Aircraft trajectory complexity metric: an image processing approach

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B. G. NGUYEN*, D. DELAHAYE†, P. MARÉCHAL‡

Abstract

In this paper we propose an image processing approach to air traffic management. We derive a method for extracting the main flows, and then develop an airspace complexity indicator along the main flows. This index can be used at the macroscopic level. Our algorithm was tested on data from the French airspace.

1 Introduction

1.1 Complexity

The operational capacity of a control sector is currently measured by the maximum number of aircraft able to cross the sector in a given time period. This measurement does not take account of the orientation of traffic and considers geometrically structured and disordered traffic in the same manner. Thus, in certain situations, a controller may continue to accept traffic even if the operational capacity is exceeded (structured traffic); in other situations, controllers may be obliged to refuse additional traffic even though operational capacity is not reached yet (disordered traffic). Thus, a measurement in terms of the number of aircraft per unit of time constitutes an insufficient metric for the representation of the level of difficulty corresponding to a particular traffic situation.

Ideally, in the context of operational control, one should find a metric which precisely measures the level of mental effort needed to manage a set of aircraft. Without going so far, it is possible to find complexity metrics which go beyond a simple measurement of the number of aircraft. We will start by clarifying two essential concepts to use in the rest of this section:

- **Control workload**: measurement of the difficulty for the traffic control system of treating a situation. This system may be a human operator or an automatic process. In the context of operational control, this workload is

*National University of Civil Engineering (NUCE). Email: giangnb@nuce.edu.vn
†Ecole national de l'Aviation Civile (ENAC). Email: delahaye@recherche.enac.fr
‡Paul Sabatier University. Email: pierre.marechal@math.univ-toulouse.fr

linked to the cognitive process involved in managing some traffic situation (conflict prediction and resolution, trajectory monitoring, etc.).

- **Traffic complexity**: intrinsic measurement of the complexity associated with a traffic situation. This measurement is independent of the system in charge of the traffic and depends only on the geometry of trajectories. It is linked to sensitivity to initial conditions and to the interdependency of conflicts. Uncertainty with respect to positions and speeds increases the difficulty of predicting future trajectories. In certain situations, this uncertainty regarding future positions can increase exponentially, making the system extremely complex in that it is virtually impossible to reliably extrapolate a future situation. When a future conflict is detected, a resolution process is launched which, in certain situations, may generate new conflicts. This interdependence between conflicts is linked to the level of mixing between trajectories.

The search for metrics of the complexity of air traffic has attracted considerable attention in recent years, particularly in the United States and in Europe. The first projects were launched in Germany in the 1970s, and since then the subject has continued to develop. Currently, NASA, MIT and Georgia Tech are involved in work on the subject within the framework of the NextGen project. In Europe, the DSNA, the DLR and the NLR are involved in similar activities linked to SESAR.

1.2 Main flow

As for previous metrics, the objective of our metric is to measure the level of complexity of a given traffic situation. Our approach is based on the notion of dominant trajectory also called major flow or main flow. In [2], the definition of the major flow was given as follows:

> *When radar tracks are observed over a long period of time in a dense area, it is very easy to identify major flows connecting major airports. The expression "major flows "is often used but never rigorously defined. Based on an exact trajectory distance and a learning classifier, it is possible to answer the following questions: Given a set of observed trajectories, can we split it into "similar "trajectory classes? If yes, classes with highest number of elements will rigorously define the major flows. Given those classes and a new trajectory, can we tell if it belongs to a major flow and which one? The principle of the major flows definition is to use shape space to represent trajectory shapes as points and to use a shape distance (the shape of a trajectory is the path followed by an aircraft, that is the projection in the 3D space of its 4D trajectory. The speed on the path has no impact).*

In order to successfully plan and accommodate the increased number of flights, one must be able to identify major flows in the airspace. In [6, 7], Histon, J.M. *et al.* indicated the importance of the standard flows crossing a sector. They also showed that complex sectors have many entry and exit points with many interacting flows. The major flows and their interactions constitute the basis for air traffic controllers to build their abstraction of a sector.

In the study [12], the authors consider that sector capacity should be based on the geometric distribution of major flows in sectors. A list of flow features is then used to describe traffic flow patterns. Based on such features, they proposed a method for computing the sector capacity. The method avoids measuring the controller workload directly and predefining the controller workload threshold.

In [11], major flows are used to study the impact of severe weather condition. Although analyzing aircraft trajectories is a vital component of such tools, individual flight scale can either be prohibitively expensive due to the large number of operations, or inappropriate for macroscopic features or trends in big airspace. Hence, it is customary for analytical tools to include algorithms that capture and aggregate the flight behavior while preserving an appropriate level of fidelity [5].

There are several algorithms used to extract the major flows in a set of trajectories. Some of them use traditional data reduction methods (e.g. Principal Component Analysis (PCA)) with clustering methods (e.g. k-means). In [3], Eckstein used PCA and k-means to build a flight taxonomy. Then, Gariel *et al.* have improved this method by increasing data dimensionality (by adding heading, angular position, etc.) and have used the DBSCAN clustering algorithm. The advantage of this algorithm is that it does not require *a priori* selection of cluster size and feature outlier identification. Marzouli *et al.* [8] also used PCA and DBSCAN to identify flows, from which a mathematical graph (network) was created. Recently, Enriquez and Kurcz [5] proposed another approach based on spectral clustering to identify flows in terminal and en-route airspaces. In [4], Enriquez extended this method to identify flows by accounting for the temporal dimension.

Based on the project called *FromDaDy* (which stands for FROM DAta to DisplaY), Marzouli *et al.* [8] have developed a visualization tool which is used to display and extract specific recorded trajectories. A two step algorithm is proposed. The first step uses the KDEEB algorithm (which stands for Kernel Density Estimation-based Edge Bundling) so as to bundle the trajectories into a less cluttered graph. Once this step is implemented, a given graph drawing is transformed into a density map using kernel density estimation. The second step collects flows through a sequence of brushing, picking, dropping algorithms.

In this paper, we propose a different approach which is based on image processing. The paper is organized as follows. Section 2 describes how to build a density map. The extraction of the main flows is given in Section 3. In Section 4, we derive a new index of complexity basing the BV norm. Finally, we provide illustrations in Section 5.

2 Generating the Density map

This section describes how the density map is built. The initial data set consists in a set of trajectories for several days of traffic in the French airspace. Each trajectory is represented by a set of samples gathering positions, time, speed vector and id which are accumulated on a map.

We set the size of the map to 800×1000 pixels. In order to increase efficiency of the algorithm, we first interpolate data. Some of the previous interpolation models have been tried and compared. For our purposes, cubic spline interpolation has been chosen as it produces the best results in term of error between models and observation. Cubic spline interpolation is easy to implement and produces a curve that appears to be seamless. Furthermore, it is an efficient and numerically stable method for determining smooth curves from a set of points. After interpolating, we scale the trajectory as a mapping from $\mathbb{R}^2$ into $[0, 1]$ (2D image). It is then quite simple to build a matrix from such a 2D map. This matrix (Map) will represent the aircraft density of the given airspace and is built with the following process.

The map is built from the interpolated trajectories as follows: the value at each square pixel of the map (indexed by the couple (i,j)) is the number of trajectories going through this pixel. We then obtain a matrix with integer values ranging from 0 to the total number of trajectories. Suppose that (x, y) is a grid point of some trajectory. Then

$$\mathrm{Map}[i][j] = \mathrm{Map}[i][j] + 1, \text{ with } i = ceil(y * length), \ j = ceil(x * width),$$

here $length \times width$ is the size of the map. The function *ceil* extract the integer part of a given real number. The Figure 1 illustrates how we can build the matrix of density map.

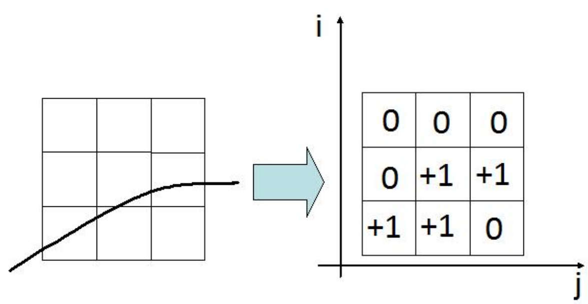


Figure 1: Establishing the matrix of density map

3 Medial axis extraction

After getting the map from the above section it is possible to generate a traffic picture. Figure 2 shows the density map of one day of traffic over France. Major air traffic flows clearly appear in the airspace. This image is quite clear but not sufficient for identifying major flows.

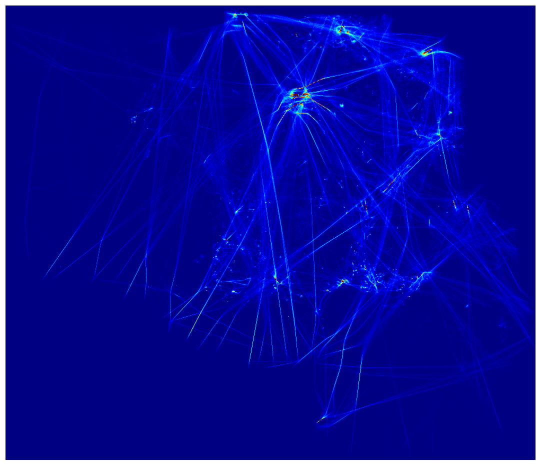
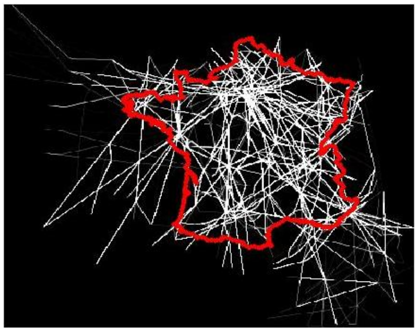


Figure 2: Traffic over France generated from the density map

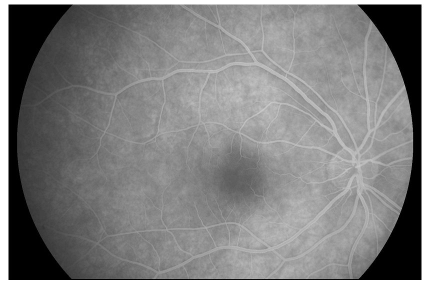
In order to produce images which are easier to understand and sharper, we scale the density map to a gray scale matrix. This allows to extract the flows without losing the structure of the airspace. The Figure 3 (a) shows a grey scale image of the traffic over France.

The map which we get from the density of traffic is analogous to the shape of retinal image (See Figure 3 (b)).

In another context, namely that of angiography of the retina [9], the local BV-norm turned out to be an interesting indicator of complexity, and provided an efficient criterion for the classification of retinal images. This encouraged us to envisage using the same criterion for the evalution of complexity in the context of air traffic management. Intuitivemly, the reason is that the BV-norm is well adapted to the measurement of local variations. As far as traffic complexity is concerned, our intuition is that the local complexity is directly related to the variations of the image whose construction was described earlier in the paper. The simulations we shall provide in the last section corroborate this intuition.



(a) An Image which was created from the gray matrix.



(b) An example of a retinal image

Figure 3: Pictures illustrate similarities between the air traffic map and the retinal image

We treat the main flows in the air traffic map as vessels in the eye image. So, the remaining task of medial axis extraction has changed to the task of detection and measurement of blood vessels in retinal images. Detection and measurements of vessel caliber have been widely studied using a variety of methods. In the retinal vessel segmentation process, the outcome is a pixel-based classification result. Any pixel is classified either as vessel or surrounding tissue. A number of methods for segmenting vascular network have been reported. In our research, we chose the algorithm of the authors Bankhead P *et al.* [1] to get the features of the major flows (the centerlines, the diameters of flow, the directions of flow, etc.). It is based on a flow representation, similar to the vessel extraction in the previous sections.

The input data set contains 104072 trajectories which were collected during one week, between 21*st* and 27*th* October 2013, in France. We applied the algorithm to each day of traffic.

The Figure 4 shows that we can effectively apply the method for getting medial axes. It also shows that almost all main flows in the air traffic were totally extracted.

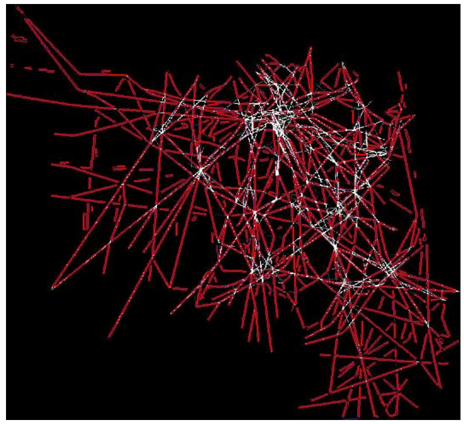


Figure 4: Major flows extraction in the French airspace

4 Application of BV norm to airspace complexity

The total variation has been introduced in Computer Vision first by Rudin, Osher and Fatemi [10], as a regularizing criterion for solving inverse problems. It has been proven to be quite efficient for regularizing images without smoothing the boundaries of the objects. Normally, one can apply it for solving differential equations as well as for image processing. By contrast, we use the total variation not for reconstruction purpose, but rather for classification. Our images are given (so we never have to minimize the TV), and we actually compute their total variation only to analyze their features. In this paper the total variation is used for measuring the change of image intensity.

We calculate BV norm at each point on centerlines of the flows in circle area or in rectangular area.

4.1 Notations and Preliminary Remarks

Let X denote the Euclidean space $\mathbb{R}^{N \times N}$. In order to define the discrete total variation, we introduce a discrete (linear) gradient operator. If $\mathbf{u} \in X$, the gradient $\nabla \mathbf{u}$ is a vector in $Y = X \times X$ given by

$$(\nabla \mathbf{u})_{i,j} = \left((\nabla \mathbf{u})^1_{i,j}, (\nabla \mathbf{u})^2_{i,j} \right)$$

with

$$(\nabla \mathbf{u})^1_{i,j} = \begin{cases} \mathbf{u}_{i+1,j} - \mathbf{u}_{i,j} & \text{if } i < N \\ 0 & \text{if } i = N \end{cases}$$

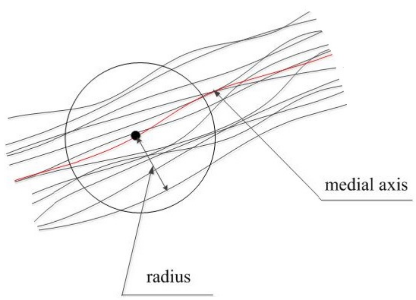
$$(\nabla \mathbf{u})^2_{i,j} = \begin{cases} \mathbf{u}_{i,j+1} - \mathbf{u}_{i,j} & \text{if } j < N \\ 0 & \text{if } j = N \end{cases}$$

for $i, j = 1, ..., N$.
Then, the total variation of $\mathbf{u}$ is defined by

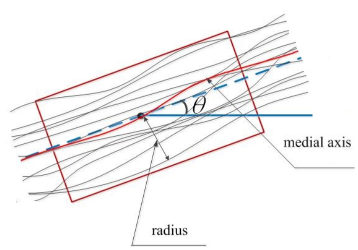
$$J(\mathbf{u}) = \sum_{1 \leq i,j \leq N} |(\nabla \mathbf{u})_{i,j}| \tag{1}$$

with $|\mathbf{y}| = \sqrt{y_1^2 + y_2^2}$ for every $\mathbf{y} = (y_1, y_2) \in \mathbb{R}^2$. We calculate the BV norm along the main flows and use it as a metric of the complexity in the airspace.

After extracting all major flows and their features by using the algorithm described in [1], we calculate the total variation on two different domains (circle and rectangular) (see Figure 5 (a) and Figure 5 (b)).



(a) BV norm computation on circle domain.



(b) BV norm computation on rectangular domain

Figure 5: Pictures illustrate the domain on which BV norm is computed

In the case of a circular domain, the total variation at each point P is given by

$$J(\mathbf{u}) = \sum_{(i,j) \in I} |(\nabla \mathbf{u})_{i,j}| \tag{2}$$

where I is the set of double indices corresponding to points in the disc of center P and radius R. Note that $R \geq r$, where r is the radius of flow at P (which is determined by experiment).

In the case of a rectangular domain, the total variation is given by

$$J(\mathbf{u}) = \sum_{(i,j) \in D} |(\nabla \mathbf{u})_{i,j}| \tag{3}$$

where D is set of double indices corresponding to points in the rectangle with center P, and width $w \geq 2r$. The domain D is rotated by an angle θ, which is the direction of the flow.

We used the color map to represent the value of BV norm at points located along centerlines. The figures below illustrate the results when we calculate the BV norm with different domains and the associated density map.

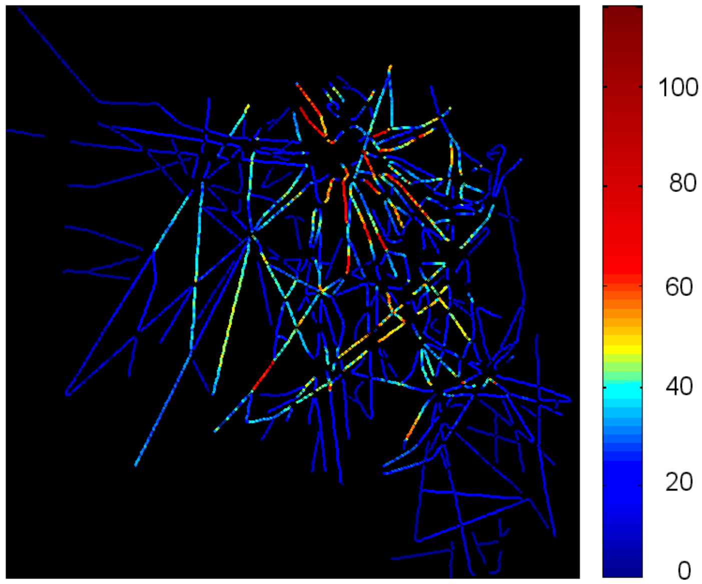


Figure 6: BV-norm values computed with circular domain

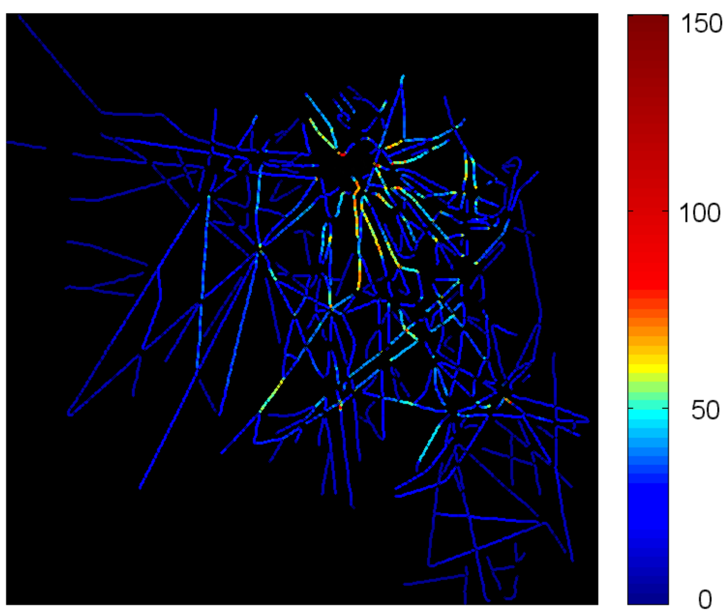


Figure 7: Density map

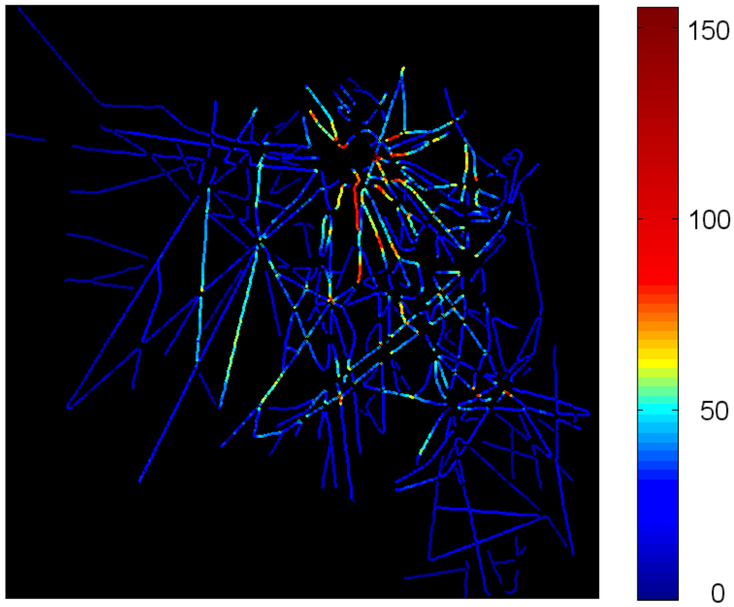


Figure 8: BV-norm values computed with rectangular domain

As expected, in both cases, the main complexity is located around the Paris area which is known to be the most complex airspace in France. The BV-norm gives more information on the complexity than the density.

5 Conclusions

In this work, we proposed a method to extract effectively main flows. And then, we introduced a novel approach for defining an air traffic complexity indicator. The algorithm was tested on data from the French airspace. It is shown that this index provides a more valuable information than density in terms of complexity. With this new indicator of air traffic complexity, we believe that it can be used to improve and upgrade the concept of dynamic density.

References

[1] Peter Bankhead, C Norman Scholfield, J Graham McGeown, and Tim M Curtis. Fast retinal vessel detection and measurement using wavelets and edge location refinement. *PloS one*, 7(3):e32435, 2012.

[2] Daniel Delahaye, Stéphane Puechmorel, P Tsiotras, and E Feron. Mathematical models for aircraft trajectory design: A survey. In *Air Traffic Management and Systems*, pages 205–247. Springer, 2014.

[3] Adric Eckstein. Automated flight track taxonomy for measuring benefits from performance based navigation. In *Integrated Communications, Navigation and Surveillance Conference, 2009. ICNS'09.*, pages 1–12. IEEE, 2009.

[4] Marco Enriquez. Identifying temporally persistent flows in the terminal airspace via spectral clustering. In *Tenth USA/Europe Air Traffic Management Research and Development Seminar (ATM2013)*, 2013.

[5] Marco Enriquez and Christopher Kurcz. A simple and robust flow detection algorithm based on spectral clustering. In *ICRAT Conference*, 2012.

[6] JM Histon, RJ Hansman, Blake Gottlieb, Howard Kleinwaks, Sarah Yenson, Daniel Delahaye, and Stéphane Puechmorel. Structural considerations and cognitive complexity in air traffic control. In *Digital Avionics Systems Conference, 2002. Proceedings. The 21st*, volume 1, pages 1C2–1. IEEE, 2002.

[7] Jonathan M Histon, R John Hansman, Guillaume Aigoin, Daniel Delahaye, and Stephane Puechmorel. Introducing structural considerations into complexity metrics. 2002.

[8] Aude Marzuoli, Vlad Popescu, and Eric Feron. Two perspectives on graph-based traffic flow management. *First SESAR Innovation Days*, 2011.

[9] B G. Nguyen, D. Delahaye, S. Puechmorel, P. Marechal, and P. Olle. Using bv-norm to classify the vasculitis in multiple sclerosis fundus angiography for ophthalmologists. *Journal of Medical and Bioengineering*, 2(1):11–15, 2013.

[10] Leonid I Rudin, Stanley Osher, and Emad Fatemi. Nonlinear total variation based noise removal algorithms. *Physica D: Nonlinear Phenomena*, 60(1-4):259–268, 1992.

[11] Lixia Song, Daniel Greenbaum, and Craig Wanke. The impact of severe weather on sector capacity. In *8th USA/Europe Air Traffic Management Research and Development Seminar (ATM2009), Napa, California, USA*, 2009.

[12] Lixia Song, Craig Wanke, and Daniel Greenbaum. Predicting sector capacity for tfm decision support. In *6th AIAA Aviation, Integration, and Operations Conference*, 2006.