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Piecewise Polynomial Model of the Aerodynamic Coefficients of the *Cumulus One* Unmanned Aircraft

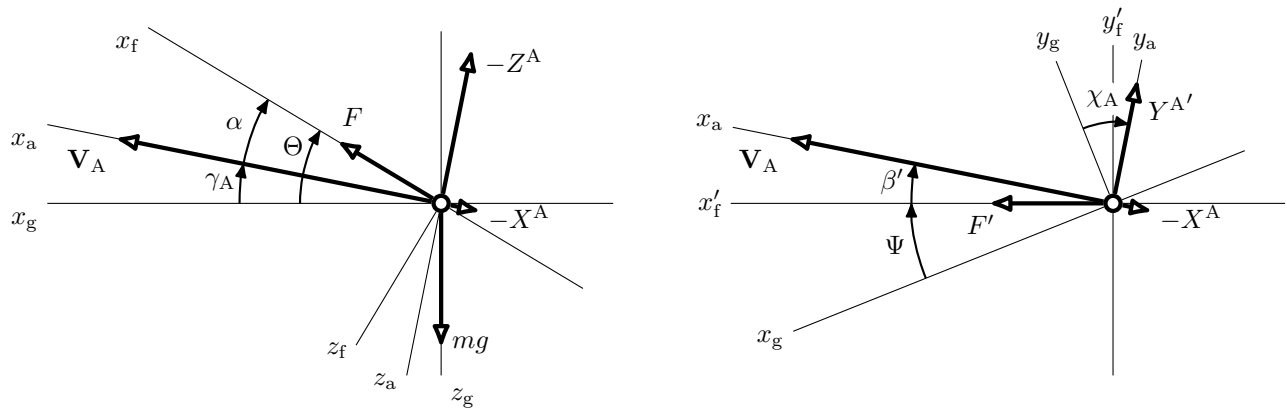
Torbjørn Cunis¹ and Anders La Cour-Harbo²

Abstract

This document illustrates a piecewise polynomial model for Cumulus One, based on continuous fluid dynamics simulation, derived using the `pwpfit` toolbox. The model has been developed in a joint project of Sky-Watch and the University of Aalborg.

PRELIMINARIES

If not stated otherwise, all variables are in SI units. We will refer to the following axis systems of ISO 1151-1: the *body axis system* (x_f, y_f, z_f) aligned with the aircraft's fuselage; the *air-path axis system* (x_a, y_a, z_a) defined by the velocity vector \mathbf{V}_A ; and the *normal earth-fixed axis system* (x_g, y_g, z_g) . The orientation of the body axes with respect to the normal earth-fixed system is given by the attitude angles Φ, Θ, Ψ and to the air-path system by angle of attack α and side-slip β ; the orientation of the air-path axes to the normal earth-fixed system is given by azimuth χ_A , inclination γ_A , and bank-angle μ_A . (Fig. ??.)



(a) Longitudinal axes ($\beta = \mu_A = 0$).

(b) Horizontal axes ($\gamma_A = 0$).

Fig. 1: Axis systems with angles and vectors. Projections into the plane are marked by $'$.

I. AERODYNAMIC COEFFICIENTS

The piece-wise polynomial models of the aerodynamic coefficients are given

$$C_{\odot}(\alpha, \beta, \xi, \eta, \zeta) = \begin{cases} C_{\odot}^{pre}(\alpha, \beta, \dots) & \text{if } \alpha \leq \alpha_0, \\ C_{\odot}^{post}(\alpha, \beta, \dots) & \text{else,} \end{cases} \quad (1)$$

where $C_{\odot} \in \{C_X, C_Y, C_Z, C_l, C_m, C_n\}$ are polynomials in angle of attack, side-slip, and surface deflections; and the boundary is found at

$$\alpha_0 = 17.949^\circ. \quad (2)$$

The polynomials in low and high angle of attack, C_{\odot}^{pre} , C_{\odot}^{post} , are sums

$$C_{\odot}^{pre} = C_{\odot\alpha}^{pre}(\alpha) + C_{\odot\xi}(\beta, \xi) + C_{\odot\eta}^{pre}(\alpha, \eta) + C_{\odot\zeta}(\beta, \zeta); \quad (3)$$

$$C_{\odot}^{post} = C_{\odot\alpha}^{post}(\alpha) + C_{\odot\xi}(\beta, \xi) + C_{\odot\eta}^{post}(\alpha, \eta) + C_{\odot\zeta}(\beta, \zeta). \quad (4)$$

In the following subsections, we present the polynomial terms obtained using the `pwpfit` toolbox. Coefficients of absolute value lower than 10^{-2} have been omitted for readability.

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A. Domain of low angle of attack

Polynomials in angle of attack:

$$C_{X\alpha}^{pre} = -2.566 \times 10^{-2} + 5.722 \times 10^{-1}\alpha + 1.496\alpha^2 - 1.148 \times 10^1\alpha^3; \quad (5)$$

$$C_{Y\alpha}^{pre} = 5.402 \times 10^{-2} - 2.345 \times 10^{-1}\alpha - 2.001\alpha^2 + 7.054\alpha^3; \quad (6)$$

$$C_{Z\alpha}^{pre} = -3.475 \times 10^{-1} - 5.467\alpha + 1.853\alpha^2 + 2.663 \times 10^1\alpha^3; \quad (7)$$

$$C_{l\alpha}^{pre} = 4.875 \times 10^{-2} - 2.190 \times 10^{-1}\alpha - 2.004\alpha^2 + 7.146\alpha^3; \quad (8)$$

$$C_{m\alpha}^{pre} = 6.214 \times 10^{-2} - 1.755\alpha - 3.427\alpha^2 + 1.256 \times 10^1\alpha^3; \quad (9)$$

$$C_{n\alpha}^{pre} = 4.748 \times 10^{-2} - 2.097 \times 10^{-1}\alpha - 2.016\alpha^2 + 7.171\alpha^3; \quad (10)$$

Polynomials in angle of attack and elevator deflections:

$$C_{X\eta}^{pre} = 4.327 \times 10^{-2}\eta - 4.458 \times 10^{-1}\alpha\eta + 3.370 \times 10^{-1}\alpha^2\eta - 4.567 \times 10^{-1}\alpha\eta^2 + 7.331 \times 10^{-2}\eta^3; \quad (11)$$

$$C_{Y\eta}^{pre} = 1.832 \times 10^{-2}\eta + 7.484 \times 10^{-2}\alpha\eta - 4.384 \times 10^{-1}\alpha^2\eta; \quad (12)$$

$$C_{Z\eta}^{pre} = -2.567 \times 10^{-1}\eta + 3.085 \times 10^{-1}\alpha\eta - 5.105 \times 10^{-2}\eta^2 - 7.394 \times 10^{-1}\alpha^2\eta + 6.936 \times 10^{-1}\alpha\eta^2 + 1.337 \times 10^{-1}\eta^3; \quad (13)$$

$$C_{l\eta}^{pre} = 1.699 \times 10^{-2}\eta + 8.936 \times 10^{-2}\alpha\eta - 4.564 \times 10^{-1}\alpha^2\eta + 1.571 \times 10^{-2}\alpha\eta^2; \quad (14)$$

$$C_{m\eta}^{pre} = -9.028 \times 10^{-1}\eta + 7.437 \times 10^{-1}\alpha\eta - 4.924 \times 10^{-2}\eta^2 - 8.415 \times 10^{-1}\alpha^2\eta + 2.210\alpha\eta^2 + 5.251 \times 10^{-1}\eta^3; \quad (15)$$

$$C_{n\eta}^{pre} = 1.532 \times 10^{-2}\eta + 8.758 \times 10^{-2}\alpha\eta - 4.513 \times 10^{-1}\alpha^2\eta + 1.487 \times 10^{-2}\alpha\eta^2. \quad (16)$$

B. Domain of high angle of attack

Polynomials in angle of attack:

$$C_{X\alpha}^{post} = 1.266 \times 10^{-2} - 3.159 \times 10^{-1}\alpha + 3.832 \times 10^{-1}\alpha^2 - 1.226 \times 10^{-1}\alpha^3; \quad (17)$$

$$C_{Y\alpha}^{post} = -2.297 \times 10^{-2}\alpha^2 + 1.337 \times 10^{-2}\alpha^3; \quad (18)$$

$$C_{Z\alpha}^{post} = -4.179 \times 10^{-1} - 2.345\alpha + 9.586 \times 10^{-1}\alpha^2 - 3.665 \times 10^{-2}\alpha^3; \quad (19)$$

$$C_{l\alpha}^{post} = 2.006 \times 10^{-2} - 8.200 \times 10^{-2}\alpha + 1.012 \times 10^{-1}\alpha^2 - 3.515 \times 10^{-2}\alpha^3; \quad (20)$$

$$C_{m\alpha}^{post} = -2.552 \times 10^{-1} - 5.131 \times 10^{-1}\alpha - 2.677 \times 10^{-1}\alpha^2 + 1.332 \times 10^{-1}\alpha^3; \quad (21)$$

$$C_{n\alpha}^{post} = 1.159 \times 10^{-2} - 3.062 \times 10^{-2}\alpha + 2.727 \times 10^{-2}\alpha^2; \quad (22)$$

Polynomials in angle of attack and elevator deflections:

$$C_{X\eta}^{post} = -1.342 \times 10^{-2}\eta - 1.663 \times 10^{-1}\alpha\eta - 1.796 \times 10^{-1}\eta^2 + 2.254 \times 10^{-2}\alpha^2\eta + 9.274 \times 10^{-2}\alpha\eta^2 + 7.331 \times 10^{-2}\eta^3; \quad (23)$$

$$C_{Z\eta}^{post} = -2.922 \times 10^{-1}\eta + 1.829 \times 10^{-1}\alpha\eta + 1.277 \times 10^{-1}\eta^2 + 2.284 \times 10^{-2}\alpha^2\eta + 1.229 \times 10^{-1}\alpha\eta^2 + 1.337 \times 10^{-1}\eta^3; \quad (24)$$

$$C_{m\eta}^{post} = -9.498 \times 10^{-1}\eta + 6.099 \times 10^{-1}\alpha\eta + 5.093 \times 10^{-1}\eta^2 + 6.456 \times 10^{-2}\alpha^2\eta + 4.264 \times 10^{-1}\alpha\eta^2 + 5.251 \times 10^{-1}\eta^3; \quad (25)$$

$$C_{Y\eta}^{post}, C_{l\eta}^{post}, C_{n\eta}^{post} \text{ are approximately zero.} \quad (26)$$

C. Full-envelope polynomials

Polynomials in side-slip and aileron deflections:

$$C_{X\xi} = 6.557 \times 10^{-2}\beta^2 + 4.214 \times 10^{-2}\beta\xi - 1.493\xi^2 + 4.264 \times 10^{-2}\xi^3 - 1.647 \times 10^{-2}\beta^4 - 2.321 \times 10^{-2}\beta^3\xi + 9.649 \times 10^{-2}\beta^2\xi^2 + 2.859 \times 10^{-2}\beta\xi^3 + 3.084 \times 10^1\xi^4; \quad (27)$$

$$C_{Y\xi} = -3.697 \times 10^{-1}\beta - 1.570 \times 10^{-1}\xi - 3.231 \times 10^{-2}\beta^2 - 6.137\xi^2 + 7.416 \times 10^{-2}\beta^3 + 1.611 \times 10^{-2}\beta\xi^2 + 2.214\xi^3 + 6.487 \times 10^{-1}\beta^2\xi^2 + 5.323 \times 10^{-2}\beta\xi^3 + 1.456 \times 10^2\xi^4; \quad (28)$$

$$C_{Z\xi} = 3.411 \times 10^{-1}\beta^2 + 4.141 \times 10^{-1}\beta\xi - 3.717\xi^2 + 4.234 \times 10^{-2}\xi^3 - 9.915 \times 10^{-2}\beta^4 - 1.922 \times 10^{-1}\beta^3\xi + 1.945 \times 10^{-1}\beta^2\xi^2 + 9.909 \times 10^{-1}\beta\xi^3 + 9.856 \times 10^1\xi^4; \quad (29)$$

$$C_{l\xi} = -5.798 \times 10^{-2}\beta - 3.929 \times 10^{-1}\xi - 2.906 \times 10^{-2}\beta^2 - 5.551\xi^2 + 1.763 \times 10^{-2}\beta^3 + 1.722 \times 10^{-1}\beta^2\xi + 4.557 \times 10^{-1}\xi^3 + 5.835 \times 10^{-1}\beta^2\xi^2 + 5.332 \times 10^{-2}\beta\xi^3 + 1.319 \times 10^2\xi^4; \quad (30)$$

$$C_{m\xi} = -3.978 \times 10^{-2}\beta^2 + 5.554 \times 10^{-1}\beta\xi - 4.689\xi^2 + 4.229 \times 10^{-2}\xi^3 + 2.585 \times 10^{-2}\beta^4 - 2.309 \times 10^{-1}\beta^3\xi + 4.077 \times 10^{-1}\beta^2\xi^2 - 5.068 \times 10^{-1}\beta\xi^3 + 1.160 \times 10^2\xi^4; \quad (31)$$

$$C_{n\xi} = 3.686 \times 10^{-2}\beta + 1.392 \times 10^{-2}\xi - 2.828 \times 10^{-2}\beta^2 - 5.410\xi^2 - 1.160 \times 10^{-1}\xi^3 + 5.678 \times 10^{-1}\beta^2\xi^2 + 5.334 \times 10^{-2}\beta\xi^3 + 1.286 \times 10^2\xi^4. \quad (32)$$

Polynomials in side-slip and rudder deflections:

$$C_{X\zeta} = -1.031 \times 10^{-2}\beta\zeta - 7.986 \times 10^{-2}\zeta^2 + 1.086 \times 10^{-2}\beta\zeta^3 + 9.739 \times 10^{-2}\zeta^4; \quad (33)$$

$$C_{Y\zeta} = -8.526 \times 10^{-2}\zeta - 2.128 \times 10^{-2}\beta^2 - 3.569 \times 10^{-1}\zeta^2 + 1.049 \times 10^{-2}\beta^2\zeta + 1.924 \times 10^{-2}\beta\zeta^2 + 7.679 \times 10^{-2}\zeta^3 + 5.663 \times 10^{-2}\beta^2\zeta^2 + 5.247 \times 10^{-1}\zeta^4; \quad (34)$$

$$C_{Z\zeta} = -2.036 \times 10^{-2}\beta^2 - 1.332 \times 10^{-2}\beta\zeta - 1.743 \times 10^{-1}\zeta^2 + 1.834 \times 10^{-2}\beta^2\zeta^2 + 3.683 \times 10^{-2}\beta\zeta^3 + 2.631 \times 10^{-1}\zeta^4; \quad (35)$$

$$C_{l\zeta} = -1.930 \times 10^{-2}\beta^2 - 3.221 \times 10^{-1}\zeta^2 + 5.111 \times 10^{-2}\beta^2\zeta^2 + 4.735 \times 10^{-1}\zeta^4; \quad (36)$$

$$C_{m\zeta} = -3.231 \times 10^{-2}\beta^2 - 1.436 \times 10^{-2}\beta\zeta - 2.672 \times 10^{-1}\zeta^2 + 1.241 \times 10^{-2}\beta^3\zeta + 1.472 \times 10^{-2}\beta^2\zeta^2 + 1.051 \times 10^{-2}\beta\zeta^3 + 4.193 \times 10^{-1}\zeta^4; \quad (37)$$

$$C_{n\zeta} = 2.158 \times 10^{-2}\zeta - 1.882 \times 10^{-2}\beta^2 - 3.137 \times 10^{-1}\zeta^2 - 1.338 \times 10^{-2}\zeta^3 + 4.978 \times 10^{-2}\beta^2\zeta^2 + 4.611 \times 10^{-1}\zeta^4. \quad (38)$$

II. EQUATIONS OF MOTION

We have the aerodynamic forces and moments in body axis system

$$\begin{bmatrix} X^A \\ Y^A \\ Z^A \end{bmatrix}_f = \frac{1}{2}\rho SV_A^2 \begin{bmatrix} C_X \\ C_Y \\ C_Z \end{bmatrix}; \quad \begin{bmatrix} L^A \\ M^A \\ N^A \end{bmatrix}_f = \frac{1}{2}\rho SV_A^2 \begin{bmatrix} b C_l \\ c_A C_m \\ b C_n \end{bmatrix} + \begin{bmatrix} X^A \\ Y^A \\ Z^A \end{bmatrix}_f \times (\mathbf{x}_{cg} - \mathbf{x}_{cg}^{\text{ref}}); \quad (39)$$

and the weight force by rotation into the body axis system

$$X_g^G = -g \sin \Theta; \quad (40)$$

$$Y_g^G = -g \sin \Phi \cos \Theta; \quad (41)$$

$$Z_g^G = -g \cos \Phi \cos \Theta; \quad (42)$$

The resulting forces lead to changes in the velocity vector, in body axis system, by

$$\dot{\mathbf{V}}_{Af} = \frac{1}{m} \begin{bmatrix} X^A + X^G + X^F \\ Y^A + Y^G \\ Z^A + Z^G \end{bmatrix}_f + \begin{bmatrix} p \\ q \\ r \end{bmatrix} \times \mathbf{V}_{Af}. \quad (43)$$

with the thrust $X_f^F = F$ (engines aligned with the x_f -axis). For a symmetric plane ($I_{xy} = I_{yz} = 0$), the resulting moments in body axis are given as

$$L_f = L_f^A - q r (I_z - I_y) + p q I_{zx}; \quad (44)$$

$$M_f = M_f^A - p r (I_x - I_z) - (p^2 - r^2) I_{zx}; \quad (45)$$

$$N_f = N_f^A - p q (I_y - I_x) - q r I_{zx}; \quad (46)$$

with the inertias I_x, I_y, I_z, I_{zx} . The changes of angular body rates are then given as

$$\dot{p} = \frac{1}{I_x I_z - I_{zx}^2} (I_z L_f + I_{zx} N_f); \quad (47)$$

$$\dot{q} = \frac{1}{I_y} M_f; \quad (48)$$

$$\dot{r} = \frac{1}{I_x I_z - I_{zx}^2} (I_{zx} L_f + I_x N_f). \quad (49)$$

Here, the normalized body rates have been used with

$$\begin{bmatrix} \hat{p} \\ \hat{q} \\ \hat{r} \end{bmatrix} = \frac{1}{2V_A} \begin{bmatrix} b p \\ c_A q \\ b r \end{bmatrix} \iff \begin{bmatrix} p \\ q \\ r \end{bmatrix} = 2V_A \begin{bmatrix} b^{-1} \hat{p} \\ c_A^{-1} \hat{q} \\ b^{-1} \hat{r} \end{bmatrix} \quad (50)$$

and

$$\begin{bmatrix} \dot{\hat{p}} \\ \dot{\hat{q}} \\ \dot{\hat{r}} \end{bmatrix} = \frac{1}{V_A} \left(\frac{1}{2} \begin{bmatrix} b \dot{p} \\ c_A \dot{q} \\ b \dot{r} \end{bmatrix} - \dot{V}_A \begin{bmatrix} \hat{p} \\ \hat{q} \\ \hat{r} \end{bmatrix} \right). \quad (51)$$

The change of attitude is finally obtained by rotation into normal earth-fixed axis system:

$$\dot{\Phi} = p + q \sin \Phi \tan \Theta + r \cos \Phi \tan \Theta; \quad (52)$$

$$\dot{\Theta} = q \cos \Phi - r \sin \Phi; \quad (53)$$

$$\dot{\Psi} = q \sin \Phi \cos^{-1} \Theta + r \cos \Phi \cos^{-1} \Theta. \quad (54)$$

III. LONGITUDINAL MODEL

The longitudinal model is restricted to the x_a - z_a -plane, assuming $\beta = \mu_A = \chi_A = 0$.

A. Aerodynamic Coefficients

The longitudinal aerodynamic coefficients, C_L, C_D, C_m , are obtained as

$$\begin{bmatrix} C_L \\ C_D \end{bmatrix} = \begin{bmatrix} + \sin \alpha & - \cos \alpha \\ - \cos \alpha & - \sin \alpha \end{bmatrix} \begin{bmatrix} C_X \\ C_Z \end{bmatrix}. \quad (55)$$

B. Equations of motion

The longitudinal equations of motion are given as

$$\dot{V}_A = \frac{1}{m} \left(T \cos \alpha - \frac{1}{2} \rho S V_A^2 C_D(\alpha, \eta, q) - mg \sin \gamma_A \right), \quad (56)$$

$$\dot{\gamma}_A = \frac{1}{m V_A} \left(T \sin \alpha + \frac{1}{2} \rho S V_A^2 C_L(\alpha, \eta, q) - mg \cos \gamma_A \right), \quad (57)$$

$$\dot{q} = \frac{1}{I_y} \frac{1}{2} \rho S c_A V_A^2 C_m(\alpha, \eta, q); \quad (58)$$

$$\dot{\Theta} = q; \quad (59)$$

with

$$\Theta = \alpha + \gamma. \quad (60)$$

NOMENCLATURE

α	=	Angle of attack (rad);
α_0	=	Low-angle of attack boundary ($^\circ$);
β	=	Side-slip angle (rad);
γ_A	=	Flight-path angle relative to air (rad);
ζ	=	Rudder deflection (rad), negative if leading to positive yaw moment;
η	=	Elevator deflection (rad), negative if leading to positive pitch moment;
Θ	=	Pitch angle (rad);
μ_A	=	Air-path bank angle (rad);
ξ	=	Aileron deflection (rad), negative if leading to positive roll moment;
ϱ	=	Air density ($\varrho = 1.200 \text{ kg m}^{-3}$);
χ_A	=	Air-path azimuth angle (rad);
Φ	=	Bank angle (rad);
Ψ	=	Azimuth angle (rad);
C_l	=	Aerodynamic coefficient moment body x_f -axis (\cdot);
C_m	=	Aerodynamic coefficient moment body y_f -axis (\cdot);
C_n	=	Aerodynamic coefficient moment body z_f -axis (\cdot);
C_D	=	Aerodynamic drag coefficient (\cdot), positive along negative air-path x_a -axis;
C_L	=	Aerodynamic lift coefficient (\cdot), positive along negative air-path z_a -axis;
C_X	=	Aerodynamic coefficient force body x_f -axis (\cdot);
C_Y	=	Aerodynamic coefficient force body y_f -axis (\cdot);
C_Z	=	Aerodynamic coefficient force body z_f -axis (\cdot);
D	=	Drag force (N), positive along negative air-path x_a -axis;
F	=	Thrust force (N), positive along body x_f -axis;
L	=	Lift force (N), positive along negative air-path z_a -axis;
q	=	Pitch rate (rad s^{-1});
b	=	Reference aerodynamic span ($b = 2.088 \text{ m}$);
c_A	=	Aerodynamic mean chord ($c_A = 2.800 \times 10^{-1} \text{ m}$);
g	=	Standard gravitational acceleration ($g \approx 9.810 \text{ m s}^{-2}$);
m	=	Aircraft mass ($m = 2.619 \times 10^4 \text{ kg}$);
p	=	Roll rate (rad s^{-1});
r	=	Yaw rate (rad s^{-1});
L_f	=	Roll moment (N m), mathematically positive around x_f -axis;
M_f	=	Pitch moment (N m), mathematically positive around y_f -axis;
N_f	=	Yaw moment (N m), mathematically positive around z_f -axis;
S	=	Wing area ($S = 5.500 \times 10^{-1} \text{ m}^2$);
V_A, \mathbf{V}_A	=	Aircraft speed and velocity <i>relative to air</i> ($V_A = \ \mathbf{V}_A\ _2, \text{ m s}^{-1}$);
X_f^A	=	Aerodynamic force along body x_f -axis (N);
X_f^F	=	Thrust force along body x_f -axis (N);
Y_f^A	=	Aerodynamic force along body y_f -axis (N);
Z_f^A	=	Aerodynamic force along body z_f -axis (N);
$(\cdot)^{post}$	=	Domain of high angle of attack;
$(\cdot)^{pre}$	=	Domain of low angle of attack;
x_a, y_a, z_a	=	Air-path axis system;
x_f, y_f, z_f	=	Body axis system;
x_g, y_g, z_g	=	Normal earth-fixed axis system;