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Global Impact Criteria for Assessing the Perturbation of an Air Traffic Control Radar by a Wind Farm

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Obstacles



Logistics Buildings



Solar Panel Fields



Wind Farm

Impact

Primary Radar

- Backscattering: saturation
- Masking: modification of the gain pattern
- Multipath: ghost tracks

Secondary Radar (cooperative target)

- Masking: modification of the gain pattern
- Multipath: ghost tracks
- Off-boresight angle errors



Study case

Objectives: Assess the perturbation caused by a wind farm on an ATC radar

Why: large vacant lots often nearby airports (2 to 15 km far from the radar)

- Industrial point of view: large wind farm settlements
- Civil aviation point of view: maintain availability, continuity and security of radar services

Characteristics of the problem

- Primary and secondary radars at S and L-band respectively
- Wind farms seen as several vertical infinite metallic cylinders
- Distance between radar and wind turbines of a few kilometres
 - ➔ Far-field assumption not verified at L-band frequencies
 - ➔ RCS not valid
- Interaction between wind turbines
 - ➔ Physical Optics (PO) cannot be used

OUTLINE

1 MODEL FORMULATION

2 PRIMARY AND SECONDARY RADARS

3 RESULTS

4 CONCLUSION

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1 MODEL FORMULATION

a HYPOTHESIS

b FORMULATION

c 3-D EXTENSION

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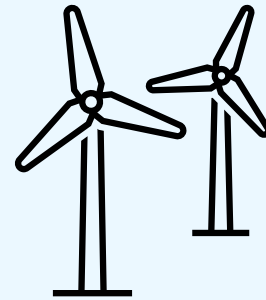
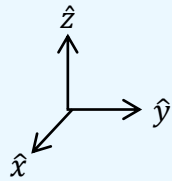
Hypothesis

- Wind turbines masts seen as vertical infinite metallic cylinders
→ 2-D approximation
- Radar beam represented by a Gaussian beam

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Real situation – 3-D

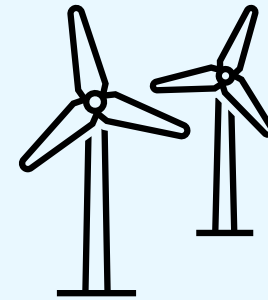
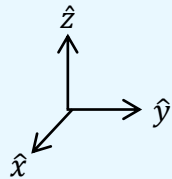


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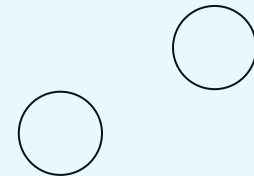
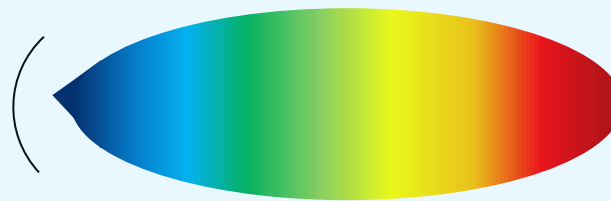
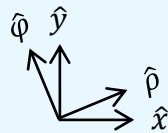
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Equivalent modelling – 2-D top view



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Formulation

Field decomposition:

$$\mathbf{E}_z = \mathbf{E}_z^i + \mathbf{E}_z^s$$

- **Incident field:**

$$E_z^i = E_z^{i,0} e^{\left(\frac{\varphi - \varphi_0}{w_\varphi}\right)^2} H_0^{(2)}(k\rho)$$

$E_z^{i,0}$ is a constant

k is the wavenumber

φ_0 is the direction of the radar beam

(ρ, φ) are the polar coordinate centred on the radar
HPBW

$$w_\varphi = \frac{\text{HPBW}}{2\sqrt{\ln(10^{-0.15})}}$$

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$$w_\varphi = \frac{\text{HPBW}}{2\sqrt{\ln(10^{-0.15})}}$$

Represented by means of out-going cylindrical harmonics in the radar coordinate system:

$$E_z^i(\rho, \varphi) = \sum_{n=-\infty}^{+\infty} a_n e^{jn\varphi} H_n^{(2)}(k\rho)$$

$$\text{with } a_n = \frac{E_z^{i,0} \times w_\varphi}{2\sqrt{\pi}} \frac{H_0^{(2)}(k\rho)}{H_n^{(2)}(k\rho)} \frac{e^{-\left(\frac{nw_\varphi}{2}\right)^2}}{e^{jn\varphi_0}}$$

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Graf addition theorem [1]

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Represented by means of in-going cylindrical harmonics in the coordinate system of the p wind turbine:

$$E_z^i(\rho_p, \varphi_p) = \sum_{n=-\infty}^{+\infty} s_n^{(p)} e^{jn\varphi_p} J_n(k\rho_p)$$

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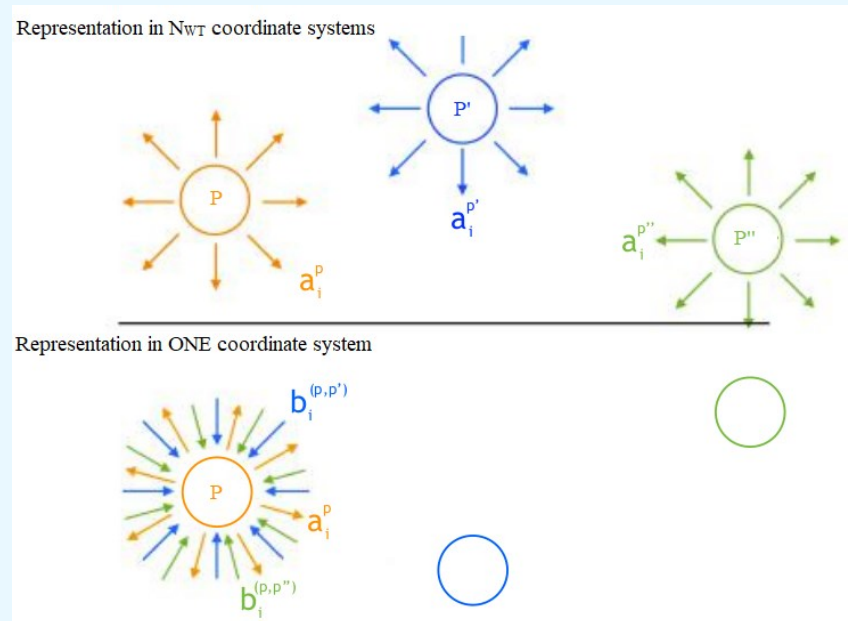
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- **Scattered field:**

Represented by means of out-going cylindrical harmonics in the coordinate system of the p wind turbine:

$$E_z^s(\rho_p, \varphi_p) = \sum_{p=1}^{N_{WT}} \sum_{n=-\infty}^{+\infty} a_n^{(p)} e^{jn\varphi_p} H_n^{(2)}(k\rho_p)$$



Formulation

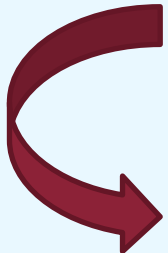
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 unknowns

- **Boundary condition at $\rho_p = A$ (mast radius): $E_z = 0$**

Using the orthogonality of the Fourier series :

$$s_n^{(p)} J_n(kA) + a_n^{(p)} H_n^{(2)}(kA) + \sum_{\substack{p'=1 \\ p' \neq p}}^{N_{WT}} b_n^{(p,p')} J_n(kA) = 0$$

- **Matrix formulation:**

$$\mathbf{M}\mathbf{a} = \mathbf{v}$$

With \mathbf{a} the unknown related to a_n

\mathbf{M} is known and a function of the problem data

\mathbf{v} is known and depends on the incident field

Formulation

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$$\mathbf{E}_z = \sum_{n=-\infty}^{+\infty} s_n^{(p)} e^{(jn\varphi_p)} J_n(k\rho_p) + \sum_{n=-\infty}^{+\infty} a_n^{(p)} e^{(jn\varphi_p)} H_n^{(2)}(k\rho_p) + \sum_{\substack{p'=1 \\ p' \neq p}}^{N_{WT}} \sum_{n=-\infty}^{+\infty} b_n^{(p,p')} e^{(jn\varphi_p)} J_n(k\rho_p)$$

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E_z is found

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3-D extension

- **Computation of the current $I(z)$ along the wind turbines:**

$$\begin{cases} I(z) = \int_0^{2\pi} J_z a d\varphi \\ J_z = \frac{1}{j\omega\mu_0} \frac{\partial E_z}{\partial \rho} \Big|_{\rho=a} \end{cases}$$

a is the wind turbine radius

- **Computation of the radiation of this wire in the far-field zone with the Stratton-Chu formula**



Work in progress

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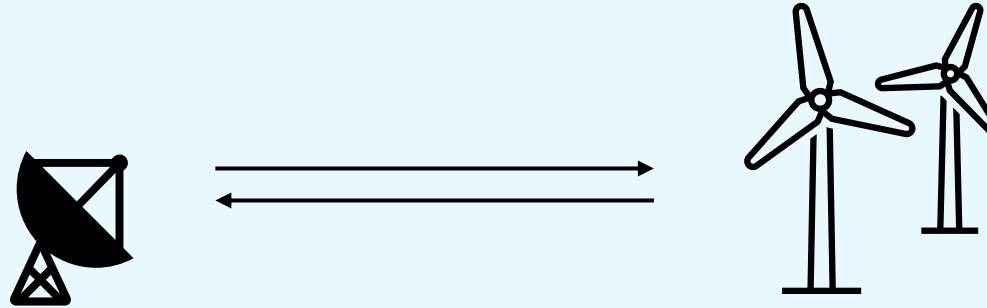
c MASKING

d $\Sigma - \Delta$ PROCESSING

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Backscattering – Primary radars only



Computation of the scattered field echoes

- Computation of the scattered field E
- Computation of the received power P_r using:

$$P_r = \frac{E^2 \lambda^2}{8\pi \zeta_0} G \text{ STC}$$

with G the antenna reception gain, STC the Sensitivity Time Control, $\zeta_0 = 120\pi$ the vacuum impedance

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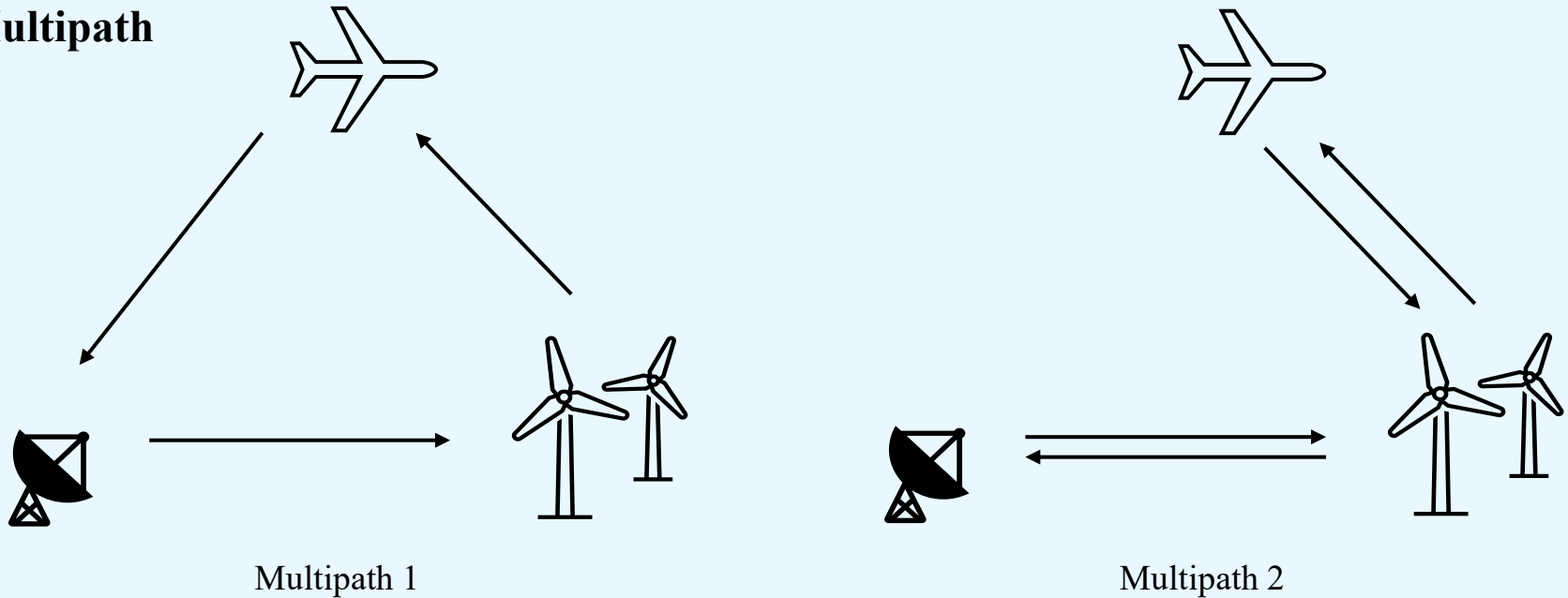
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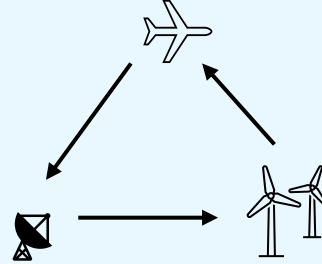
Multipath



Modelling methodology

- Computation of the scattered electric field at the aircraft
- Estimation of the received power at the radar following two scenarii:
 - Multipath 1: Radar → wind turbine → aircraft → radar;
 - Multipath 2: Radar → wind turbine → aircraft → wind turbine → radar.
- Primary radar: multipaths 1 and 2
- Secondary radar: multipath 2 only

Multipath 1: Radar → wind turbine → aircraft → radar



Primary radar method

- Computation of the scattered electric field at the aircraft
- Modelling of the aircraft by its bistatic RCS σ_a
- Propagation towards the radar at a distance R
- Radar gain G_a in the direction of the aircraft

Final expression

$$P_r = \frac{\|E\|^2}{2\zeta_0} \sigma_a G_a \left(\frac{\lambda}{4\pi R} \right)^2$$

Negligible impact if:

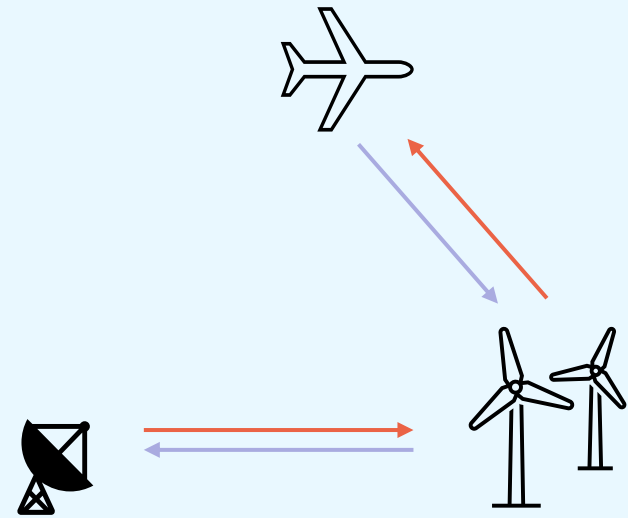
$$\|E\| < \frac{4\pi R}{\lambda} \sqrt{\frac{2\zeta_0 P_{r0}}{\sigma_a G_a}}$$

P_{r0} the radar sensitivity

Multipath 2: Radar → wind turbine → aircraft → wind turbine → radar

Primary radar method

- Computation of the scattered electric field
- Reciprocity theorem, considering both following states:
 1. The radar transmits, the aircraft receives
 2. The aircraft transmits, the radar receives
- This theorem imposes: $P_e^{(1)} P_r^{(2)} = P_e^{(2)} P_r^{(1)}$
 - $P_e^{(1)}$ the power transmitted by the radar in state 1
 - $P_r^{(1)}$ the power received by the aircraft in state 1
 - $P_e^{(2)}$ the power transmitted by the target in state 2
 - $P_r^{(2)}$ the power received by the radar in state 2



Final expression

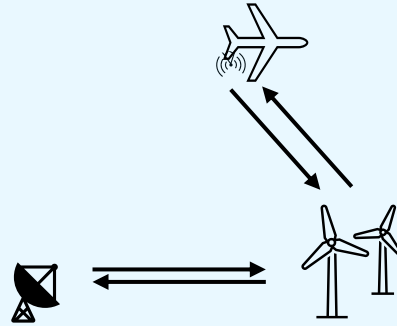
$$P_r^{(2)} = \frac{P_e^{(2)}}{P_e^{(1)}} P_r^{(1)} = \frac{\sigma_a \|E\|^2 / 2\zeta_0}{P_e^{(1)}} \frac{\lambda^2 \|E\|^2}{4\pi 2\zeta_0}$$

Negligible impact if:

$$\|E\| < \sqrt[4]{\frac{16\pi\zeta_0^2 P_e^{(1)} P_{r0}}{\lambda^2 \sigma_a}}$$

P_{r0} the radar sensitivity

Multipath 2: Radar → wind turbine → aircraft → wind turbine → radar



Secondary radar method

- Computation of the scattered electric field at the aircraft
- Comparison to the threshold of the aircraft transponder $P_{\text{transp}} = -74$ dBm

Final expression

Considering $G_{\text{transp}} = 0$ dBi

$$P_r = \frac{\lambda^2}{8\pi\zeta_0} \|E\|^2$$

No transponder's answer if:

$$\|E\| < \frac{\sqrt{8\pi\zeta_0 P_{\text{transp}}}}{\lambda}$$

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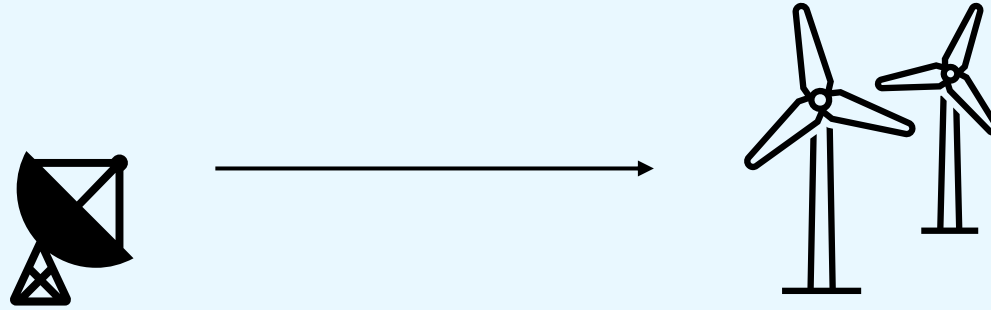
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Masking



Method

- Computation of the total field E around the radar, along the main axis, in the presence of the wind farm
- Comparison to the nominal gain pattern

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$\Sigma - \Delta$ processing – Secondary radar only

Method

- Computation of the total field E around the radar, along the main axis, in the presence of the wind farm, for Σ and Δ patterns
- Computation of the off-boresight angle error

Mathematical expressions

Off-boresight angle (OBA) in azimuth

$$\text{OBA}_{az} = \frac{\Re(E_{\Delta})}{\Re(E_{\Sigma})}$$

OBA error

$$\text{Err} = \text{OBA}_{az} - \text{OBA}_0$$

with OBA_0 computed without obstacle.

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Configuration 1:

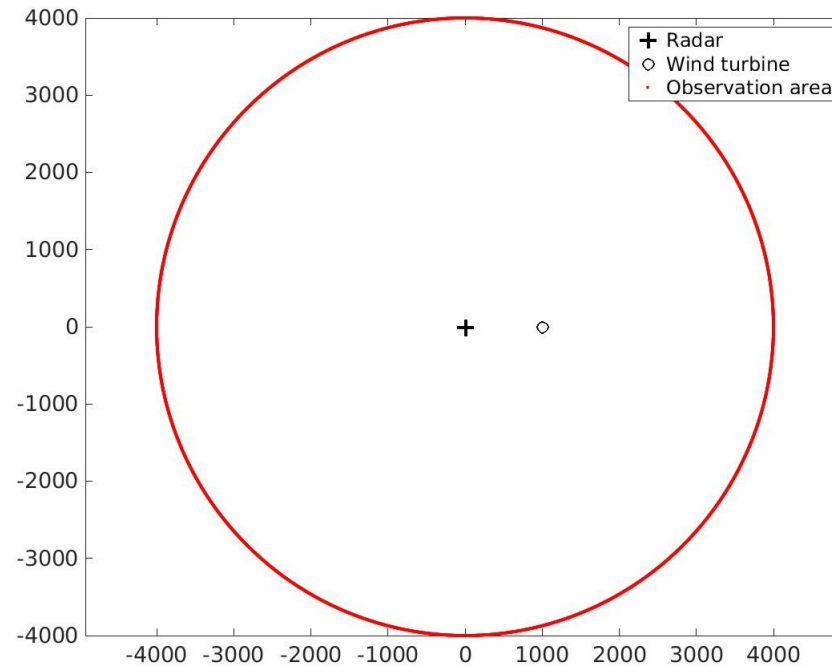
Radar: L-band (1030 MHz emission), HPBW = 5°

Unique wind turbine: 2.5 m in radius, distance to radar 1 km

Radar directions: 1. towards the center of the wind turbine

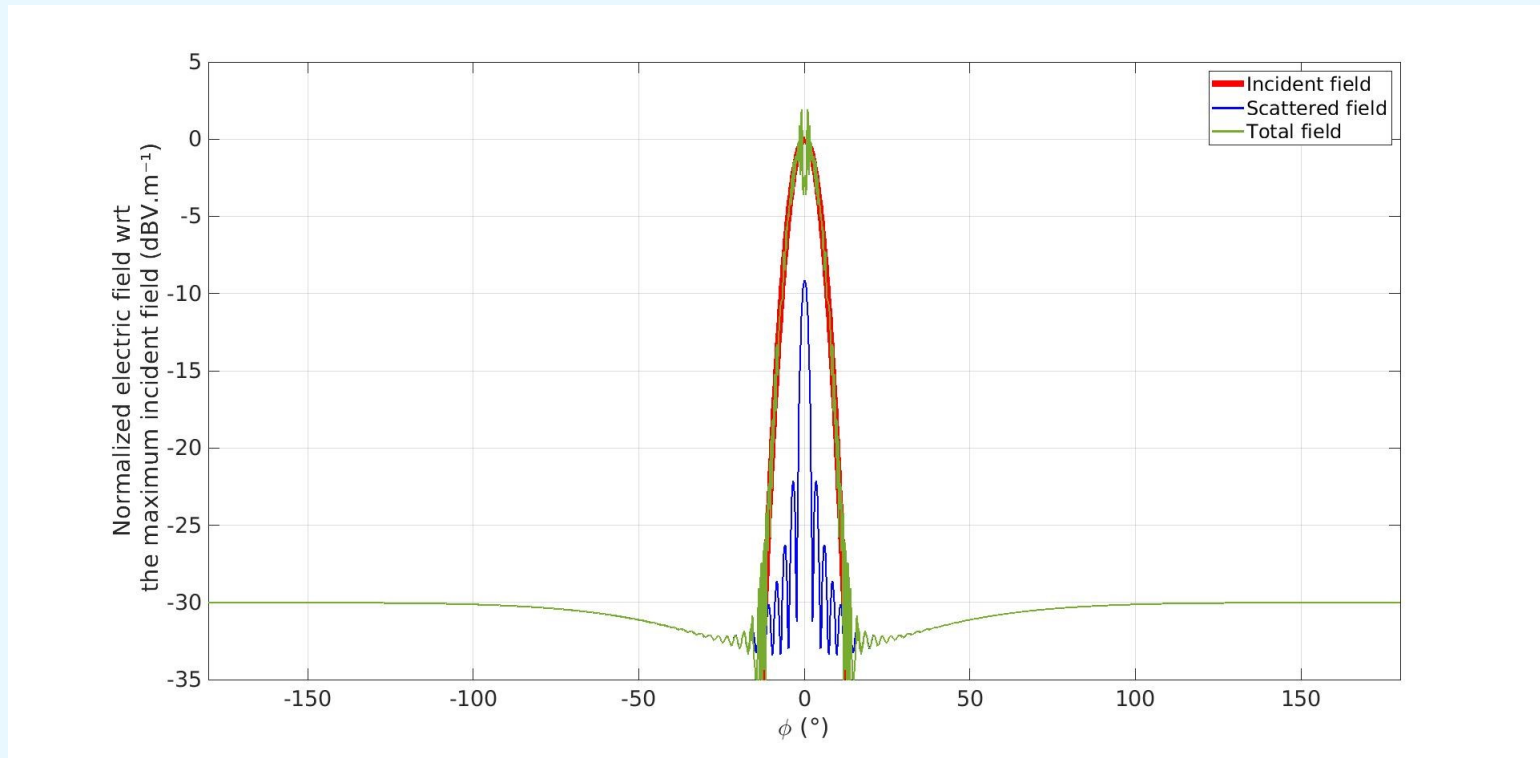
2. towards $\phi = 2.5^\circ$

Observation: $\rho = 4$ km, $\phi \in [-\pi, \pi]$



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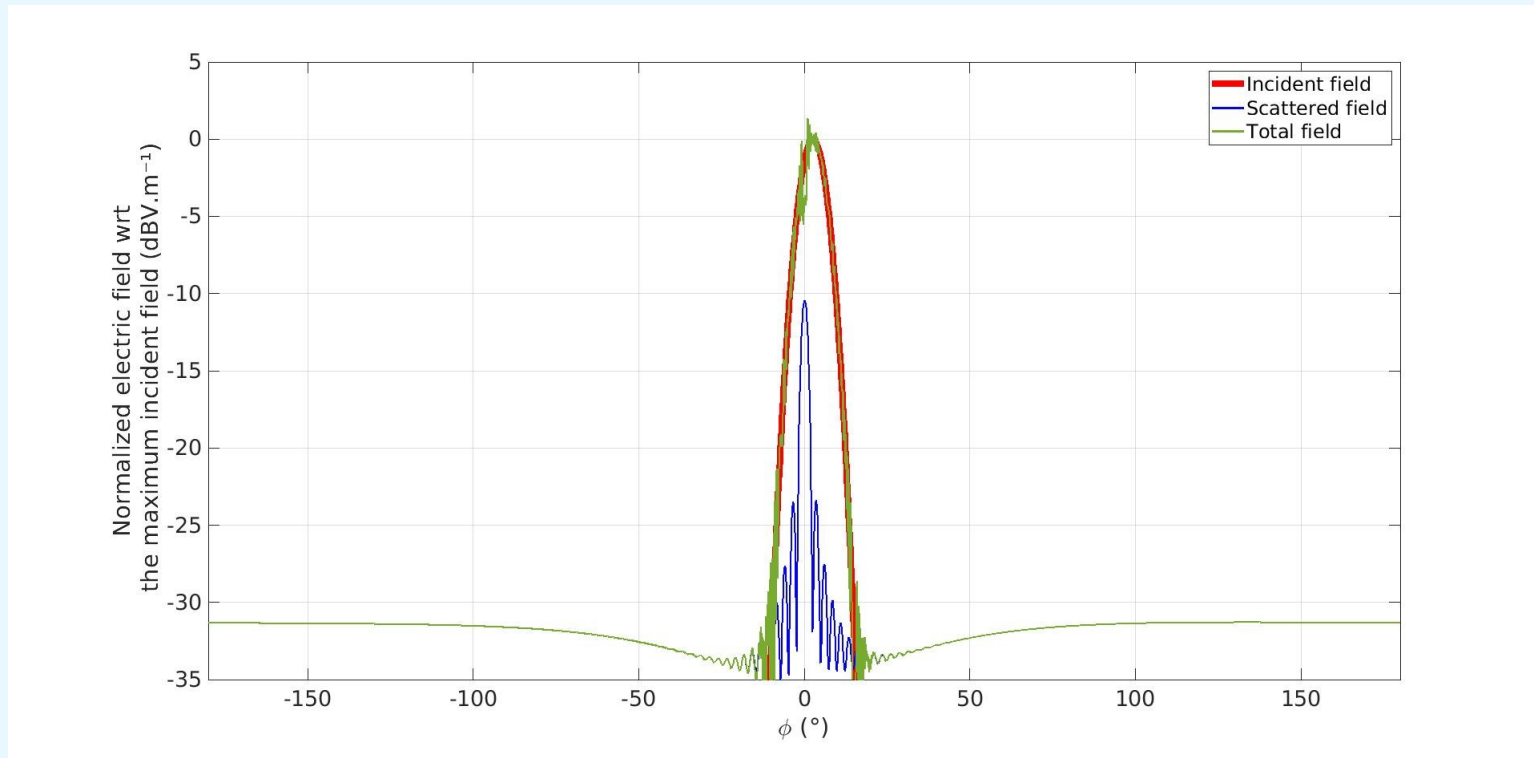
Radar direction: 1. towards the center of the wind turbine



- ➔ No multipath
- ➔ No backscattering
- ➔ Small masking effect : loss about 2.5 dB towards $\phi = 0^\circ$

Configuration 1:

Radar direction: 2. towards $\phi = 2.5^\circ$



- ➔ No multipath
- ➔ No backscattering
- ➔ Small masking effect : loss about 3.8 dB towards $\phi = -0.4^\circ$

Configuration 2:

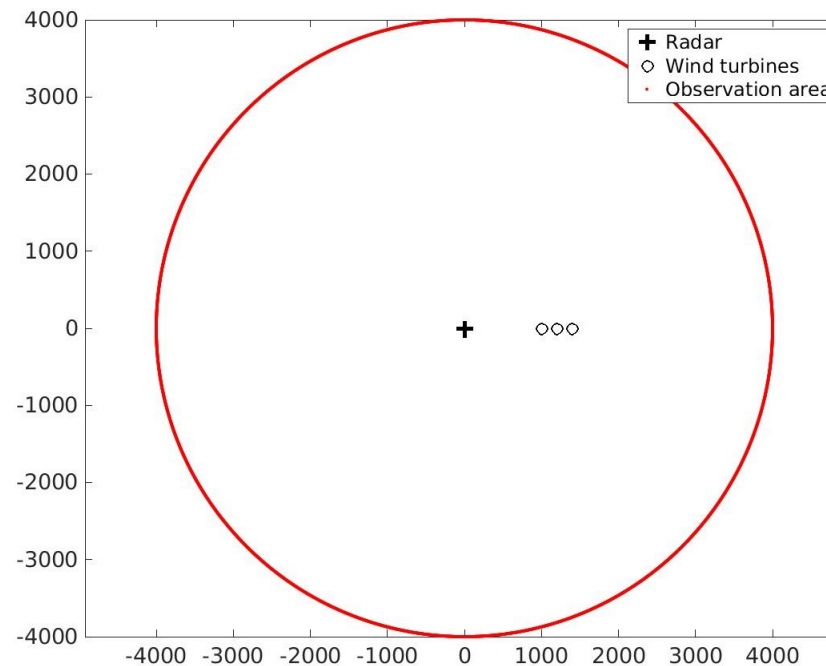
Radar: L-band (1030 MHz emission), HPBW = 5°

3 wind turbines: 2.5 m in radius, distance to radar 1 km, each 200 m apart

Radar directions: 1. towards the center of the wind turbine

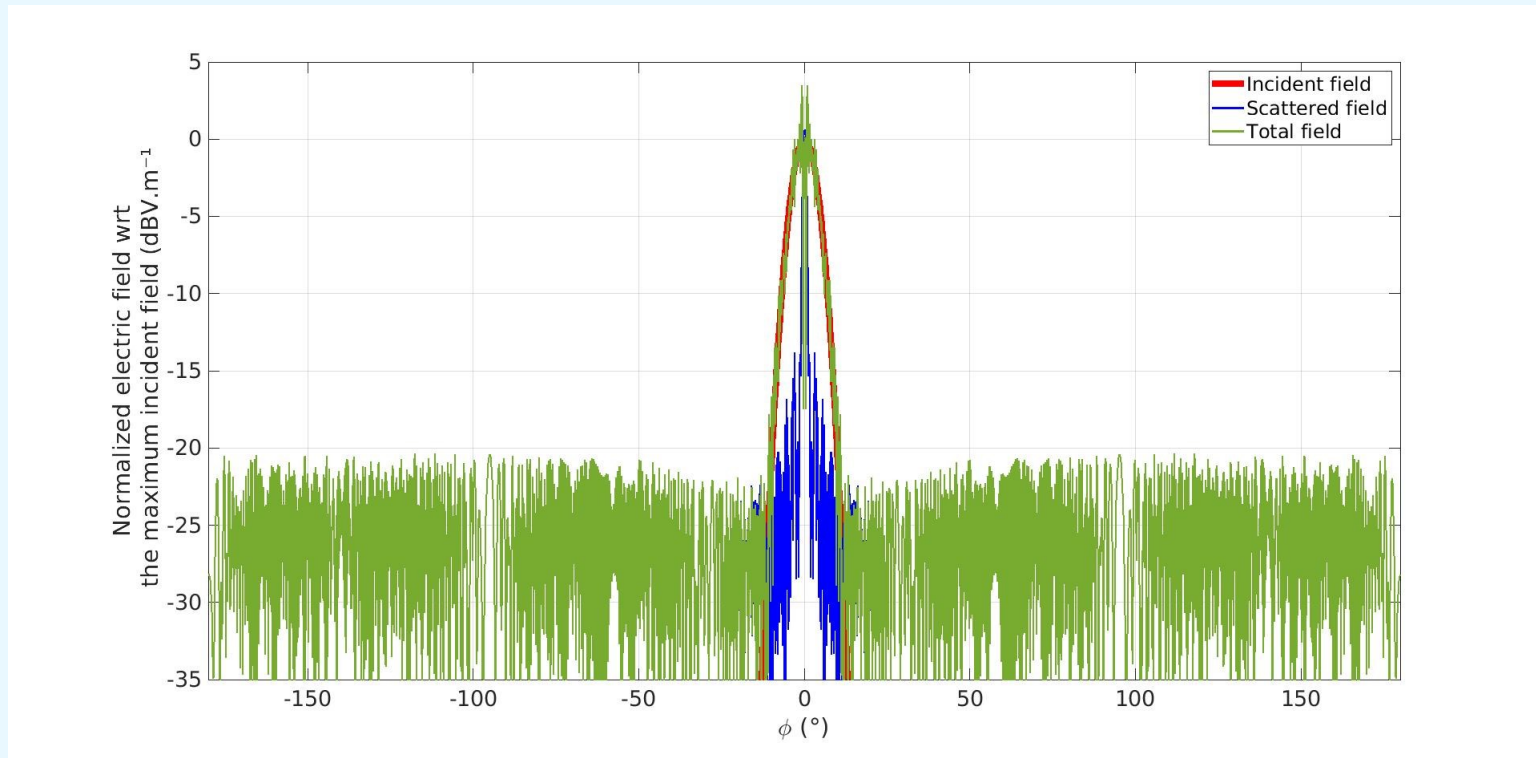
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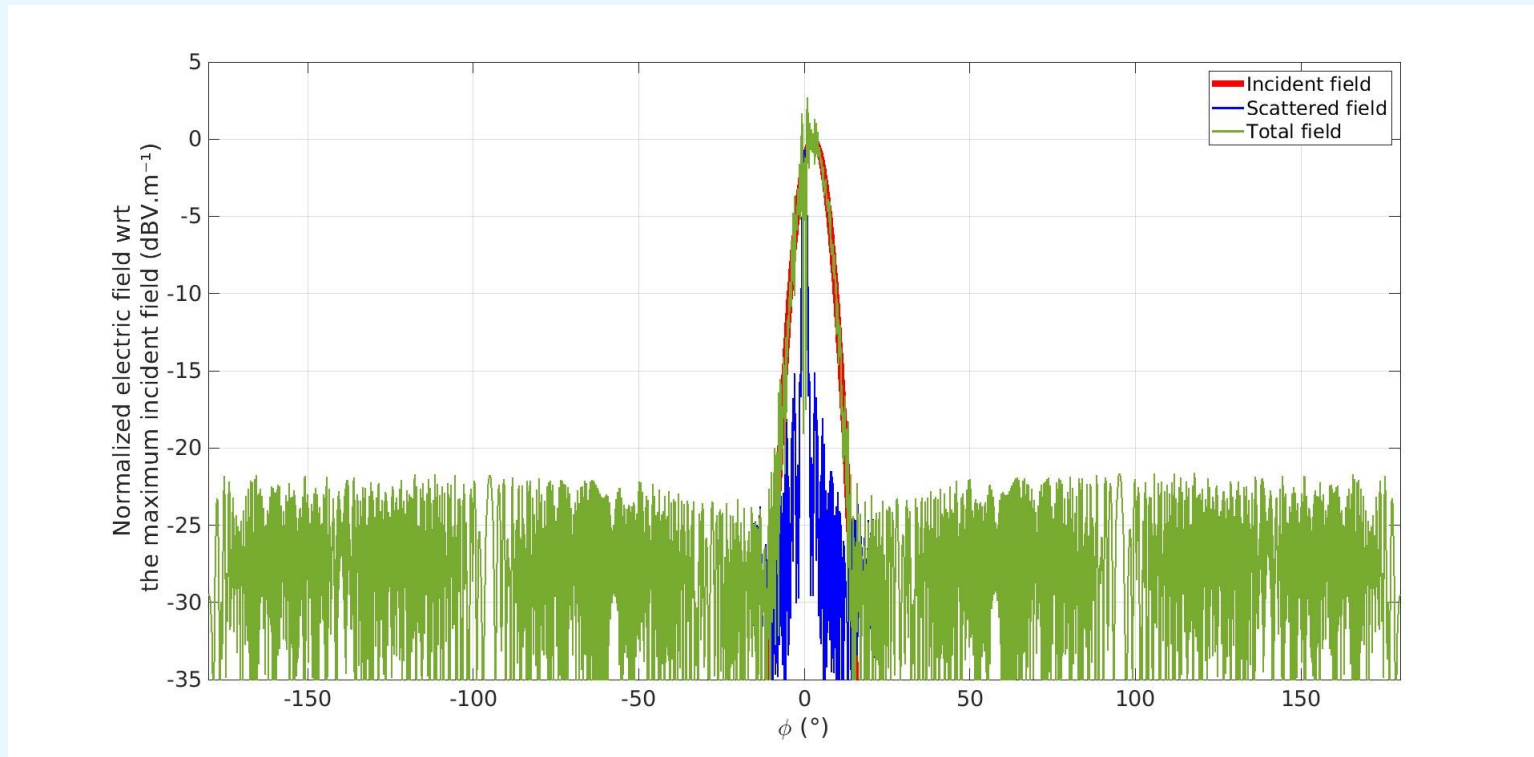
Radar direction: 1. towards the center of the wind turbine



- ➔ No multipath
- ➔ No backscattering
- ➔ Masking effect: loss about 18 dB towards $\phi = 0^\circ$

Configuration 2:

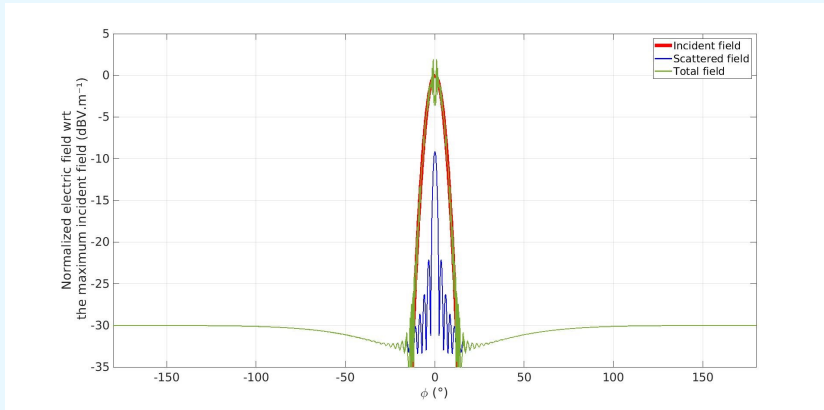
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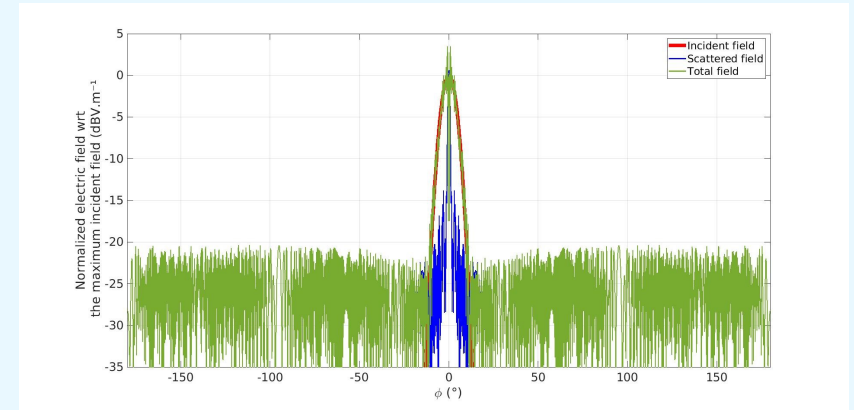
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Radar direction: 1. towards the center of the wind turbine



Configuration 2:

Radar direction: 1. towards the center of the wind turbine



Increasing the number of wind turbines increases the masking effect towards them and the perturbations in the other direction.

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Methodology

Methodology for assessing the impact of wind turbines on ATC primary and secondary radars:

- Modelling the antenna with a Gaussian beam
- Computation of the scattered field using the cylindrical harmonics to consider the multiple interactions between the wind turbines
- Direct backscattering for primary radars
- Multipath and masking effect for both ATC radars

Future works

- Validation of the tool for finite cylinders
- Implementation of the different criteria
- Implementation of the real radar patterns (both Σ and Δ for the secondary radar)
- Developing an optimisation tool to configure a wind farm yielding the weakest perturbation



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QUESTIONS ?

