

Analytic Formulation of the Doppler VOR Multipath Error for a Digital IQ Receiver Model with a Quadrature FM Demodulator

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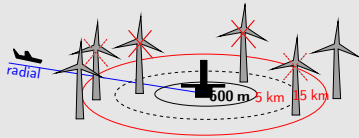


VOR system

- VOR gives an aircraft its bearing (azimuth) with respect to a ground beacon
- VHF frequency band (108-118 MHz), horizontal polarization
- Two components : a reference signal (REF) and a variable azimuth-dependent signal (VAR)
- Two types of VOR station : Conventional VOR and Doppler VOR

Building restricted areas

- Multipath yields bearing errors in the VOR system
- Surrounding objects can be a potential major contributor to this error and particularly wind turbines
- ICAO recommendations exist but the applied rules differs between countries



Challenge

- Develop a tools to assess the impact of wind turbines on bearing errors

Introduction

ENAC-EMA contribution

- Static VOR error accuracy has been given in [Morlaas et al., ITAES 2008]
- In [Morlaas et al., Eucap 2010], a hybridization of parabolic equation with physical optics has been proposed, called VERSO for VOR Error SimulatOR
- Improvements of this method have been discussed in [Claudepierre et al., Eucap 2015]
- In [Ben-Hassine et al., EuCAP 2019], a digital IQ receiver model including filtering and demodulation responses, has been developed
 - Reproduce the response of a VOR receiver in a realistic way
 - Take into account the variations in time of multipath in the bearing error

Previous study : Doppler multipath analysis

- Analysis of Bredemeyer measurements in [Bredemeyer, FCS 2015] indicates that the REF signal must be considered in the Doppler VOR error computation, contrary to the Odunaiya model proposed in [Morlaas et al., ITAES 2008]

Objectives

- Present the digital IQ receiver model
- Develop and validate an alternative expression of the analytic Doppler VOR multipath error which is consistent with our FM demodulator type \Rightarrow Show that the bearing error is sensitive to the type of FM demodulator for a Doppler VOR signal

Plan

- 1 Introduction
- 2 VOR Receiver Model
- 3 Analytic Doppler VOR multipath error : Theory
- 4 Analytic Doppler VOR multipath error : Validation
- 5 Conclusion

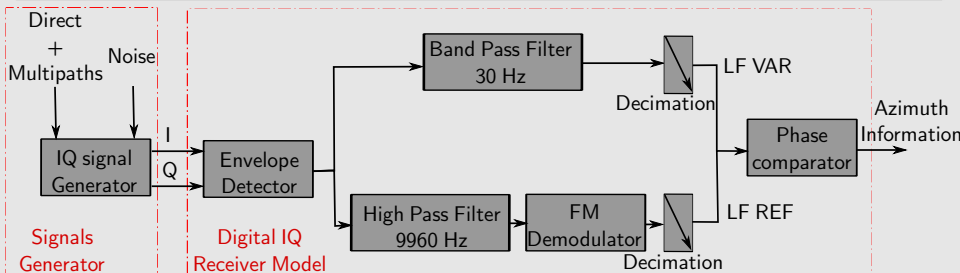
VOR Receiver Model

Objective

- Present the digital VOR receiver model

Proposed model

- Developed model based on I/Q signals
- **Signal generator**
 - Generate I/Q channels from time series of multipath parameters
- **Digital IQ receiver model**
 - Envelope detector
 - REF channel
 - VAR channel
 - Azimuth estimation



Times Series VOR Multipath Simulator

Times series generator

- **Goal** : sample a realistic aircraft trajectory to obtain time series of multipath parameters (amplitude a_n , phase θ_n and azimuth φ_n)
- Aircraft trajectory is a series of straight or circular elements separated by way-points
- Time series give the evolution of multipath along a sampled trajectory
- Two consecutive samples must be separated by a sufficiently short distance (Δx) with respect to the wavelength at VHF frequency

$$\Delta x < \frac{\lambda}{2} \quad (1)$$

- Time step of sampling is fixed according to the space step and the maximum speed (V_{max}) along the trajectory

$$\Delta t_{min} = \frac{\Delta x}{V_{max}} \quad (2)$$

- **Example**

$$F_{VOR} = 113.8 \text{ MHz}, \Delta x = \frac{\lambda}{5} \simeq 0.526 \text{ m}, V_{max} = 500 \text{ km/h} \Rightarrow \Delta t_{min} \simeq 3 \text{ ms}$$

Extraction of multipath parameters

- **Method 1** : canonical extraction
- **Method 2** : VERSO simulator [Chabory et al., EMWT 2019]

VOR Receiver Model

I/Q signals Generator

- **Goal** : obtain the data received by a VOR receiver
- Sample the time series of multipath parameters so as to fulfill the Nyquist criterion at 10440 Hz
- I/Q signals are defined by

$$I(t_k) = \sum_{n=0}^N M_n(t_k) \cos(\theta_n(t_k)) + n_I(t_k), \quad (3)$$

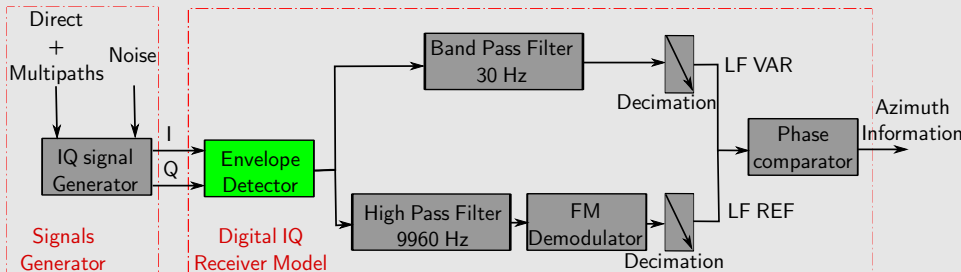
$$Q(t_k) = \sum_{n=0}^N M_n(t_k) \sin(\theta_n(t_k)) + n_Q(t_k). \quad (4)$$

- The n -th multipath signal M_n is defined by

$$M_n(t_k) = a_n(t_k)[1 + REF(t_k) + VAR_n(t_k)] \quad (5)$$

- $REF(t_k) = m_a \cos(\omega_{sc} t_k + m_f \sin(\omega_{LF} t_k))$
- $VAR_n(t_k) = m_a \cos(\omega_{LF} t_k - \varphi_n(t_k))$
- $n_I(t_k)$, $n_Q(t_k)$ the AWGN noises associated to the I/Q signals, respectively
- ω_{sc} the subcarrier pulse (9960 Hz) in rad/s
- ω_{LF} the LF pulse (30 Hz) in rad/s
- m_a the modulation index (= 0.3)
- m_f the modulation rate (= 16)

VOR Receiver Model

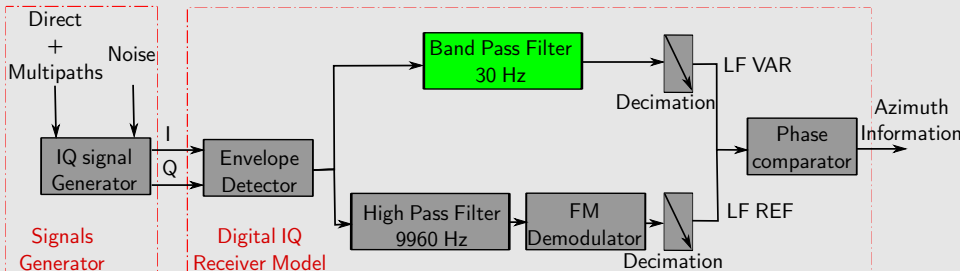


Envelope detection

- Carry out an AM demodulation in order to recover the VAR and REF signals
- The complex envelope is defined by

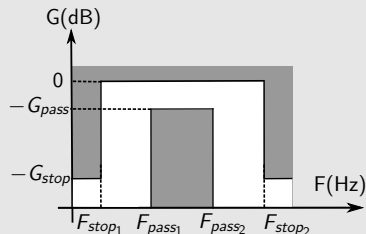
$$A(t_k) = \sqrt{I^2(t_k) + Q^2(t_k)}$$

VOR Receiver Model

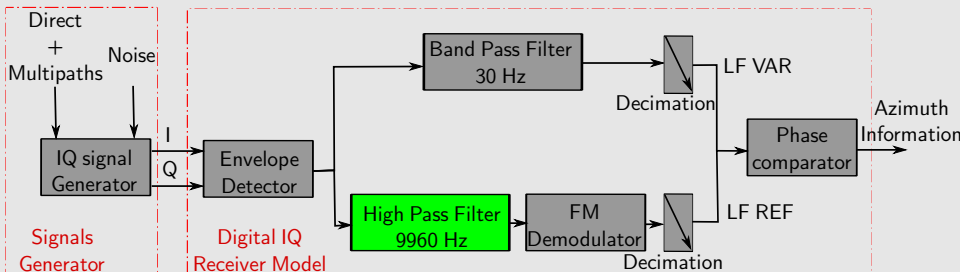


Band pass filtering

- Extract the 30 Hz VAR signal
- Parameters
 - Type : Butterworth
 - Passband frequency : $F_{pass} = [F_{pass1}, F_{pass2}] = [29, 31]$ Hz
 - Stopband frequency : $F_{stop} = [F_{stop1}, F_{stop2}] = [24, 36]$ Hz
 - Passband ripple : $R_{pass} = 0.1$ dB
 - Stopband ripple : $R_{stop} = 30$ dB

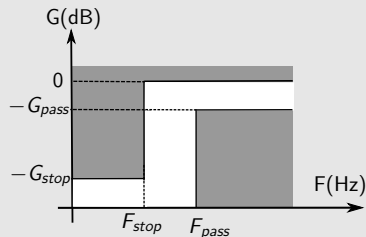


VOR Receiver Model

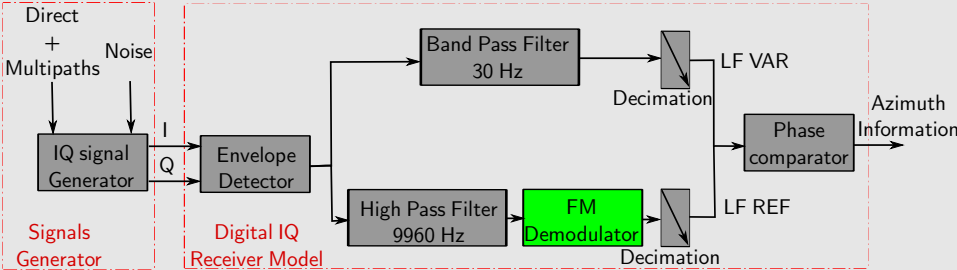


High pass filtering

- Extract the narrowband REF signal 9960 ± 480 Hz
- Parameters
 - Type : Kaiser
 - Passband frequency : $F_{pass} = 8000$ Hz
 - Stopband frequency : $F_{stop} = 7000$ Hz
 - Passband ripple : $R_{pass} = \emptyset$ dB
 - Stopband ripple : $R_{stop} = 60$ dB

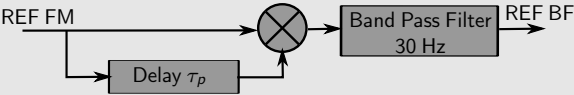


VOR Receiver Model

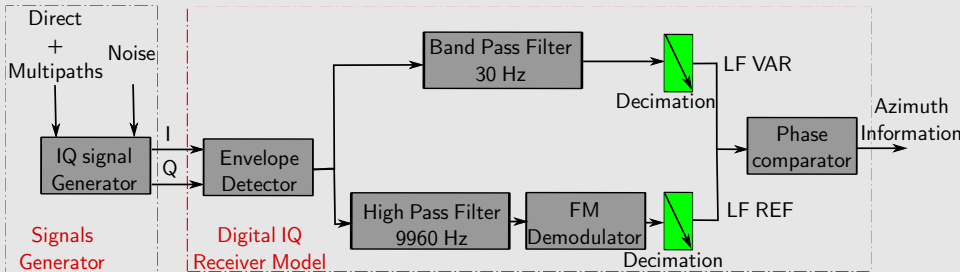


FM demodulation

- Quadrature demodulation
- Extract the LF modulating signal
- Band pass filter parameters
 - Type : Butterworth
 - $F_{pass} = [29, 31]$ Hz
 - $F_{stop} = [24, 36]$ Hz
 - $R_{pass} = 0.1$ dB
 - $R_{stop} = 30$ dB



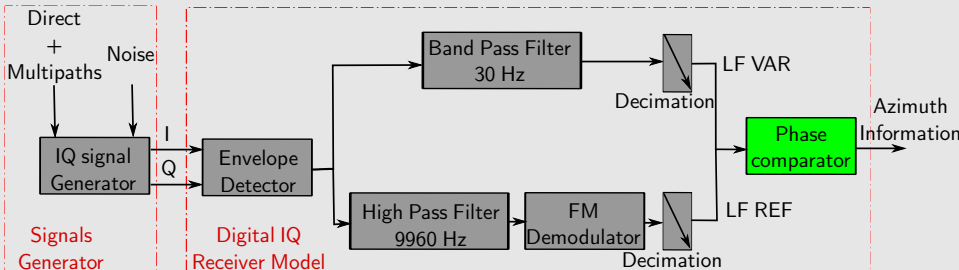
VOR Receiver Model



Decimation

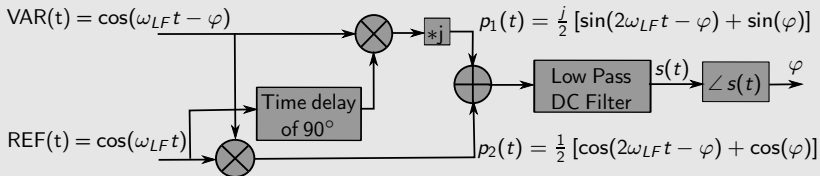
- Reduce the computation time
- Keep a number of samples as small as possible (the Nyquist criterion at 30 Hz)
- There are sampling criteria applied before and after decimation

VOR Receiver Model

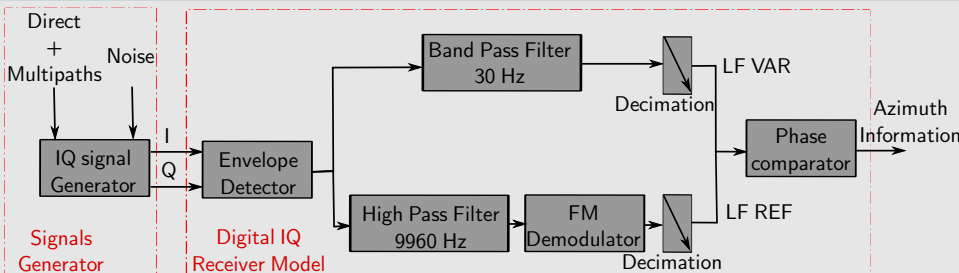


Phase comparator

- Calculate the phase difference between the AM and FM 30 Hz signals



VOR Receiver Model



Used model parameters

- IQ sampling frequency = 25 kHz
- Decimation rate = 210
- Filtering parameters

	Type	F_{pass} (Hz)	F_{stop} (Hz)	R_{pass} (dB)	R_{stop} (dB)
Band pass filter 30 Hz	Butterworth	[29,31]	[24,36]	0.1	30
Low pass filter DC	Butterworth	1	2	0.1	20
High pass filter 9960 Hz	Kaiser	8000	7000	∅	60

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Odunaiya expression (1)

Objectives

- Present and analyze the Odunaiya VOR Doppler error
- Illustrate the hypotheses of Odunaiya's demonstration

Expression and hypotheses

- Static model
- **Expression** [Odunaiya *et al.*, DASC 2004]

$$\varepsilon_{Ody}^D = \frac{2}{m_f} \sum_{n=1}^N \frac{a_n}{a_0} \cos(\theta_n - \theta_0) J_1(2m_f \sin(\frac{\varphi_n - \varphi_0}{2})) \cos(\frac{\varphi_n - \varphi_0}{2}) \quad (6)$$

J_1 is the Bessel function of the first kind

- **hypotheses**
 - Weak amplitude multipath
 - Ideal FM demodulation \Rightarrow The demodulated signal is proportional to the phase derivative of input signal

Odunaiya expression (2)

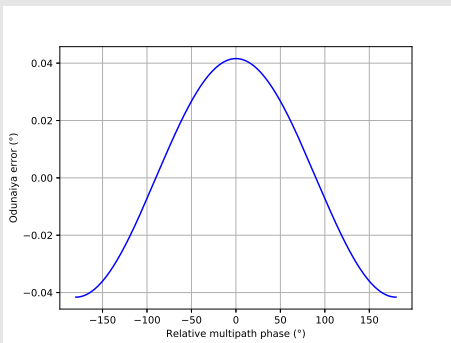


Figure 2 – Odunaiya response for $(\varphi_1 - \varphi_0) = 6.5^\circ$ and $\frac{a_1}{a_0} = 0.01$

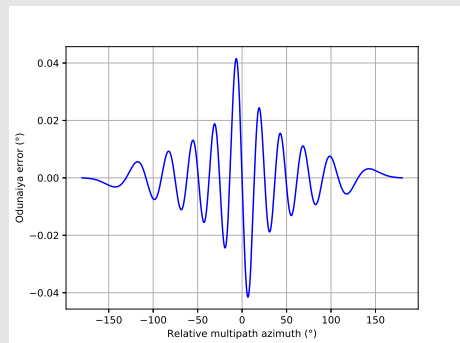


Figure 3 – Odunaiya response for $(\theta_1 - \theta_0) = 0^\circ$ and $\frac{a_1}{a_0} = 0.01$

Analysis

- The error is maximum when the multipath and direct signal are in phase or in phase opposition
- The error is maximum when the relative azimuth of multipath $(\varphi_n - \varphi_0)$ tends to $\pm 6.5^\circ$

Illustration test : IQ receiver model vs Odunaiya expression

Objective

- Illustrate a comparison test between the receiver model and Odunaiya expression by processing a canonical multipath for a Doppler VOR

Comparison test

- Number of multipath : $N = 1$
- Relative multipath amplitude : $\frac{a_1}{a_0} = 0.01$
- Relative multipath phase : $\theta_1 - \theta_0 = 0^\circ$
- Relative multipath azimuth : $\varphi_1 - \varphi_0$ varies between 180° and -180°
 \Rightarrow Model a receiver rotation around the VOR station

Analysis

- Disagreement between the results for a Doppler VOR signal processing !

\Rightarrow Is it due to the FM demodulator type???

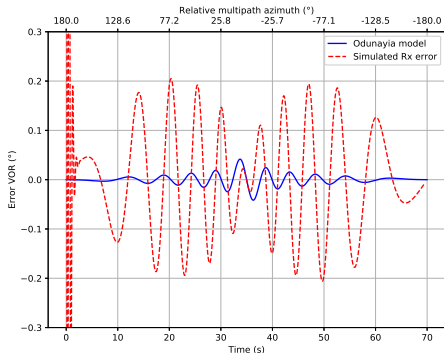
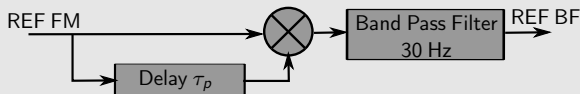


Figure 4 – Doppler VOR error

Analytic Doppler error with a quadrature FM demodulator : Theory (1)

FM demodulator

- Quadrature demodulation
- Extract the LF modulating signal



Hypotheses

- Single weak multipath
- Ideal high pass filtering
- Absence of noise

Equation setting

- The variable signal at the FM demodulator input

$$U_{Var}(t) = R(t) \cos(\omega_{sc}t + \phi_{LF}(t)) \quad (7)$$

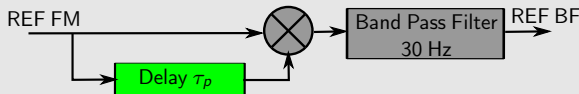
- $R(t) = \sqrt{A_0^2 + A_1^2 + 2A_0A_1 \cos(\phi_{sc}^1(t) - \phi_{sc}^0(t))}$
- $\phi_{LF}(t) = \phi_{sc}^0 + \frac{A_1}{A_0} \sin(\phi_{sc}^1(t) - \phi_{sc}^0(t))$

- $A_0 = m_a a_0 \cos(\theta_0)$
- $A_1 = m_a a_1 \cos(\theta_1)$
- $\phi_{sc}^0(t) = m_f \sin(\omega_{LF}t - \varphi_0)$
- $\phi_{sc}^1(t) = m_f \sin(\omega_{LF}t - \varphi_1)$

Analytic Doppler error with a quadrature FM demodulator : Theory (2)

FM demodulator

- Quadrature demodulation
- Extract the LF modulating signal



Step 1 : delay operation

- Delay the FM input signal by τ_p (in second) \Rightarrow Introduce a quadrature phase shift on the cosine argument

$$\omega_{sc}\tau_p = \omega_{sc}\frac{p}{F_e} = \frac{\pi}{2}[\pi] + \varepsilon, (p \in \mathbb{N}) \quad (8)$$

ε is the quadrature phase shift error

- τ_p is taken small compared to the period of LF signal

$$\omega_{LF}\tau_p \simeq 0 \quad (9)$$

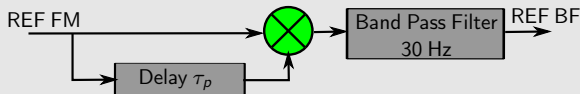
- **Output**

$$U_{Var}(t - \tau_p) = R(t - \tau_p) \cos(\omega_{sc}(t - \tau_p) + \phi_{LF}(t - \tau_p)) \quad (10)$$

Analytic Doppler error with a quadrature FM demodulator : Theory (3)

FM demodulator

- Quadrature demodulation
- Extract the LF modulating signal



Step 2 : mixer

- Apply a product between the FM signal and its delayed version \Rightarrow Appear the difference term of the LF arguments

• Output

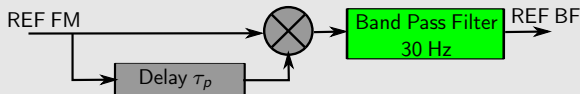
$$p(t) = \frac{R(t)R(t - \tau_p)}{2} [\cos(2\omega_{sc}t - \omega_{sc}\tau_p + \phi_{LF}(t) + \phi_{LF}(t - \tau_p)) + \cos(\omega_{sc}\tau_p + \delta\phi_{LF})] \quad (11)$$

- $\delta\phi_{LF} = \phi_{LF}(t) - \phi_{LF}(t - \tau_p)$

Analytic Doppler error with a quadrature FM demodulator : Theory (4)

FM demodulator

- Quadrature demodulation
- Extract the LF modulating signal



Step 3 : Band pass filter

- Filter the product signal \Rightarrow obtain the demodulated LF signal

• Output

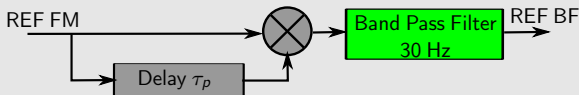
~~$$p(t) = \frac{R(t)R(t - \tau_p)}{2} [\cos(2\omega_{sc}t - \omega_{sc}\tau_p + \phi_{LF}(t) + \phi_{LF}(t - \tau_p)) + \cos(\omega_{sc}\tau_p + \delta\phi_{LF})] \quad (12)$$~~

- $\delta\phi_{LF} = \phi_{LF}(t) - \phi_{LF}(t - \tau_p)$

Analytic Doppler error with a quadrature FM demodulator : Theory (5)

FM demodulator

- Quadrature demodulation
- Extract the LF modulating signal



Hypotheses and simplifications (2)

$$p(t) = \frac{R(t)R(t - \tau_p)}{2} \cos(\omega_{sc}\tau_p + \delta\phi_{LF}) \quad (13)$$

- Weak amplitude multipath \Rightarrow apply the Taylor series expansion on the term $R(t)R(t - \tau_p)$

$$R(t)R(t - \tau_p) \simeq A_0^2 \left[1 + \frac{A_1}{A_0} \left[\cos(\phi_{sc}^1(t) - \phi_{sc}^0(t)) + \cos(\phi_{sc}^1(t - \tau_p) - \phi_{sc}^0(t - \tau_p)) \right] \right] + o(A_1) \quad (14)$$

- Simplification of the computation using Fourier developments expressed with Bessel functions

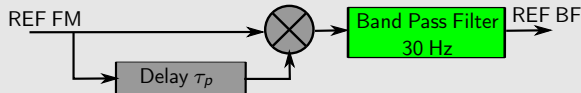
$$\cos(\phi_{sc}^1(t) - \phi_{sc}^0(t)) = J_0(z) + 2 \sum_{n=1}^{\infty} (-1)^n J_{2n}(z) \cos(2n\omega_{LF}t - n(\varphi_1 + \varphi_0)) \quad (15)$$

$$z = -2m_f \sin\left(\frac{\varphi_1 - \varphi_0}{2}\right)$$

Analytic Doppler error with a quadrature FM demodulator : Theory (6)

FM demodulator

- Quadrature demodulation
- Extract the LF modulating signal



Hypotheses and simplifications (1)

$$p(t) = \frac{R(t)R(t - \tau_p)}{2} \cos(\omega_{sc}\tau_p + \delta\phi_{LF}) \quad (16)$$

- τ_p is taken small compared to the period of LF signal

$$\cos(\omega_{sc}\tau_p + \delta\phi_{LF}) \simeq \tau_p \dot{\phi}_{LF}(t - \frac{\tau_p}{2}) + \varepsilon + o(\varepsilon) + o(\tau_p^2) + o(\delta\phi_{LF}^3) \quad (17)$$

- LF phase derivative of the input signal is given by

$$\dot{\phi}_{LF}(t) = \dot{\phi}_{sc}^0(t) + \frac{A_1}{A_0}(\dot{\phi}_{sc}^1(t) - \dot{\phi}_{sc}^0(t)) \cos(\phi_{sc}^1(t) - \phi_{sc}^0(t)) \quad (18)$$

- Simplification of the computation using Fourier developments expressed with Bessel functions

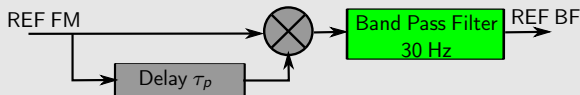
$$\cos(\phi_{sc}^1(t) - \phi_{sc}^0(t)) = J_0(z) + 2 \sum_{n=1}^{\infty} (-1)^n J_{2n}(z) \cos(2n\omega_{LF}t - n(\varphi_1 + \varphi_0)) \quad (19)$$

$$z = -2m_f \sin\left(\frac{\varphi_1 - \varphi_0}{2}\right)$$

Analytic Doppler error with a quadrature FM demodulator : Theory (7)

FM demodulator

- Quadrature demodulation
- Extract the LF modulating signal



Hypotheses and simplifications (3)

$$p(t) = \frac{R(t)R(t - \tau_p)}{2} \cos(\omega_{sc}\tau_p + \delta\phi_{LF}) \quad (20)$$

- The high frequencies are filtered, only the first term of the Fourier series is considered

⇒ The demodulated variable signal is expressed by

$$p_f(t) = A' \cos(\omega_{LF}t - \tilde{\varphi}^D) \quad (21)$$

- $A' = \frac{-\tau_p m_f \omega_{LF}}{2} A_0^2$
- $\sin(\tilde{\varphi}^D) = 2 \frac{A_1}{A_0} J_1'(z) \sin(\varphi_1 - \varphi_0)$
- $\cos(\tilde{\varphi}^D) = 1 + 2 \frac{A_1}{A_0} J_1'(z) [1 + \cos(\varphi_1 - \varphi_0)]$
- $z = -2m_f \sin\left(\frac{\varphi_1 - \varphi_0}{2}\right)$

Analytic Doppler error with a quadrature FM demodulator : Theory (8)

Analytic Doppler error

- Extract the VOR error from the demodulated variable signal

$$p_f(t) = A' \cos(\omega_{LF}t - \tilde{\varphi}^D) \quad (22)$$

- The Doppler VOR error for single multipath is expressed by

$$\varepsilon_{R_x}^{D,1} = \tan^{-1} \left(\frac{\sin(\tilde{\varphi}^D)}{\cos(\tilde{\varphi}^D)} \right) = \tan^{-1} \left(\frac{2 \frac{A_1}{A_0} J'_1(z) \sin(\varphi_1 - \varphi_0)}{1 + 2 \frac{A_1}{A_0} J'_1(z) [1 + \cos(\varphi_1 - \varphi_0)]} \right) \quad (23)$$

$$z = -2m_f \sin \left(\frac{\varphi_1 - \varphi_0}{2} \right)$$

⇒ The Doppler VOR error for N weak multipath is given by

$$\varepsilon_{R_x}^{D,N} = 2 \sum_{n=1}^N \frac{a_n}{a_0} \cos(\theta_n - \theta_0) J'_1 \left(-2m_f \sin \left(\frac{\varphi_n - \varphi_0}{2} \right) \right) \sin(\varphi_n - \varphi_0) \quad (24)$$

Analytic Doppler error with a quadrature FM demodulator : Theory (9)

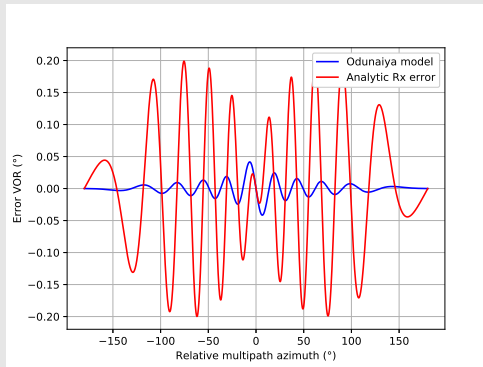


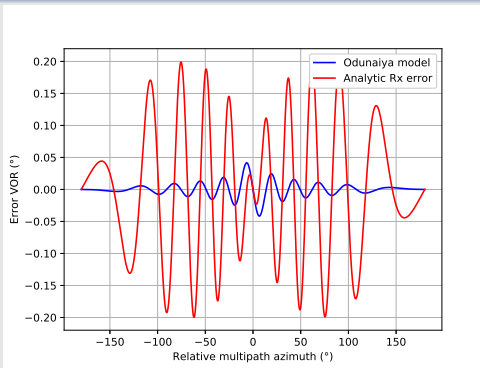
Figure 5 – Doppler VOR error for $\theta_n - \theta_0 = 0^\circ$

Receiver model vs Odunaiya

- Analytic receiver error : $\varepsilon_{Rx}^{D,N} = 2 \sum_{n=1}^N \frac{a_n}{a_0} \cos(\theta_n - \theta_0) J_1' \left(-2m_f \sin \left(\frac{\varphi_n - \varphi_0}{2} \right) \right) \sin(\varphi_n - \varphi_0)$
- Odunaiya error : $\varepsilon_{Ody}^D = \frac{2}{m_f} \sum_{n=1}^N \frac{a_n}{a_0} \cos(\theta_n - \theta_0) J_1 \left(2m_f \sin \left(\frac{\varphi_n - \varphi_0}{2} \right) \right) \cos \left(\frac{\varphi_n - \varphi_0}{2} \right)$

⇒ Quadrature phase shift between the two responses on maximum value of error

Analytic Doppler error with a quadrature FM demodulator : Theory (10)



Receiver model vs Odunaiya

- Analytic receiver error : $\varepsilon_{Rx}^{D,N} = 2 \sum_{n=1}^N \frac{a_n}{a_0} \cos(\theta_n - \theta_0) J_1' \left(-2m_f \sin \left(\frac{\varphi_n - \varphi_0}{2} \right) \right) \sin(\varphi_n - \varphi_0)$
- Odunaiya error : $\varepsilon_{Ody}^D = \frac{2}{m_f} \sum_{n=1}^N \frac{a_n}{a_0} \cos(\theta_n - \theta_0) J_1 \left(2m_f \sin \left(\frac{\varphi_n - \varphi_0}{2} \right) \right) \cos \left(\frac{\varphi_n - \varphi_0}{2} \right)$

⇒ Difference in error level : maximum of analytic receiver error is 5 times greater than the one of Odunaiya error

	$\varphi_n - \varphi_0 = \pm 6.5^\circ$	$\varphi_n - \varphi_0 = \pm 62^\circ, \pm 76^\circ$
Maximum error	Odunaiya	Receiver model

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Analytic Doppler error with a quadrature FM demodulator : Validation (1)

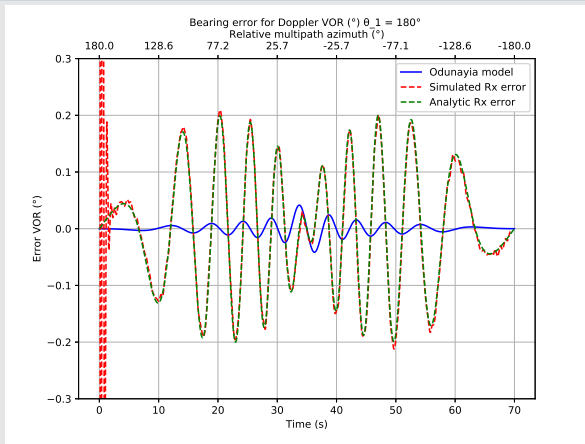


Figure 7 – Doppler VOR error for canonical multipath

Analysis

- Good agreement between the analytic Doppler error estimated by the receiver and the simulation
- Bearing error is sensitive to the type of FM demodulator for a Doppler VOR signal

Analytic Doppler error with a quadrature FM demodulator : Validation (2)

Configuration in multipath variation context

- DVOR station
 - $F = 113 \text{ MHz}$
 - $P = 50 \text{ W}$
- 1 wind turbine is placed at 1 km from the VOR
 - Simplified model
 - Based on ENERCON E82 size
 - Mast : ground diameter = 7 m, top diameter = 3 m, height = 90 m
 - Nacelle : $3 \text{ m} \times 3 \text{ m} \times 3 \text{ m}$
 - Blades : length = 10 m
- VOR Receiver
 - Circular trajectory
 - Departure = $[4500, 0, 1000] \text{ m}$
 - Speed = $0 \rightarrow 180 \text{ km/h}$
 - Space step = $\frac{\lambda}{5} \simeq 0.526 \text{ m}$
 - Motionless for the first 10 s

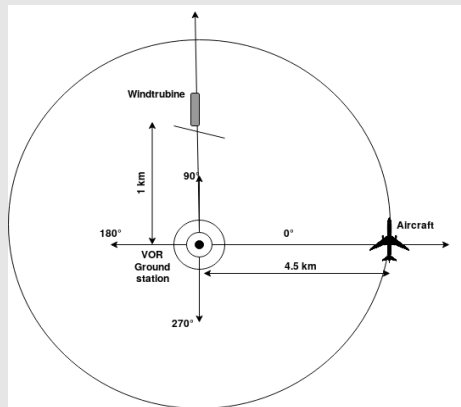


Figure 8 – Configuration

Analytic Doppler error with a quadrature FM demodulator : Validation (3)

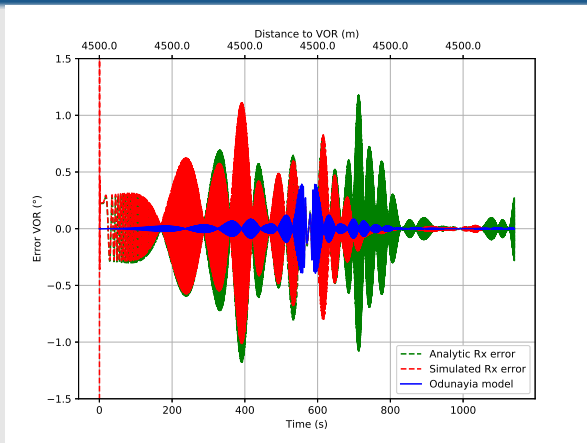


Figure 9 – Doppler VOR error for realistic complex scenario

Analysis

- Slowly moving aircraft \Rightarrow Slow multipath variation \Rightarrow analytical error = simulated error
- Faster aircraft at the end of trajectory \Rightarrow Fast multipath variation \Rightarrow analytical error \neq simulated error \Rightarrow Dynamic effect of multipath [*Ben-Hassine et al.*, EMWT 2019], [*Ben-Hassine et al.*, EuCAP 2020]

Conclusion

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- Presentation of the architecture of the digital IQ receiver model
- Showing the sensitivity of the FM demodulator type on the error model of a Doppler VOR receiver
- Development and of an alternative expression of Doppler bearing consistent with the quadrature FM demodulator
- Validation for canonical and dynamic simulation

Future works

- Develop a PLL (Phase Locked Loop) demodulator
- Confrontation with calibration (Rohde & Schwarz EVS300) and avionic receivers \Rightarrow To identify the receiver behavior
- Complete analysis of time variation of multipath on VAR and REF signals