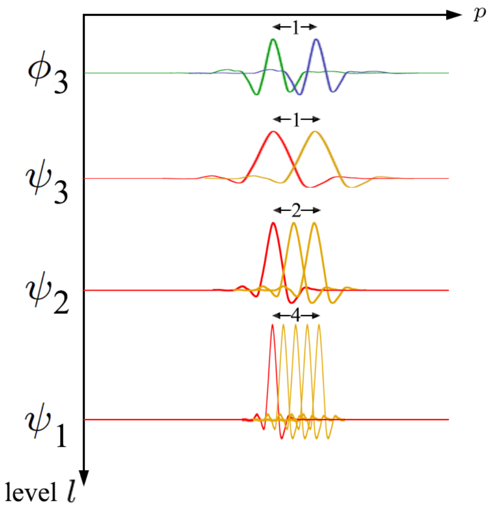


Fig. 1. A wavelet basis.

i.e.,

$$\int_{-\infty}^{\infty} \psi(z) dz = 0, \quad (1)$$

with $\|\psi\|_2 = 1$. From this mother wavelet, the wavelet family can be defined by

$$\mathcal{F} = \{\psi_{l,p}(z) = 2^{-l/2}\psi(2^{-l}z - p)\}_{(l,p)\in\mathbb{N}\times\mathbb{Z}'}, \quad (2)$$

where p corresponds to the translations needed to cover the domain. Besides, l corresponds to the dilation level. The greater l, the more dilated the wavelets are. Thus slower variations of the signal are captured by greater values of l. In practice, the decomposition is only computed for $l \in [1, L]$, L being the maximum level of decomposition but the lowest part of the spectrum is not covered then and the family is not a basis anymore. In order to obtain an orthonormal basis, the scaling function $\phi_{L,p} \in \mathcal{L}^2(\mathbb{R})$, of non-zero mean is thus added to the family. The translated versions of the wavelets and scaling function allow to cover the entire spatial domain. Wavelets are also localized in the spectral domain. In Fig. 1, a wavelet basis with a maximum level $L = 3$ is plotted.

To conclude, we obtain an orthonormal basis of $\mathcal{L}^2(\mathbb{R})$ with functions localized in both space and spectrum.

III. Sampling theorem on a sphere

Continuous and discrete wavelet theories are well established in cartesian coordinates in 1D and 2D spaces [24]. Over the sphere, a specific wavelet theory has recently been developed for which the approach for the continuous case has been combined with sampling theorems so as to obtain spherical wavelet transforms for sampled data. There are significant differences between classical and spherical wavelets. For instance, translations become rotations on the sphere.

Besides, a major problem encountered when computing spherical harmonics and spherical wavelets on a sphere is the sampling and meshing of this sphere. Several sampling theorems have been proposed on that purpose [25], [22]. They define θ-ϕ grids associated with an exact representation of band limited signals, *i.e.*, signals which spherical harmonics representation is limited to a maximal order. McEwen and Wiaux [22] have developed such a sampling theorem on the sphere and corresponding fast algorithms for spherical harmonics by associating the sphere with the torus through a periodic extension of θ to the domain [0,2π] to ease the computation. The proposed sampling is equiangular with sample positions given by

$$\begin{aligned} \theta &= \frac{\pi(2p_\theta + 1)}{2N_{\text{max}} - 1}, \quad \text{for } p_\theta \in \{0, 1, ..., N_{\text{max}} - 1\}, \\ \varphi &= \frac{2\pi p_\phi}{2N_{\text{max}} - 1}, \quad \text{for } p_\phi \in \{0, 1, ..., 2N_{\text{max}} - 2\}, \end{aligned} \quad (3)$$

where N_{max} is the maximal spherical harmonic order, *i.e.*, the band limit of the signal. Spherical wavelets used in this article are based on this sampling. Besides, in this article, for a band limited signal, the indices associated with spherical harmonics coefficients are denoted (n, m) with $n \in \{0, \cdots, N_{\text{max}}\}$ and $m \in \{-n, \cdots, +n\}$.

IV. Spherical wavelets

This section presents the theoretical basis of spherical wavelets as developed in [20]. Furthermore, in this article, we use scale-discretised axisymmetric wavelets. Axisymmetric means that wavelets are azimuthally symmetric when centred on the poles.

As for the 1D wavelet transform, the basic elements of a spherical wavelet transform are a scaling function Φ and wavelets functions at various scales Ψ_j for $j \in [J_0, J_{max}]$, as shown in Fig. 2. The wavelets and scaling functions are well-localised both spatially on the sphere and also in harmonic space. We can see the contrast with spherical harmonics in Fig. 3 that are way less localised on the sphere. Consequently, wavelets on the sphere can be used to extract spatially localised, scale-dependent features in signals of interest.

Each of these functions is associated with a part of the spherical harmonic spectrum in n. As illustrated in Fig. 4 the scaling function corresponds to the lowest components of the spectrum and the wavelets to higher components. The greatest the scale index j, the fastest the variations of the signal.

Here are few details to perform a multiresolution spherical wavelet analysis from these functions. The first step is to compute the spherical harmonic transform of the signal, in our case, the radiation. Then this spectrum is multiplied by the spectrum of the scaling function and of each wavelet function, which are known beforehand. Doing so, we obtain spherical harmonic coefficients associated with the scaling function W^{Φ}_{nm} and with each scale of wavelets $W^{\Psi^j}_{nm}$. To obtain the multiresolution wavelets analysis over the sphere, inverse spherical harmonic transforms of these coefficients are performed. Since W^{Φ}_{nm} and $W^{\Psi^j}_{nm}$ have a smaller band limit than the initial signal, this inverse transform can be realized at a lower resolution, without introducing any error because of the sampling theorem. For example, from Fig. 4, scaling coefficients have the smallest band limit, so that they can be represented by the lowest resolution. Then comes $W^{\Psi^1}_{nm}, W^{\Psi^2}_{nm}, \cdots$.

Finally, as with 1D wavelets, the multiresolution analysis should yield a sparse representation of the signal. Thresholding can thus be used to produce data compression.

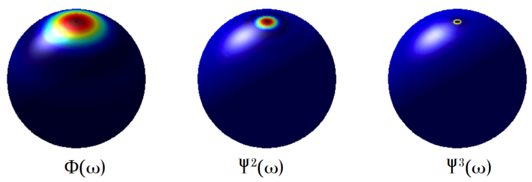


Fig. 2. Axisymmetric scale discretised spherical scaling and wavelet functions for scales $j \in \{2, 3\}$. This plot comes from [20] and was produced with a Matlab demo included in the s2let package [26].

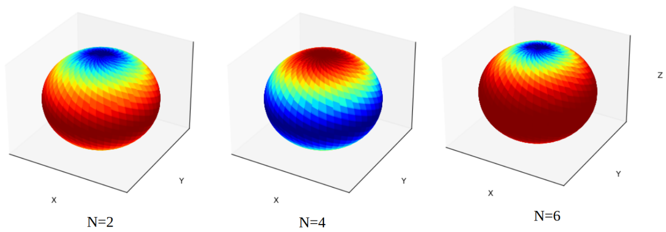


Fig. 3. Spherical harmonics for $n = 1$ and $m = 0, 1, 2$.

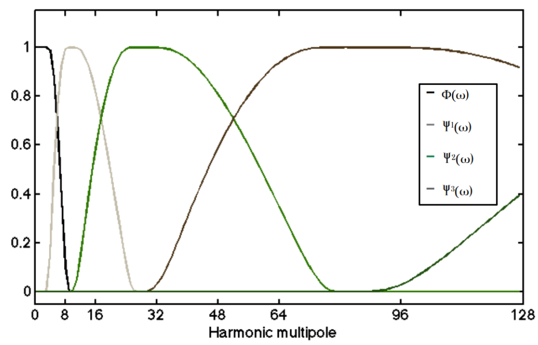


Fig. 4. Spectral partitioning in n for different types of wavelets [26].

V. Multiresolution analysis of usual antenna radiation pattern

In this section, the multiresolution spherical wavelets analysis is applied on two antenna radiations. All the simulations of this section make use of the freely available package s2let for spherical wavelets [26].

A. Uniform aperture

The first radiation analysis is a scalar far-field from a 0.4 m × 0.6 m uniform aperture at 10 GHz. As a reminder the far field radiation of a uniform aperture of dimension $[a, b]$ is expressed by

$$\begin{aligned} E(r, \theta, \phi) = & A \cos\theta \frac{e^{-jkr}}{kr} \text{sinc}\left(\frac{k \sin\theta \cos\phi d_x}{2}\right) \\ & \text{sinc}\left(\frac{k \sin\theta \sin\phi d_y}{2}\right), \end{aligned} \quad (4)$$

where k is the wavenumber, d_x, d_y the transverse size of the aperture and A a constant. The McEven and Wiaux sampling (3) is used with $N_{\text{max}} = 256$. This far-field on the sphere for this uniform aperture is displayed in Fig. 5.

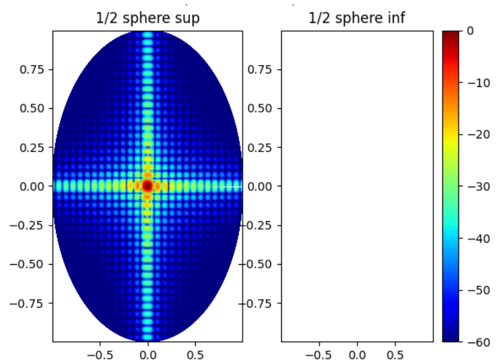


Fig. 5. Initial far-field of the uniform aperture on the sphere.

Then the wavelet coefficients associated with the scaling function and the different wavelet levels are computed. Axisymmetric scale discretized wavelets are used with $j \in \{1, \cdots, 6\}$. Besides, a compression is applied by thresholding coefficients that are -40 dB below the maximal value. In Fig. 6 we observe the results of the spherical wavelet transform, *i.e.*, patterns at various resolution levels. As expected the different levels in the spherical wavelet analysis corresponds to the different scales of variations of the signal. The main lobe is represented in scales $j = 4, 5, 6$ while secondary lobes corresponds to scales 5 and 6. In this case, the compression rate is about 98%.

B. Horn antenna

In this section, the radiation pattern of a pyramidal horn is studied: this horn is excited by a TE10 mode rectangular guide, at 1.645 GHz. The dimension of the aperture is 0.55 m × 0.428 m. The radiation pattern is simulated with Altair Feko, using the method of moments and calculated on a sphere of 180 × 359 points, consistently located according to the sampling theorem of McEwen and Wiaux [19], which means that N_{max} is set at 180. In Fig. 7 the radiation pattern is represented using the third Ludwig definition for co and cross-polarisations.

From the simulated field, we can obtain the wavelets coefficients, see Fig. 8, in co and cross-polarisations. As in the previous section, axisymmetric scale discretized wavelets are used with $j \in \{1, \cdots, 8\}$. Here as well, we can notice that the higher the order of the wavelet, the smaller are the variations.

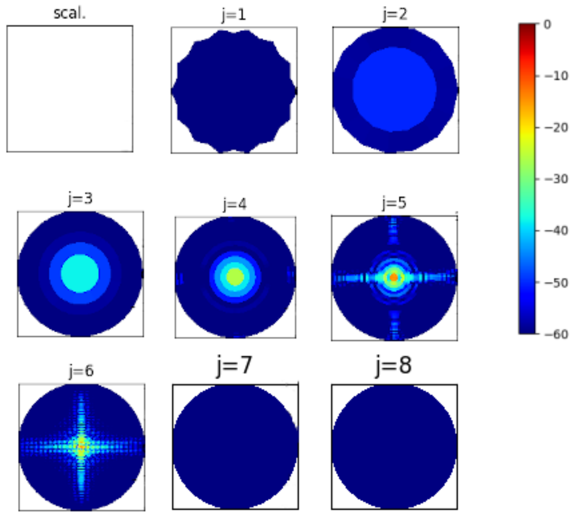
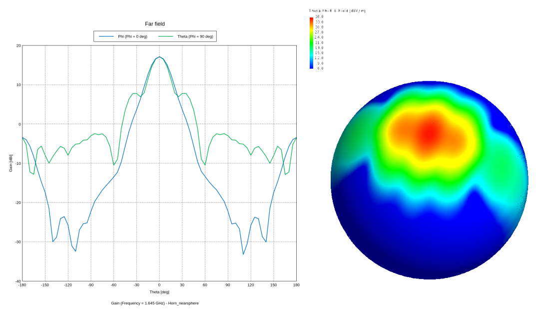


Fig. 6. Wavelet coefficients associated with the scaling function and the different wavelet levels for the uniform aperture (dB).



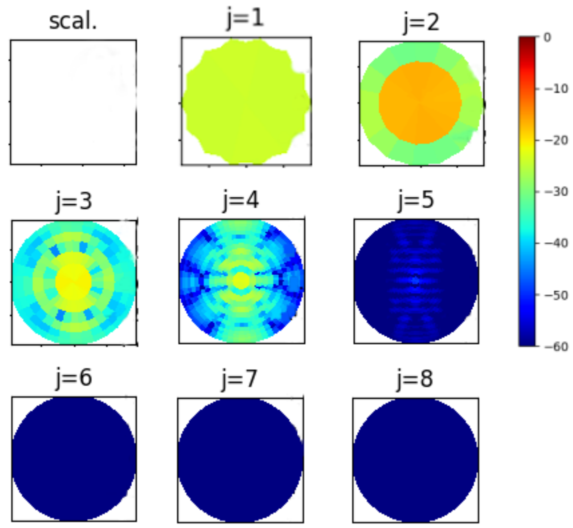
(a) E and H planes (b) 3D

Fig. 7. Radiation pattern (dB) of the horn antenna.

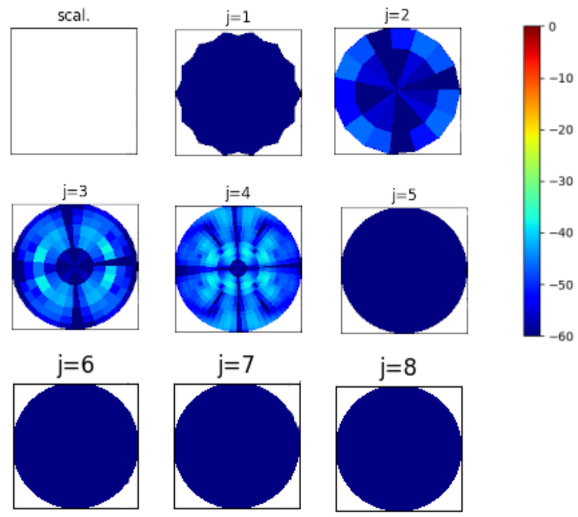
Compression rate has been computed and is of 98% , about as in previous section.

VI. Conclusion

In this paper, the objective was to introduce spherical wavelet transform as a new tool to analyse antenna radiation and to apply it to typical antennas via numerical experiments. In Section II a review about 1D wavelets has been done. Then, in Section III this work has been placed in the framework of the sampling theorem related to the exact representation of band-limited signals in terms of spherical harmonics. In Section IV spherical wavelets and their relation with spherical harmonics have been presented. Finally in Section V, the multiresolution analysis using spherical wavelets has been aplied to two types of simulated scalar radiations. This analysis has shown that using wavelets may be suitable to describe scalar radiations and has good location and scaling properties. Furthermore, since wavelets are deconvolution and denoising tools that are used in numerous domains of engineering and physics [24], [9], this type of analysis might be particularly useful for removing spurious reflections and noise in the



(a) Co-polarisation



(b) Cross-polarisation

Fig. 8. Wavelet coefficients associated with the scaling function and the different wavelet levels for the horn antenna (dB).

context of post-processing of antenna measurements. Further works will be led to compare compression rates with the analysis in terms of usual spherical harmonics. Besides, only scalar radiations have been considered here. For electromagnetic fields, vector spherical wavelets should be developed.

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