Preliminary Assessment of Fuel-Efficient Arrival Trajectories Accounting for Current Aircraft State

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The continued increase of air traffic, which doubles every 15 years, has produced economic benefits but poses environmental issues that put at risk the sustainable development of air transport. Factors such as jet fuel prices volatility, the introduction of new environmental regulations and intense competition in the airline industry incentive the research on flight efficiency topics. Despite fuel costs in descent and approach phases are minor than those in cruise phase, flight crew high-workload during these phases often lead to energy mismanagement resulting in inefficient flight operations. This paper proposes an optimization algorithm that generates fuel-efficient arrival trajectories accounting for the current aircraft position. Results show 13% fuel savings compared with a best-in-class Flight Management System and, at the same time, descent time is reduced by 1%. A preliminary assessment has been conducted in the flight simulator with the aim of validating the operational feasibility of the computed trajectory. Data post-analysis suggest that the trajectory is flyable with current flight controls and guidance laws although idle margins shall be readjusted to account for accelerations during descent. This paper may constitute a solid background for the likely development of flight efficiency functions for the future generation of avionic systems.

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Nomenclature

δ_{ab} = Airbrakes position
$\dot{V}$ = Aircraft acceleration
γ = Aerodynamic flight path angle
γ_T = Total flight path angle
D = Aerodynamic drag force
ds = Distance span
E_{k_s} = Specific potential energy
E_{p_s} = Specific kinetic energy
E_{t_s} = Specific total energy
ESF = Energy Share Factor
FF = Fuel flow
g_0 = Gravitational constant
h = Geometric altitude
m = Aircraft mass
s = Aircraft distance
t = Flight time
Thr = Thrust force
V_w = Wind airspeed
CI = Cost Index
V = True airspeed

I. Introduction

Air traffic continued growth in the last decades and predictions for the coming years [1] require a modernization of current Air Transportation System (ATS). The increased number of operations results in the congestion of airspaces with the consequent propagation of delays [2]. The environmental footprint of aviation industry already constitutes a 3.6 % of the total European greenhouse emissions [3]. Noise levels in the vicinity of airports have decreased by a 14% per flight due to technological improvements on aircraft, nevertheless the average noise exposure grows [3] because of air traffic increase. Moreover, other factors such as jet fuel prices volatility [4] and airlines strong market competition promote flight efficiency as a central topic for research.

In recent years, flight efficiency has been largely improved through initiatives such as Free Route Airspace (FRA) [5] or the introduction of Continuous Descent Operations (CDO) [6], which have demonstrated to be generally more efficient than conventional step-down operations in terms of noise and gas emissions [7]. In major airports surrounded by large residential areas, airspace designers have introduced noise abatement procedures based on CDO that are operated during certain periods of time, usually in nighttime. Similarly, novel Area Navigation (RNAV) design procedures in the United States contain a set of window altitude constraints located higher than regular altitudes restrictions and define a descent corridor that enables CDO. Furthermore, the complex environment in descent and approach phases represents a high level of workload for pilots, and complicates the optimal management of aircraft trajectory with the consequent impact on flight efficiency. This paper proposes an on-boarded function that generates permanent optimal trajectories based on enhanced energy management. The term permanent refers to the continuous trajectory computed from the destination airport up to the current aircraft state regardless of the current guidance mode. Hence, the calculations take into account the aircraft current position and energy condition with the aim of supporting flight crew decision-making processes and pave the route for advanced automation capabilities in the future.

The Flight Management System (FMS) entered into service in the early 1980s [8] and decreased navigation workload in a manner that reduced the flight crew from three members to two. The system performs relevant functions [9] such as navigation, flight planning (both lateral and vertical), performance and provision of guidance and display commands. This paper focuses on descent and approach operations where state-of-the-art FMS compute a vertical profile based on the lateral path defined by the arrival procedure and other parameters such as the Cost Index (CI), cruise flight level and an estimation of the mass at the destination. The profile is constructed upstream from the runway threshold until the cruise altitude and consist of a concatenation of idle and geometric segments, the latter being constructed as soon as an altitude constraint restricts the construction of the idle path. Depending on the nature of the altitude constraints, geometric segments require either auto-thrust adjustments to maintain a speed target in shallow path or airbrakes extension in steep segments, while the elevator guides the aircraft through the vertical path. In contrast, idle segments set auto-thrust to idle whilst the elevator maintains the target speed. The path is constructed through the integration of the equations of motion and decelerations are computed by a fixed Energy Share Factor (ESF), which distributes the available energy between altitude and speed, permitting the aircraft to descend and decelerate at the same time. In traditional step-down approaches, aircraft deceleration to approach speed is performed in a level-flight located at glide-slope capture altitude whereas, in the frame of CDO, aircraft decelerate and change flap configurations as they descend. The main limitation of this approach is that depending on the aircraft performance and the arrival procedure, deceleration to approach speed might be initiated too soon, in certain cases well above 7000 feet, which requires the anticipation of flap changes leading to long approach procedures that, in some cases, require more fuel than conventional step-down profiles. Therefore, current FMS hypotheses are valid for the construction of any arrival procedure but the resulting trajectory is not necessarily optimal. The design could be enhanced by taking into account other types of segments for the profile construction, which depend on the selected arrival procedure. These segments may require to fly at other speeds than those used by the traditional Mach/CAS descent law.

The provision of radar vectors from ATC, unexpected wind errors or biased mass estimations are the most probably causes for profile deviations from the intended routes. In these situations, the FMS profile is not recomputed to take into consideration the current aircraft energy condition, so flight crews adapt the flight strategy according to their criterion. The aircraft energy condition is defined as the sum of potential and kinetic energy so that high-energy occurs when the aircraft is too fast, too high or both, while low-energy state implies that the aircraft is low or below its target speed. In flight operations, the term energy management refers to the continuous transformation of energy that occurs due to the use of flight controls as the aircraft descends to the destination. In the absence of a valid reference trajectory, pilots are responsible for managing aircraft energy state through power and control devices such as thrust levers, airbrakes, landing gear and flap settings that modify the aircraft energy rate. The second enhancement of FMS design proposed in this paper is the continuous recomputation of the optimal trajectory on the basis of current aircraft position. This is the concept of the permanent trajectory, where the calculation always reaches the aircraft position independently of the current energy state or flight mode, and applies the necessary energy management strategy to dissipate any excess of energy during the approach. The permanent trajectory is a relevant concept as not only helps pilots in energy management decision-making processes but also would permit to completely automate the approach phase and then constitute a significant improvement to efficient operations.

Trajectory optimization is usually formulated as an Optimal Control Problem (OCP) [10] solved through direct, indirect or dynamic programming methods [11]. Past works have solved the problem through pseudo-spectral methods, as it is the case for the Time and Energy Management Operations (TEMO) [12] function that minimizes the number of energy corrections to compensate wind errors and satisfy a time constraint. Indirect methods are implemented in the real-time algorithm [13], whose results are compared with a FMS under different wind conditions [14]. While most works optimize fuel consumption other focus on the minimization of noise and pollutant emissions [15]. Low Noise Augmentation System (LNAS) presented in [16] computes dynamically airbrakes, landing gear and flap settings extension in order to stabilize the aircraft whilst minimizing noise impact. The function has been embarked on an Electronic Flight Bag (EFB) and has been successfully flight tested. A* algorithm [17] has been successfully implemented in [18], [19] and [20] to compute optimal trajectories but they rely on simple heuristic functions that represent no advantage with regards to classic Dijkstra's algorithm [19]. Other methods like Energy-Optimal Path-Tracking algorithm introduced in [21] and [22] optimize the descent and approach due to the minimization of the energy path and proposes a relation between flight time and fuel consumption [23]. The main drawback of most works found in the literature is that they solve typical descent profiles, based on a series of consecutive segments, instead of particular arrival procedures. This lack of generality implies that the algorithm may not applicable to certain operational procedures. This is the goal of the algorithm presented in this paper that provides an optimal trajectory for any existing arrival procedure as long as a solution exists, and accounts for the current aircraft position. The objective of this paper is to extend the previous work [24] by comparing the obtained results with a real FMS and, then, perform a preliminary operational assessment of in the flight simulators, with the aim of validating the representativeness of the computed profile and fuel savings.

The outline of this paper is the following; the mathematical formulation of the problem is presented in section II whereas the main functioning principles of the algorithm are succinctly described in section III. Then, section IV discusses the results obtained for the presented case study, whose trajectory is compared with that of a FMS and tested in the simulator. Finally, section V concludes the study and sets the future line of work.

II. Mathematical Formulation

A. Optimization Problem

The aircraft motion in the vertical plane is represented by a point-mass model that provides a sufficient level of representativeness [25] from a performance perspective. The generation of the trajectory focuses on slow dynamics variables and disregards fast dynamics such as flight path angle rate ($\dot{\gamma}$). The objective function is the minimization of fuel consumption along the trajectory, which is given by the following expression:

$$J = \min \int_{s_0}^{s_f} \left(\mathrm{FF} + \frac{\mathrm{CI}}{60}\right) \frac{1}{V\cos\gamma + V_w}\, ds \tag{1}$$

The fuel flow FF is expressed in $\frac{kg}{s}$ and the CI in $\frac{\$/min}{\$/kg} = \frac{kg}{min}$, which equals to zero for the fuel minimization problem. The algorithm computes optimal trajectories by means of an Airbus genuine Performance database (PDB) containing engine, aerodynamic and other performance data. However, for confidentiality reasons, any data related to the aircraft performance is not displayed in this paper.

B. Aircraft Model

The aircraft model is described in the PDB, which contains engine, aerodynamic and performance-related data for the given aircraft type. The engine model contains thrust maximum and minimum (idle) settings whose use depends on the flight phase. The thrust setting parameter (TSP) is a scalable factor independent of the engine type, which is defined by:

$$TSP_{min,max} = f(h, M) \tag{2}$$

Thrust minimum and maximum ratings are computed by means of Eq. (2).

$$Thr_{min,max} = f(TSP, M) \tag{3}$$

In certain cases, thrust is computed by general equations of motion and TSP value is iterated from Eq. (3). Eventually, fuel flow is computed through the following expression:

$$FF = f(TSP, h, M) \tag{4}$$

This paper presents a mathematical formulation based on the total energy of the aircraft, which directly relates the altitude and speed with the control variables. Aircraft specific energy is defined as the sum of kinetic and potential energy independent of aircraft weight. The derivation of the term with respect to time gives the energy rate or energy height:

$$\dot{E}_{T_s} = \dot{E}_{k_s} + \dot{E}_{p_s} = V \sin\gamma + \frac{V\dot{V}}{g_0} \tag{5}$$

The ESF is defined as the percentage of total energy rate attributed to kinetic energy whilst the remaining goes to potential energy:

$$ESF = \frac{\dot{E}_{k_s}}{\dot{E}_{k_s} + \dot{E}_{p_s}} \tag{6}$$

The combination of Eqs.(5) and (6) lead to the following expression:

$$\sin\gamma = \frac{(1 - ESF)}{ESF} \, \frac{\dot{V}}{g_0} \tag{7}$$

The total flight path angle or total energy angle (γ_T) is the available energy budget to be distributed between potential and kinetic energy, thus is represented as the sum of the aerodynamic flight path angle (γ) and the resulting acceleration:

$$\sin\gamma_T = \sin\gamma + \frac{\dot{V}}{g_0} = \frac{\sin\gamma}{1 - ESF} = \frac{Thr - D\big(h, V, \delta_{ab}\big)}{m g_0} \tag{8}$$

In this paper, the classic time-dependent equations of motion are converted into distance-dependent, since it simplifies the constraint management and both initial and final distance are known, contrary to the final time. The introduction of Eqs.(7) and (8) combined with the small angle approximation, $\sin\gamma_T \approx \gamma_T$, result in the following formulation:

$$\begin{cases} h' = \dfrac{V\ (1 - ESF)\ \gamma_T}{V\cos\gamma + V_w} \\ V' = \dfrac{g_0\ ESF\ \gamma_T}{V\cos\gamma + V_w} \\ m' = \dfrac{dm}{ds} = \dfrac{-FF}{V\cos\gamma + V_w} \\ t' = \dfrac{dt}{ds} = \dfrac{1}{V\cos\gamma + V_w} \end{cases} \tag{9}$$

The previous equations of motion in Eq.(9) describe the state variables of the problem:

$$x(s) = \{h, V, m, t\} \tag{10}$$

Then, the control variables that generate the states of Eq.(10) are defined by:

$$u(s) = \{\gamma_T, ESF, \delta_{ab}, Conf\} \tag{11}$$

C. Problem Constraints

Constraints on state variables are given by the arrival procedure design and the associated altitude and speed constraints contained in the Navigation Database (NDB) [26]:

$$\begin{cases} \text{AT OR ABOVE} \rightarrow \quad h \geq h_{CSTR} \\ \text{AT OR BELOW} \rightarrow \quad h \leq h_{CSTR} \\ \text{WINDOW} \rightarrow \quad h_{CSTR} \geq h \leq h_{CSTR} \\ \text{AT} \rightarrow \quad h = h_{CSTR} \end{cases} \tag{12}$$

Similarly, a speed constraint limits aircraft speed below a certain value:

$$V_{CAS} \leq V_{CAS_{CSTR}} \tag{13}$$

Furthermore, ATC regulation generally imposes a maximum speed of 250 kt Calibrated Airspeed (CAS) for all aircraft below FL100:

$$V_{CAS} \leq V_{CAS_{SPDLIM}} \quad \forall h \leq FL100 \tag{14}$$

In addition to the previous boundaries, aircraft speed shall remain within the flight envelope defined by the stall speed (V_{LS}) and the maximum operating speed (V_{MO}):

$$V_{LS} \leq V_{CAS} \leq V_{MO} \tag{15}$$

During the approach phase, flap changes decrease the stall speed and the next configuration is limited to the maximum flap extended speed (V_{FE}) in order to avoid structural damage:

$$V_{LS} \leq V_{CAS} \leq V_{FE} \quad \forall Conf \neq clean \tag{16}$$

Longitudinal accelerations are limited to take into account passengers comfort [27] as follows:

$$\left|\frac{dV}{dt}\right| \leq 0.07 \cdot g_0 \tag{17}$$

The control variables γ_T, ESF and δ_{ab} are bounded between a maximum and minimum value. The thrust require to perform a level-off at constant speed defines the upper limit of γ_T, whilst idle thrust plus full airbrakes set the lower bound:

$$\gamma_{T_{min}}\Big|_{\substack{Thr=Thr_{idle}\\ D=\delta_{ab_{Full}}}} \leq \gamma_T \leq \gamma_{T_{max}}\Big|_{Thr=D} \tag{18}$$

Climb segments ($\gamma > 0$) are forbidden per design so that any $\gamma_T > 0$ corresponds to a speed acceleration. The ESF is limited operationally:

$$ESF \in \{-0.5, ..., 1\} \tag{19}$$

It can be observed from Eq.(6) that $ESF = 1$ results in a decelerated level-off segment; for an $ESF = 0$, the energy budget is dedicated to the descent while the true airspeed remains constant. Then, any $ESF < 0$ provides a steep segment where part of the potential energy is transformed into kinetic. Finally, airbrakes extension is limited between zero and the maximum deflection angle, which varies as a function of altitude and speed:

$$0 \leq \delta_{ab} \leq \delta_{ab_{Full}} \tag{20}$$

Finally, flap settings are changed according to the aerodynamic change configuration speed ($V_{CC_{ap}}$) for each flap deflection, as defined in the PDB:

$$Conf \in \{\text{clean}, 1, 2, 3, \text{Full}\} \tag{21}$$

III. Trajectory Optimization Algorithm

The algorithm implemented for the computation of optimal trajectories is a version of A* [17] completely adapted and generalized [28] to comply with any existing arrival procedure, and assures that the path reaches the aircraft position. The stabilization point is the initial state of the algorithm and is generally defined as the point of a trajectory located 1000 feet above the runway [29] , where the aircraft is in landing configuration as indicated in Fig. 1. Therefore, the calculation is performed upstream from that stabilization gate to the current aircraft position, which defines a permanent optimal trajectory linking two particular energy states.

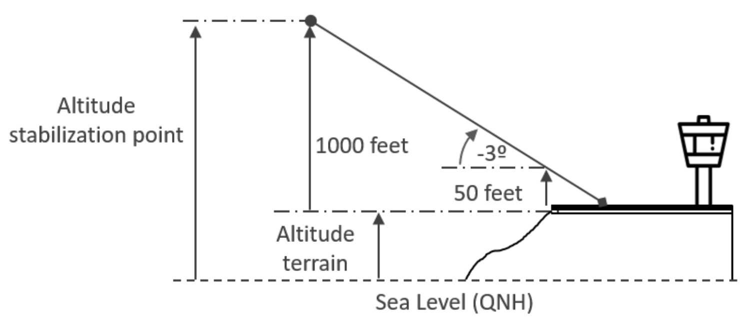


Figure 1. Definition of the stabilization point.

The heuristic function is constructed on the basis of a constraint-less idle trajectory maintaining $V_{CAS_{Gdot}}$, which provides the best lift-to-drag ratio for a certain altitude and mass. Both admissibility and consistency properties are satisfied as the function is optimistic (estimated cost is always lower than the actual optimal) and monotonically decreasing along the path. The search space is incrementally generated from the initial node until the neighborhood area around the target state is reached. The generation of nodes is done through the discretization of control variables and accounts for the distribution of constraints for the construction and pruning of the search space. Hence, the number of combinations and candidate states is reduced and nodes are generated at relevant locations of the search space. The resulting search space at the end of the computation is displayed in Fig. 2, which terminates when a node falls within the neighborhood zone around the target and there is no more promising node in the Open list. From that node, parents are retrieved until the start node is attained, so the projections on the XZ-plane form the altitude profile whilst the speed profile is defined by the XY-plane. The constraints (magenta triangles) prune the search space and the resulting trajectory complies with the procedure design.

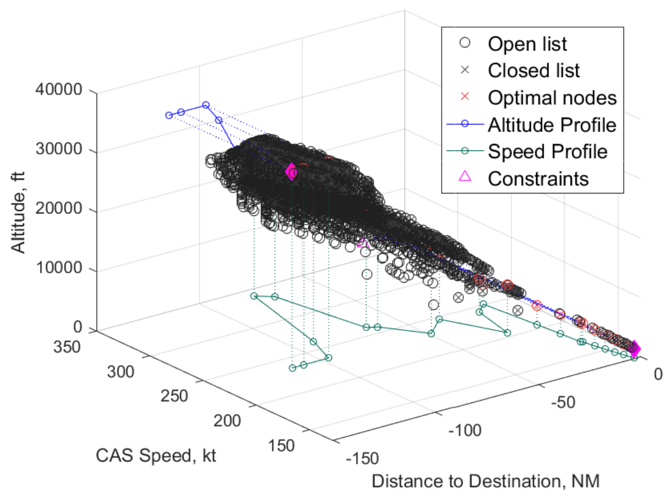


Figure 2. Search Space after calculation.

From an operational point of view, flight crew would follow the computed path manually or automatically except when any circumstance forces the aircraft to deviate from the intended route. In that case, the calculation shall be relaunched to adapt the flight strategy to the dynamic aircraft state (tactical approach). In case that a solution does not exist due to the complexity of the procedure design and aircraft performance, the algorithm explores all candidate nodes and produces an error message, resulting in large computation times.

IV. Results and Findings

A. Trajectory Optimization at Los Angeles (KLAX) airport

This chapter presents and discusses the results for the case study performed at Los Angeles (KLAX) airport. The trajectory is computed assuming that the aircraft is still on cruise phase. This trajectory is compared with a real FMS and has been tested in the simulator to assess the operational concept. The selected aircraft model is the A320, since fuel savings in descent and approach phases represent a higher percentage for short-haul aircraft than for long-haul, and the figures are multiplied as soon as the number of operations per day and the airline fleet are taken into account. The following Table 1 defines a set of parameters used for the calculation:

Table 1. General parameters of the calculation.

Parameters	
Aircraft type	A320
Cost Index, kg/min	0
Gross weight, t	60
Wind, m/s	0
ΔISA, °C	0

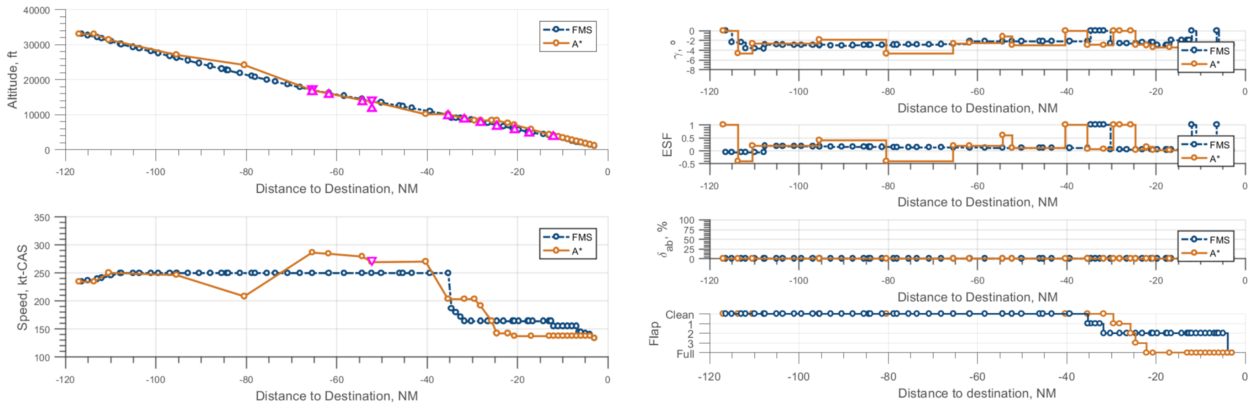
The aircraft is relatively heavy as the landing weight of 60 tonnes corresponds to 90% of the maximum landing weight (MLW), a representative value of airlines operations. The selected Standard Terminal Arrival Route (STAR) procedure corresponds to that of a Required Navigation (RNAV) procedure at KLAX airport, called SEAVU2. This procedure has been selected due to the number of constraints and the fact that track variations are small from the entry point to the runway. A standard precision approach (ILS-24L) to runway 24L has been selected. The initial and final conditions are resumed in Table 2:

Table 2. KLAX case study initial and final conditions.

	Initial state	Final state
Distance to destination, NM	-2.95	-116.5
Altitude, ft	1125	33000
Speed, kt	133.8	235
Flap settings	Full	Clean

B. Trajectory comparison with a certified FMS

The final state determined by the FMS computation permits to compare with the calculation provided by the A* algorithm. Nevertheless, A* computes the optimal trajectory for any final state while the FMS always calculates a top of descent, regardless of the actual aircraft position. The altitude and speed profile computed by the algorithm is compared to that of a real FMS as displayed in Fig. 3(a). A vertical discontinuity is clearly seen at -35 NM in the speed profile; the aircraft does not accelerate –backwards– and descend simultaneously to satisfy the constraints. As to the A* trajectory, initially the aircraft remains in landing configuration until eventually it accelerates to 270 knots in two consecutive level-offs, Fig. 3(b), at the same time as the altitude constraints are satisfied. On the contrary, the FMS constructs a geometric segment with a shallow path, which fails to accelerate to 250 knots. The fuel consumption is reduced by 13% with respect to the FMS calculation whereas arrival time is as well decreased by 1%. In general, fuel savings are well localized; in this case, the shorter approach path and the removal of the geometric path, which occurs between -35 and -61 NM, during the descent are the causes of this decrease. The A* design splits geometric paths into several segments, which yields a more efficient energy repartition that maintains thrust idle as long as possible, and only adds thrust at the most favorable altitudes.



(a) Altitude and speed profile. (b) Flight control values.

Figure 3. Trajectory comparison between real FMS and A* calculation for KLAX.

C. Assessment of the trajectory in A320 simulator

The computed trajectory was flown in a simulator at Airbus facilities to verify that the behavior of the aircraft was consistent with the calculations. The integration points used by the algorithm were entered manually into the flight plan through their latitude and longitude coordinates, which helped to change the flight path targets at the correct distance for the trajectory monitoring.

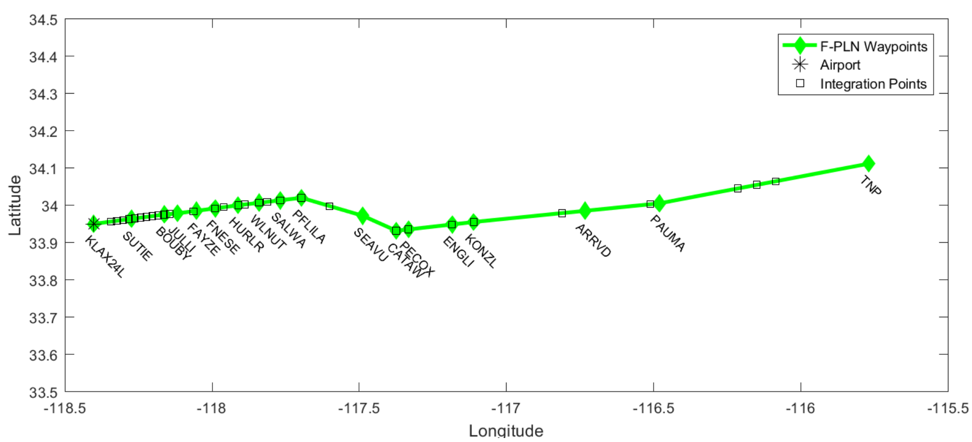


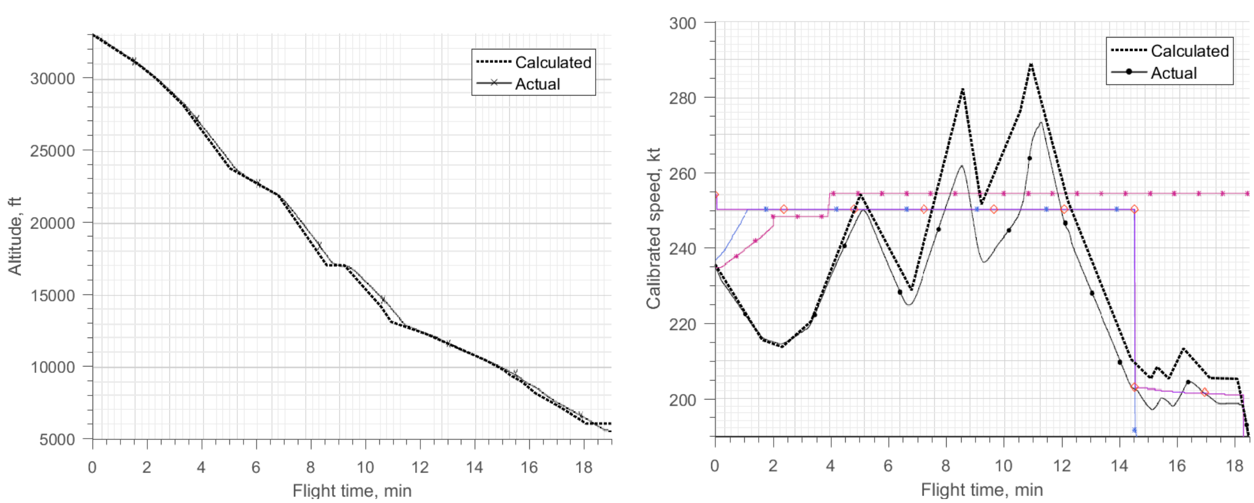
Figure 4. Flight plan preparation for KLAX arrival procedure.

Figure 4 shows the STAR procedures waypoints (green-diamond) and those computed by the A* (black-squared) that were stored in the FM memory and entered in the flight-plan. As of today, there is no guidance mode that follows automatically the computed trajectory. Thus, the trajectory was flown with autothrust off, thrust levers manually adjusted at idle setting and the auto-pilot switched on in order to follow the lateral path. The vertical motion was managed by means of FPA adjustments. Figure 5 gives an example of the autoflight and thrust configurations that were set during the tests:

(a) Flight Control Unit (FCU) adjustment. (b) Idle thrust levers.

Figure 5. Autoflight and thrust levers setting for the simulation tests.

The trajectory was followed properly through successive FPA changes at the proper integration points. The pilot-flying (PF) adjusted the FPA values on the FCU whereas the pilot non-flying (PNF) checked that the changes were done at the proper distance and both the altitude and speed profiles were followed. Note that neither the PF nor the PNF are professional flight test pilots. As a general principle, transitions from shallow to steep paths were anticipated to limit over-shooting.



(a) Altitude profile. (b) Speed profile.

Figure 6. Trajectory comparison between A* calculation and simulator flight.

The comparison between the trajectory calculated by the algorithm and that flown in the simulator is given in Fig. 6 as a function of time. The speed profile observed in Fig. 6(b) is not the same than that of Fig. 3, since it corresponds to a previous calculation where the 270 knot speed constraint was misplaced with the purpose of observing higher speed variations. Whereas the altitude profile was followed correctly, speed deviations are observed in Fig. 6(b); the aircraft decelerates as calculated but accelerates less than expected. This is due to the existence of idle margins that over-estimate the actual idle rating of the engines, which are only used for the calculation of the profile. Thrust was manually added to mitigate the lack of acceleration, as it can be observed in Fig. 6(b) between minutes 11 and 12. The altitude error at 5000 ft is about 400 ft, which is relatively small compared with the speed error of 8 knots, approximately. From an operational perspective, this preliminary assessment suggests that the concept is not operational as of today, due to the increased workload resulting from the continuous changes of FPA target, but is physically flyable from a performance perspective. The design and implementation of a guidance mode on the basis of the energy-sharing concept could enable this type of flight operations. From an air traffic perspective, the acceptance of variable optimal speed descent profiles instead of traditional Mach/CAS may depend upon the implementation of the trajectory information sharing from an aircraft to ground.

V. Conclusion

This paper proposes an algorithm that computes, upstream, the optimal trajectory for any published arrival procedure (STAR). The trajectory is permanent in the sense that it accounts for the current aircraft state regardless of the energy condition and guidance mode. Results show that 13% of fuel savings can be obtained with respect to state-of-the-art FMS computation at the same time as flight time is reduced by 1%. The trajectory was flown in an Airbus A320 simulator, equipped with a certified FMS, in order to assess the operational representativeness of the calculation. Data post-analysis suggest that the aircraft is capable of following the calculated altitude profile with current guidance modes but the workload is increased as a result of FPA changes. Speed deviations occur as the aircraft accelerates worse than calculated, thus idle margins used in descent shall be refined to better model accelerations during this phase. Besides, simulations confirmed that vertical discontinuities on the flight plan could be simply solved through appropriate energy management. The calculation of the descent and approach profile allows to accelerate or decelerate as required and does not follow a traditional Mach/CAS construction. In order to be more representative of current flight operations, additional constraints can be implemented by giving-up optimality. As a result, it could encompass the implementation of certain concepts disclosed in this paper in current FMS standards. In conclusion, the algorithm proposed in this paper yields the most fuel-efficient trajectory that reaches the current aircraft state, no matter what the energy condition or guidance mode is, and that could be followed with state-of-the-art guidance laws. Nonetheless, it is planned to test the function on other aircraft types with reduced idle margins in order to corroborate the fuel and time benefits. Besides, further investigation and evaluation with flight tests pilots is required in order to mature the operational concept. The generation of an optimal trajectory which takes into account the current aircraft position on a real-time basis is an enabler for the development of more automated cockpits in the light of improving the efficiency of the flight.

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