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# ***SECOND INTERNATIONAL AIRPORT CONFERENCE: PLANNING, INFRASTRUCTURE & ENVIRONMENT***

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## **ADDING A NEW DIMENSION TO AIRSIDE CAPACITY AT AIRPORTS**

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## **ABSTRACT**

With the sustained development of air transportation over the last decades, airport capacity has been a permanent concern for airport planners and operators. Until recently, airport capacity was considered only at its two traditional bottlenecks: the runways system and the passenger's terminals. However, today, aircraft ground traffic at airports has become also a critical question with important influences on security and efficiency levels and new ground traffic management and control systems including a higher degree of automation have been introduced. Traditionally, with respect to airside airport capacity, a distinction has been done between theoretical and practical capacity, depending if level of service thresholds and operational practices are taken into account or not. In general, practical capacity, which is of main interest for airport managers, has been estimated on statistical grounds while cumbersome simulation models have been developed to perform some scenario based capacity predictions.

In this communication, a new dimension is introduced in the airside capacity analysis: the amount of grounded aircraft present in the airside active areas. An approach based on the solution of successive optimization problems is proposed to perform the 3D estimation of the practical airside capacity of an airport. For given mean in-bound and out-bound flows and a current ground traffic situation, different minimal cost flow problems are formulated. When the interaction of aircraft flows at ground intersections (taxiways and aprons) is taken explicitly into consideration, this leads to a non convex optimization problem.

The proposed approach has been applied to different case studies. In this communication, some numerical results relative to its application to the case of Toulouse-Blagnac Airport and Portland International Airport, are displayed.

## **KEY WORDS**

Airport capacity, airport modeling, flows in networks, large scale optimization

## **INTRODUCTION**

With the sustained development of air transportation over the last decades, airport capacity has remained a permanent issue for airport planners and operators. Until recently, airport capacity was considered only at its two traditional bottlenecks: the runways system capacity and the passenger's terminals capacity. However, today, aircraft ground traffic at airports has become also a critical question with important influences on security and efficiency levels and new ground traffic management and control systems including a higher degree of automation have been introduced. Traditionally, with respect to airside airport capacity, a distinction has been done between theoretical and practical capacity, depending if level of service thresholds and operational practices are taken into account or not. In general, practical capacity, which is of main interest for airport managers, has been estimated on statistical grounds while cumbersome simulation models have been developed to perform some scenario based capacity predictions. However, it appeared early that since in-bound and out-bound flights are competing to use the same airport facilities, the nature of this problem is multi criteria and capacity must be defined in terms of Pareto frontiers.

In this communication, a new dimension is introduced in the airside capacity analysis: the amount of grounded aircraft present in the airside active areas. This parameter can be significant with respect to airside capacity at major airports such as London-Heathrow, Chicago-O'Hare or Amsterdam-Schiphol, but also at some domestic airports such as São Paulo-Congonhas, Hannover airport or Paris-Orly.

The 3D estimation of the practical airside capacity of an airport is based on the solution of successive optimization problems of increasing complexity. For given mean in-bound and out-bound flows and a current ground traffic situation, a set of minimal cost flow problems, taking into account additional operations constraints, is considered. When the interaction of aircraft flows at ground intersections (taxiways and aprons) is taken explicitly into consideration, this leads to a non convex optimization problem for which different optimization schemes can be used. The proposed approach has been applied in preliminary form to the case of Toulouse-Blagnac Airport and to Portland International Airport for which some numerical results are displayed.

## **AIRSIDE TRAFFIC MODELING**

The airside at airports is composed of three main components: the traffic network, composed of runways, taxiways, aprons and parking areas, the flows of aircraft and the ground traffic signaling and control system. The main objective of ground traffic control is to allow ground traffic operations at minimum costs, avoiding saturation problems while insuring high standards of security. Many management and control issues in ground traffic operations lead to decision problems whose solutions can be based on suitable mathematical models. The various models which have been built with this purpose are different with respect to the time period considered (long term models are devoted to the design of the system, medium term models are involved with airport operations planning and short term models are related with activity control issues), to the level of traffic detail (macroscopic, intermediate or microscopic for each of the main components of the traffic system) and to the degree of determinism adopted. However, none of them allows the estimation of airside capacity at airports taking into account the aircraft ground traffic conditions.

### Airside network modeling

Here a reference period of time of an hour is retained so that arrival and departure rates can be considered to be rather steady. The dynamics of the waiting queues present at different stages of the traffic system are not considered explicitly but the storage capacities in different sections of this traffic system are taken into account. Aircraft ground traffic is then considered to flow continuously from runways to parking positions and from parking positions to runways, while the stock of parked aircraft is also considered. An oriented graph  $G=(N,U)$ , is used to represent the airport aircraft ground circulation network:

- The set of nodes  $N$  of this graph represent different connection points and limits of the circulation ways and can be classified according to their functions : mere connecting points between two successive traffic segments, crossing point with competing traffic, decision points. Some nodes represent the runways boundaries: runway entries, runway exits and crossing points
- The set of arcs  $U$  of the graph is composed of five different sub-sets : runways, runways exit segments, taxiways, aprons and parking areas. Some of these arcs can be bidirectional, while the orientation of some others are dependent of the current mean wind direction. This arc orientation can obey to either a group logic (runways arcs) or a local one (bidirectional arcs and parking positions). To each arc different parameters are attached : length and width, maximum wingspan, maximum weight, storage capacity and orientation:
- The geometry of the parking positions, of the taxiways crossings and of the aprons leads to limitations for the use of determined types of aircraft in some areas of the traffic system. Thus to each aircraft type a circulation sub graph can be considered.
- The set of arcs can also be partitioned in three classes when considering arriving and departing traffics, then :

$$U = U_{ad} \cup U_a \cup U_d \quad (1)$$

where  $U_a$  is the set or arcs involved related with arriving traffic,  $U_d$  is the set of arcs related with departing traffic,  $U_{ad}$  is the set of arcs associated with circulation segments used both for departures as for arrivals,  $I$  is the set of runways under operation during the time period considered,  $R_i^d$ ,  $i \in I$ , is the set of arcs associated to the runway exits,  $R_i^a$ ,  $i \in I$ , is the set of arcs associated to the run ways entries,  $J$  is the set of available parking areas,  $P_j^s$ ,  $j \in J$ , is the set of arcs associated to the exits from the parking areas,  $P_j^e$ ,  $j \in J$ , is the set of arcs associated to the entries of the parking areas.

### Traffic flow modelling

The traffic flow through an arc is defined here as the number of aircraft movements per period of time through the corresponding circulation link. Arriving and departing flows are considered separately :  $\Phi^a$  is the total arriving flow from the runways and  $\Phi^d$  is the total departing flow towards the runways,  $\varphi_u^a$  and  $\varphi_u^d$ , are the arriving and departing flows using arc  $u$ . To each arc  $u$  is attached a capacity whose level  $\varphi_u^{max}$  is related with its geometric characteristics and to the size and operational characteristics (taxiing speed). This leads to a set of arc capacity and positive ness constraints:

$$0 \leq \varphi_u^a + \varphi_u^d \leq \varphi_u^{max}, u \in U_{ad}, \quad 0 \leq \varphi_u^a \leq \varphi_u^{max}, u \in U_a, \quad 0 \leq \varphi_u^d \leq \varphi_u^{max}, u \in U_d, \quad (2)$$

$$0 \leq \varphi_u^a \leq \varphi_u^{max}, u \in R_i^s, i \in I, \quad 0 \leq \varphi_u^d \leq \varphi_u^{max}, u \in R_i^d, i \in I \quad (3)$$

$$0 \leq \varphi_u^a \leq \varphi_u^{max}, u \in P_j^e, j \in J, \quad 0 \leq \varphi_u^d \leq \varphi_u^{max}, u \in P_j^s, j \in J \quad (4)$$

The network definition is completed by the addition of flow conservation constraints at the different crossings and decision points. So the following constraints are introduced:

$$\sum_{u \in \omega^-(i)} (\varphi_u^a + \varphi_u^d) + \sum_{v \in \omega^-(i)} \varphi_v^d + \sum_{w \in \omega^-(i)} \varphi_w^a = \sum_{u \in \omega^+(i)} (\varphi_u^a + \varphi_u^d) + \sum_{v \in \omega^+(i)} \varphi_v^d + \sum_{w \in \omega^+(i)} \varphi_w^a \quad (5)$$

where  $u \in U, v \in U_d, w \in U_a$ ,  $\omega^-(i)$  is the set of incident arcs to node  $i$ ,  $\omega^+(i)$  is the set of leaving arcs from node  $i$ . Other flow conservation constraints are attached to the exits and entries of the runways and parking areas :

$$\sum_{u \in R_i^s} \varphi_u^a = \Phi_i^a, \forall i \in I, \text{ where } \Phi_i^a \text{ is the landing flow at runway } i \quad (6)$$

$$\sum_{u \in R_i^e} \varphi_u^d = \Phi_i^d, \forall i \in I, \text{ where } \Phi_i^d \text{ is the take-off flow from runway } i \quad (7)$$

$$\sum_{u \in P_j^s} \varphi_u^d - \Psi_j^d = 0, \forall j \in J, \text{ where } \Psi_j^d \text{ is the departing flow from parking area } j \quad (8)$$

$$\sum_{u \in P_j^e} \varphi_u^a - \Psi_j^a = 0, \forall j \in J, \text{ where } \Psi_j^a \text{ is the arriving flow at parking area number } j. \quad (9)$$

The transfer from one link to another happens at intersections where other concurrent flows may be present, this is taken into account through the following constraints :

$$\sum_{u \in \omega^-(l)} (\varphi_u^a + \varphi_u^d) \theta_l \leq T, \quad l \in L \quad (10)$$

where  $L$  is the set of such intersections,  $\theta_l$  is the mean crossing time of intersection  $l$ . In the case of double oriented arcs, the capacity constraints are written as :

$$\sum_{u \in U'} (\varphi_u^a + \varphi_u^d) \tau_u + \sum_{u' \in U''} (\varphi_{u'}^a + \varphi_{u'}^d) \tau_{u'} \leq T \quad (11)$$

where  $\tau_u$  is the arc occupancy time in one direction and  $\tau_{u'}$  is the arc occupancy time in the other direction. Other flow conservation constraints are respectively for parking areas exits, for parking areas entries, for entries to runways and for runways exits:

$$\sum_{j \in J} \Psi_j^d = \Phi^d, \quad \sum_{j \in J} \Psi_j^a = \Phi^a, \quad \sum_{i \in I} \Phi_i^d = \Phi^d, \quad \sum_{i \in I} \Phi_i^a = \Phi^a \quad (12)$$

If the global flows  $\Phi^a$  and  $\Phi^d$  are input parameters for the capacity study, it is the model which should distribute them between the different parking areas and runways. Let  $I_0$  be the set of runways whose landing and departing activities are independent from each other. The corresponding runways capacity constraints, defining a convex set, can be written as :

$$\lambda_\alpha^i \Phi_i^a + \gamma_\alpha^i \Phi_i^d \leq C_i, \alpha \in A_i, i \in I_0 \quad (13)$$

where  $\lambda_\alpha^i, \gamma_\alpha^i \in C_j$  are real positive parameters,  $A_i$  is the set of indexes for the constraints defining a convex capacity domain for runway  $i$ . For the set  $I_k$  of interdependent runways, the convex capacity domain equations can be written as :

$$\sum_{i \in I_k} (\lambda_\alpha^{il} \Phi_i^a + \gamma_\alpha^{il} \Phi_i^d) \leq C_l, \alpha \in A_{kl}, l \in L_k, k \in K \quad (14)$$

where  $A_{kl}$  and  $L_k$  are the set of indexes related with the  $k^{th}$  sub set of the  $K$  interdependent runways. The same modeling approach can be adopted in relation to the parking areas, however, here it will be assumed that the parking areas are independent. Then the parking capacity constraints can be written as :

$$0 \leq N_j^0 + \Psi_j^a - \Psi_j^d \leq S_j, j \in J \text{ et } \sum_{j \in J} N_j^0 = N^0 \quad (15)$$

where  $S_j$  is the capacity of the  $j^{th}$  parking area,  $N_j^0$  is the number of occupied positions in the  $j^{th}$  parking area at the beginning of the period,  $N^0$  is the total number of occupied parking positions at the beginning of the period.

## THEORETICAL CAPACITY EVALUATION

The theoretical capacity is given by the  $(\Phi^a, \Phi^d)$  Pareto frontier with  $\underline{N}$  as parameter. It can be obtained by solving repeatedly problem  $P(\Phi^a, \underline{N}^0)$  given by :

$$\max_{\underline{\varphi}, \underline{\Phi}, \underline{\Psi}, \underline{N}} \Phi^d \text{ with } \underline{\varphi} = \begin{pmatrix} \varphi^a \\ \varphi^d \end{pmatrix}, \underline{\Phi} = \begin{pmatrix} \Phi^a \\ \Phi^d \end{pmatrix}, \underline{\Psi} = \begin{pmatrix} \Psi^a \\ \Psi^d \end{pmatrix}, \underline{N} = \begin{pmatrix} N_1^0 \\ \vdots \\ N_J^0 \end{pmatrix} \quad (16)$$

and with the constraints :

$$\sum_{j \in J} \Psi_j^d = \Phi^d, \sum_{j \in J} \Psi_j^a = \Phi^a, \sum_{i \in I} \Phi_i^d = \Phi^d, \sum_{i \in I} \Phi_i^a = \Phi^a, \sum_{j \in J} N_j^0 = N^0, N_j^0 \geq 0, \forall j \in J \quad (17)$$

$$\sum_{i \in I_k} (\lambda_{\alpha}^{il} \Phi_i^a + \gamma_{\alpha}^{il} \Phi_i^d) \leq C_l, \alpha \in A_{kl}, l \in L_k, k \in K \quad (18)$$

$$0 \leq N_j^0 + \Psi_j^a - \Psi_j^d \leq S_j, j \in J \quad (19)$$

$$\text{and } \underline{\varphi} \in F(\underline{\Phi}, \underline{\Psi}, \underline{N}) \quad (20)$$

where  $F(\underline{\Phi}, \underline{\Psi}, \underline{N})$  is the convex set defined by the traffic flow capacity constraints expressed with  $\varphi_u^a, \varphi_u^d, u \in U$  for given levels of  $\underline{\Phi}, \underline{\Psi}$  et  $\underline{N}$ . Here the flow variables are taken real so that large scale integer linear programming problems are avoided. Let us notice that the set of flow related constraints  $\underline{\varphi} \in F(\underline{\Phi}, \underline{\Psi}, \underline{N})$ , is decoupled from the runway and parking area capacity constraints of problem  $P(\Phi^a, \underline{N}^0)$ . The connection between the two sets of variables is realized by the global flows  $\underline{\Phi}, \underline{\Psi}$  and  $\underline{N}$ . If in a first step, the flow related constraints which are a capacity limiting factor, are let apart, the following relaxed problem can be formulated.  $\tilde{P}(\Phi^a, \underline{N}^0)$ :

$$\max_{\underline{\Phi}, \underline{\Psi}, \underline{N}} \Phi^d$$

under the constraints :

$$\sum_{j \in J} \Psi_j^d = \Phi^d, \sum_{j \in J} \Psi_j^a = \Phi^a, \sum_{i \in I} \Phi_i^d = \Phi^d, \sum_{i \in I} \Phi_i^a = \Phi^a, \sum_{j \in J} N_j^0 = N^0 \quad (21)$$

$$\sum_{i \in I_k} (\lambda_{\alpha}^{il} \Phi_i^a + \gamma_{\alpha}^{il} \Phi_i^d) \leq C_l, \alpha \in A_{kl}, l \in L_k, k \in K \quad (22)$$

$$0 \leq N_j^0 + \Psi_j^a - \Psi_j^d \leq S_j, j \in J \quad (23)$$

$$\text{with } \Psi_j^a \geq 0, \Psi_j^d \geq 0, j \in J, \Phi_i^a \geq 0, \Phi_i^d \geq 0, i \in I, N_j^0 \geq 0, j \in J \quad (24)$$

The solution of this problem,  $\tilde{\Phi}^d$ , is an upper bound for the solution of problem  $P(\Phi^a, N^0)$ . If there is a feasible flow  $\underline{\varphi}$  for  $F(\tilde{\Phi}, \tilde{\Psi}, \tilde{N})$  where  $(\tilde{\Phi}, \tilde{\Psi}, \tilde{N})$  is the solution of  $\tilde{P}(\Phi^a, N^0)$ , then the  $\Phi^d$  solution of problem  $P(\Phi^a, N^0)$  is such that:  $\Phi^{d*} = \tilde{\Phi}^d$ . In this case,  $(N^0, \Phi^a)$ , the circulation system is not a limiting factor for the airside capacity which is then only dependant of the capacities of the runways systems and parking areas. In the case where  $F(\tilde{\Phi}, \tilde{\Psi}, \tilde{N})$  is an empty set, the circulation flow constraints must be taken into account to evaluate the airside capacity, then it is necessary to cope with the global linear programming problem  $P_\varphi(\Phi^a, N^0)$  which is equivalent to problem  $P(\Phi^a, N^0)$ , but where the circulation flows appear explicitly in the global constraints related with runways and parking areas capacities. Problem  $P_\varphi(\Phi^a, N^0)$  is then written :

$$\max_{\underline{\varphi}, \underline{N}} \Phi^d \text{ with :}$$

$$\sum_{j \in J} \sum_{u \in P_j^d} \varphi_u^d = \Phi^d \quad \sum_{j \in J} \sum_{u \in P_j^a} \varphi_u^a = \Phi^a \quad \sum_{i \in I} \sum_{u \in R_i^d} \varphi_u^d = \Phi^d \quad \sum_{i \in I} \sum_{u \in R_i^a} \varphi_u^a = \Phi^a \quad (25)$$

$$\sum_{i \in I_k} \left( \lambda_\alpha^{il} \sum_{u \in R_i^a} \varphi_u^a + \gamma_\alpha^{il} \sum_{u \in R_i^d} \varphi_u^d \right) \leq C_l, \alpha \in A_{kl}, l \in L_k, k \in K \quad 0 \leq N_j^0 + \sum_{u \in P_j^a} \varphi_u^a - \sum_{u \in P_j^d} \varphi_u^d \leq S_j, j \in J \quad \sum_{j \in J} N_j^0 = N^0 \quad (26)$$

$$\sum_{u \in \omega^-(i)} (\varphi_u^a + \varphi_u^d) + \sum_{v \in \omega^-(i)} \varphi_v^d + \sum_{w \in \omega^-(i)} \varphi_w^a = \sum_{u \in \omega^+(i)} (\varphi_u^a + \varphi_u^d) + \sum_{v \in \omega^+(i)} \varphi_v^d + \sum_{w \in \omega^+(i)} \varphi_w^a, \text{ where } u \in U_{ad}, v \in U_d, w \in U_a \quad (27)$$

$$\sum_{u \in U'} (\varphi_u^a + \varphi_u^d) r_u + \sum_{u' \in U'} (\varphi_{u'}^a + \varphi_{u'}^d) r_{u'} \leq T \quad (28)$$

and with the arc capacity and positive ness constraints (Eq.(2), Eq.(3) and Eq.(4).

Since the above problem can be solved easily with a variation of the « out of kilter » algorithm, a possible solution scheme for the estimation of airside theoretical capacity is to solve repeatedly problem  $P(\Phi^a, N^0)$  for  $N^0 \in \{0, \dots, N_{max}^0\}$  and  $\Phi^0 \in \{0, \dots, \Phi_{max}^0\}$  where  $N_{max}^0$  is the maximum number of parking positions that can be used simultaneously in the whole airport.  $\Phi_{max}^a$  is the solution of problem  $P(\Phi^a)$  given by :

$$\max (\Phi^a)$$

$$\text{with } \sum_{i \in I_k} \lambda_\alpha^{il} \Phi_i^a \leq C_l, \alpha \in A_{kl}, l \in L_k, k \in K \quad , \quad \sum_{i \in I} \Phi_i^a = \Phi^a \quad \text{and} \quad \Phi_i^a \geq 0, i \in I \quad (29)$$

The output of this heavy process is the theoretical airside capacity which can be represented in a three dimensions space  $(\Phi^a, \Phi^d, N^0)$  :

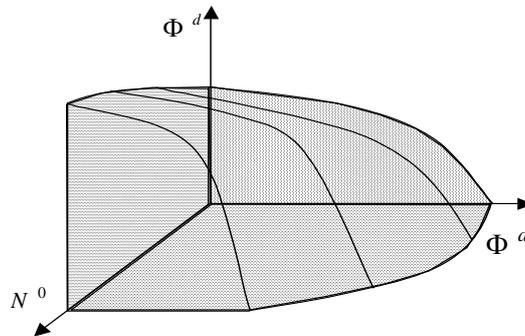


Figure 1: theoretical airside capacity envelope

The evaluation of the theoretical capacity has not taken into account all the interactions between arriving and departing flows and the capacity of the ground traffic control sectors (The airside is divided in control sectors, each of them being operated by a ground controller). Then, practical airside capacity is, by far, smaller than theoretical capacity.

## PRACTICAL CAPACITY EVALUATION

Here an operational situation, given by the arriving flows and the initial airside occupancy,  $\Phi_i^a, i \in I$  et  $N_j^0, j \in J$ , is considered. Here, practical capacity is considered to be reached when for a global departing flow  $\Phi^d$ , the mean ground delay is greater than a given threshold (15 minutes in general). Then, delays must be estimated to achieve the evaluation of airside practical capacity.

### Traffic delay model

Considering that the mean travel time for an aircraft along a given arc is an increasing function of the flow through this arc, a model such as [Gazis et Potts, 1963] :

$$t_p(\varphi) = t_0 \left( 1 + c \cdot (\varphi / \varphi^m)^\alpha \right) \quad (30)$$

with  $\alpha > 1$  and  $c > 0$ , could be adopted. When the flow reaches the capacity of the arc, a queue has build up over the whole arc and traffic is stopped. When the flow of the arc interacts with other flows, it is necessary to take into account the resulting delay. A mean delay model which can be adopted, since its complexity is medium while taking into account the main phenomena, is the following :

$$t_u(\varphi) = \frac{l_u}{\tilde{v}_u} (1 + \alpha_u \varphi_u)^{\gamma_u} + \sum_{j \in \omega^-(u)} \varphi_j \tau_j \quad (31)$$

where  $l_u$  is the length of arc  $u$ ,  $\tilde{v}_u$  is the standard aircraft ground free speed for this class of arc,  $\alpha_u$  and  $\gamma_u$  are parameters characteristic of proper congestion effects,  $\omega^-(u)$  is the set of incident arcs towards  $u$ ,  $\tau_j$  is the mean additional delay resulting from side flow  $\varphi_j$  (this parameter should allow to take into account wait time at crossings, minimum separation standards and the relative orientation of arcs at crossings). In a more generic way, the mean travel delay along an arc  $u$  could be given by :

$$f_u(\varphi_u, \mathbf{F}_u^-) \text{ where } \mathbf{F}_u^- = \{ \varphi_v | v \in \omega^-(u), v \neq u \} \quad (32)$$

### The airside traffic flow optimisation problem

Here, it is considered that the total arriving flow  $\Phi^a$  is already given and that the initial distribution of grounded aircraft over the different parking areas is known. The problem is here to maximize the total departing flight while insuring that the travel delays remain under a given threshold. This problem,  $P_{\Phi^d}(\Phi^a, N^0)$ , is written as :

$$\max_{\Phi^a, N_j^0} \Phi^d \quad \text{with} \quad \sum_{u \in U} f_u(\varphi_u, \mathbf{F}_u^-) / \sum_{u \in U} \varphi_u \leq d_{max} \quad (33)$$

and with the flow constraints within the ground traffic network written in a summarized way as :

$$\underline{0} \leq \underline{\varphi} \leq \underline{\varphi}^{max} \quad \text{and} \quad \mathbf{A}\underline{\varphi} = 0 \quad (34)$$

where  $\mathbf{A}$  is the arc-node incident matrix and  $d_{max}$  is an upper limit for the mean delay. Since this problem, due to the delay constraint, is not a standard max flow problem over a network, progressively increasing  $\Phi^d$  until problem  $P_{\Phi^d}(\Phi^a, \underline{N}^0)$  has no more a feasible solution, problem  $P_f(\Phi^a, \underline{N}^0, \Phi^d)$ , given by :

$$\min \sum_{u \in U} f_u(\varphi_u, \mathbf{F}_u^-) \quad (35)$$

with

$$0 \leq \varphi_u^a + \varphi_u^d \leq \varphi_u^{max}, u \in U_{ad} \quad 0 \leq \varphi_v^d \leq \varphi_v^{max}, \varphi_v^a = 0, v \in U_d \quad 0 \leq \varphi_w^a \leq \varphi_w^{max}, \varphi_w^d = 0, w \in U_a \quad (36)$$

$$\sum_{u \in \omega^-(i)} (\varphi_u^a + \varphi_u^d) + \sum_{v \in \omega^-(i)} \varphi_v^d + \sum_{w \in \omega^-(i)} \varphi_w^a = \sum_{u \in \omega^+(i)} (\varphi_u^a + \varphi_u^d) + \sum_{v \in \omega^+(i)} \varphi_v^d + \sum_{w \in \omega^+(i)} \varphi_w^a \quad (37)$$

$$\sum_{u \in \omega^-(l)} (\varphi_u^a + \varphi_u^d) \theta \leq T, l \in L \quad \sum_{u \in U'} (\varphi_u^a + \varphi_u^d) \tau_u + \sum_{u' \in U'} (\varphi_{u'}^a + \varphi_{u'}^d) \tau_{u'} \leq T \quad (38)$$

$$\sum_{u \in R_i^a} \varphi_u^a = \Phi_i^a, \forall i \in I \quad \sum_{u \in R_i^d} \varphi_u^d = \Phi_i^d, \forall i \in I \quad (39)$$

$$\sum_{u \in P_j^a} \varphi_u^a - \Psi_j^a = 0, \forall j \in J \quad \sum_{j \in J} \Psi_j^d = \Phi^d \quad (40)$$

$$0 \leq \Psi_j^a \leq \Psi_j^{a,max}, j \in J \quad \sum_{u \in P_j^a} \varphi_u^a - \Psi_j^a = 0, \forall j \in J \quad \sum_{j \in J} \Psi_j^a = \Phi^a \quad 0 \leq \Psi_j^d \leq \Psi_j^{d,max}, j \in J \quad (41)$$

$$0 \leq N_j^0 + \Psi_j^a - \Psi_j^d \leq S_j, j \in J \quad (42)$$

$$\sum_{u \in \Omega_k^a} (\varphi_u^a + \varphi_u^d) + \sum_{v \in \Omega_k^d} \varphi_v^d + \sum_{w \in \Omega_k^a} \varphi_w^a \leq Z_k, k \in \{1, 2, \dots, K\} \quad (43)$$

where  $\Phi_i^a$  is the total demand for arrival at runway  $i$ ,  $\Phi_i^d$  is the total demand for departure at runway  $i$ ,  $\varphi_u^{max}$  is the maximum flow for arc  $u$ ,  $\varphi_v^{max}$  is the maximum flow for arc  $v$ ,  $\varphi_w^{max}$  is the maximum flow for arc  $w$ ,  $\Psi_j^a$  is the arriving flow at parking area number  $j$  and  $\Psi_j^d$  is the departing flow from parking area number  $j$ ,  $\Psi_j^{a,max}$  and  $\Psi_j^{d,max}$  are the maximum arriving and departing flows at parking area number  $j$ ,  $\Omega_k = \Omega_k^{ad} \cup \Omega_k^d \cup \Omega_k^a$  is the sub set of traffic links controlled by the  $k^{\text{th}}$  ground traffic controller, the capacity of the corresponding control is  $Z_k$ .  $K$  is the total number of ground control sectors in the airport.

## APPLICATION

The proposed approach has been tested using data from two medium size airports: Toulouse-Blagnac Airport and Portland International Airport. Figure 2 shows the layout of Toulouse-Blagnac Airport, figures 3 and 4 represent respectively the theoretical and the practical tri dimensional airside capacities envelopes.

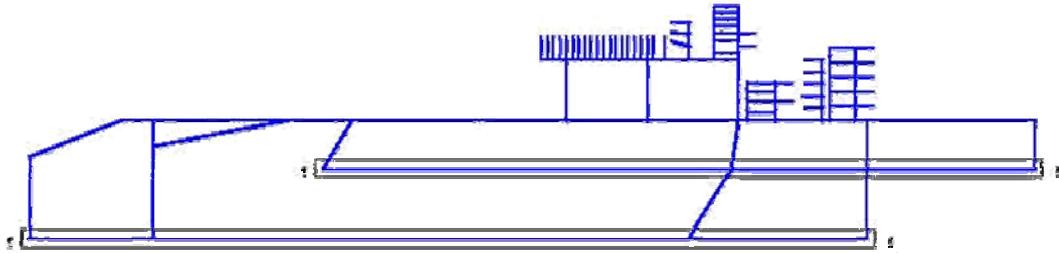


Figure 2: Airside layout at Toulouse-Blagnac Airport

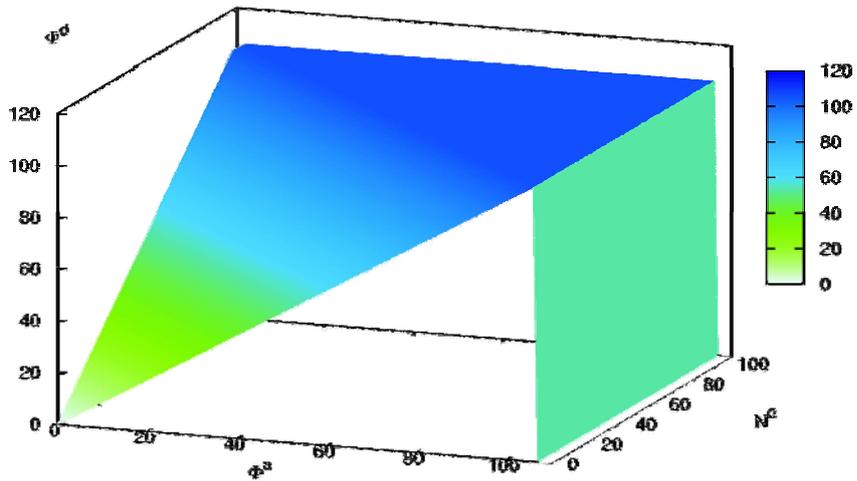


Figure 3: Theoretical airside capacity envelope at Toulouse-Blagnac Airport

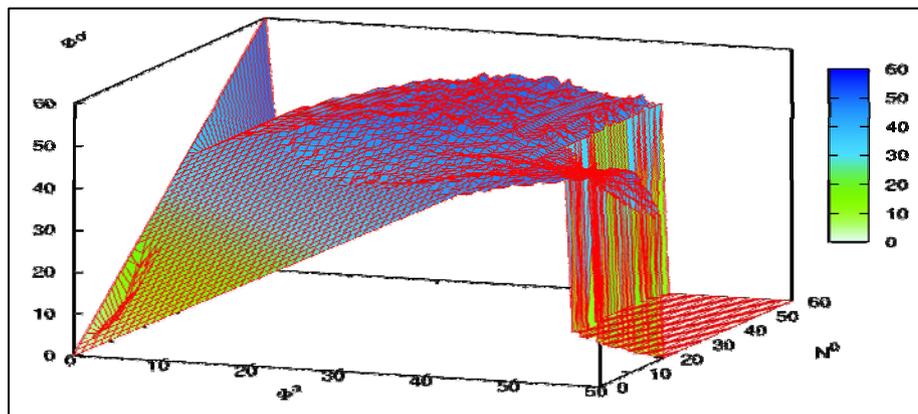


Figure 4: Practical airside capacity envelope at Toulouse-Blagnac Airport

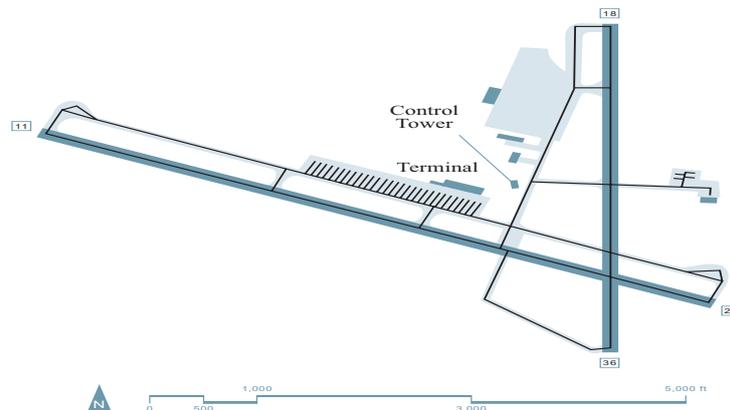


Figure 5: Ground traffic system at Portland International Airport

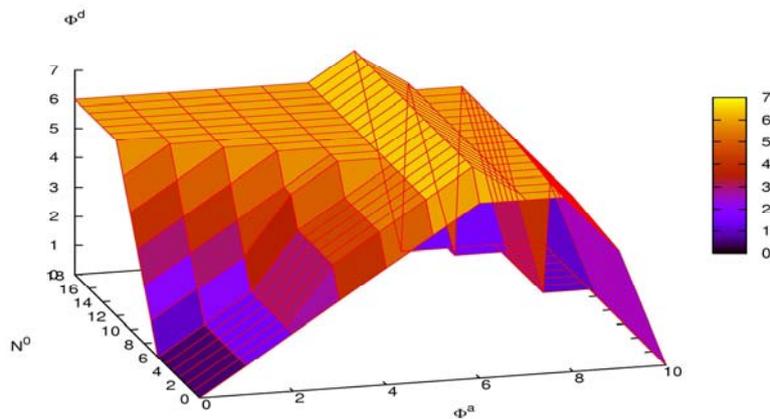


Figure 6 : 3D representation of the Portland International Airport practical capacity

## CONCLUSIONS AND FURTHER WORK

In this communication, a new dimension has been introduced to assess the practical airside capacity at airports by taking into account ground traffic delays and capacities. The estimation of the airside practical capacity is based on a flows network model of ground traffic and on the solution of successive optimization problems. The level of detail chosen for the representation of aircraft ground traffic flows at airports appears to be compatible with the study of ground operational and planning problems such as the definition of the aircraft circulation plan, the redesign of pre existent taxiways and apron areas and the assignment of parking areas to different airlines. However it seems interesting to try to introduce in some way the effect of the traffic of other ground vehicles and the new traffic patterns resulting from the use of advanced ground traffic management and control systems.

A detailed model validation for different sizes and configurations of airports seems necessary while improved mathematical programming techniques should be tailored for this set of optimization problems.

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