Measuring Airline Networks: Comprehensive Indicators

Chantal Roucolle¹, Tatiana Seregina², Miguel Urdanoz³

Abstract

The literature on airlines presents few studies analyzing the airlines network evolution and its impact on prices, costs or profitability. We believe that this gap is due to the difficulty of capturing the network complexity in a simple manner. This paper proposes new simple and continuous indicators to measure the airlines network structure. The methodology to build them is based on graph theory and principal component analysis. We apply this approach to the US domestic market for 2005-2015, and obtain three network indicators. The first one measures how close the network is to a hub-and-spoke structure. The second indicator measures the airline’s ability to provide alternative routes. The third indicator captures the network size. We analyze how the carriers’ network evolution can be described by those indicators. We show that low-cost carriers (LCCs) and legacy carriers’ network choices differ for the second indicator, while our results exhibit no difference in strategies for the other two indicators. We also show that economic conditions affect differently the three indicators and the magnitude of the impact depends on the airline type.

Keywords: Airline; Graph theory; Hub; Network; Principal Component Analysis (PCA)

Highlights:

● Combine graph theory and principal component analysis
● Obtain three indicators to characterize airline network structure for US domestic market
● Compare these indicators for low-cost and legacy carriers
● Estimate evolution in the indicators over time
● Analyze the impact of the main US mergers on the network structure

¹ Chantal Roucolle, École Nationale de l’Aviation Civile, 7 avenue Edouard Belin 31055 Toulouse, France. chantal.latge-roucolle@enac.fr
² Tatiana Seregina, École Nationale de l’Aviation Civile, 7 avenue Edouard Belin 31055 Toulouse, France, and Toulouse Business School – Université Toulouse I. ts.tseregina@gmail.com
³ Corresponding author. Miguel Urdanoz. Toulouse Business School – Université Toulouse I, 20, boulevard Lascrosses BP 7010 – 31068 Toulouse Cedex 7, France. m.urdanoz@tbs-education.fr, Tel: +33 561 29 47 43
1. Introduction

The air passenger industry is extremely complex and dynamic, and continues to grow rapidly. Since 1970, worldwide air traffic has doubled every 15 years, and according to Airbus (2017) forecasts this growth rate will persist over the next 20 years. In this context, each air carrier adapts its network structure, i.e., airports served and flight schedules, aircraft type, number of assigned seats, and frequency on each flight leg. Scheduled flights form a complex network of connected cities whose organization depends on airlines’ strategies of expansion. This study applies a methodology based on graph theory and principal component analysis to build new indicators that reflect an airline’s individual network structure. Then we analyze, through these indicators, how airlines’ strategies of expansion are related to intrinsic airline characteristics and to economic conditions in the US domestic market.

The evolution of airline networks has been a central concern in the airline literature since the emergence of the first hubs in the U.S. in the 1970s. The hub-and-spoke network seemed at that time to be a more profitable structure due to economies of density and scope (Caves, Christensen, and Tretheway 1984; Brueckner and Spiller 1994; Nero 1999). Therefore, airlines were expected to implement this type of structure while increasing their size.

However, new operators in the industry, namely low-cost carriers (LCCs), adopted a totally different network organization, with highly connected or point-to-point networks, achieving higher profits in some cases than the legacy carriers. The attention of several authors focused on comparing these two distinct network structures, as in Brueckner (2004), Alderighi et al. (2005), Barla and Constantatos (2005), Flores-Fillol (2009), and Silva, Verhoef, and Van den Berg (2014). In most of the cases, perfect hub-and-spoke networks were compared with fully-connected networks. However, reality is more complex: airline network organization lies between these two extreme cases. Ryerson and Kim (2013) presents a methodology defining tiers for hubs. Wojahn (2001) shows that a network combining hub-and-spoke and fully-connected structures could be optimal and that multi-hub networks are not cost minimizing for airlines. Moreover, hubbing is only one of several possible network dimensions and should not be the only point of focus. To our knowledge, no studies in the literature attempt to analyze the evolution of airline networks from an economic perspective. We believe that this gap is due to the difficulty of capturing the network complexity in a simple manner. The large number of interconnected routes, and the diversity of relationships between those routes, make it difficult to construct an appropriate model and thereby analyze the structural and dynamical properties of the networks.

In this study, we build continuous indicators of network structures combining graph theory measures with a principal component analysis. Building continuous indicators have a double purpose. First, it allows to study the airline’s network complex evolution in a simple manner and to measure how this evolution is affected by external factors such as the economic environment, petrol prices or regulatory constraints. Second, it will allow to study the impact of the network structure over profits, costs or delays. Both purposes are relevant to determine the optimal airline strategies and in consequence the optimal regulation. This article considers the first objective while the second one is left for future research.

To begin with, we consider airlines’ networks as graphs and we select graph theory measures relevant to describe the network structure, which are commonly used in the literature on airlines. Given the large number of graph measures, we propose to apply a principal component analysis (PCA) to synthesize the information. PCA will allow to obtain simple and continuous indicators, the principal components, to characterize the airline networks. We apply this methodology to the airlines operating on the US domestic market. We retain three principal components that reflect most of the information contained in the selected graph measures. The retained components allow characterizing the network structure and help to represent its evolution. The first component is an indicator of the airline network structure related to the hub presence that we will denote as NetHub. However, we study only the spatial dimension of hubs and leave for further research the inclusion of the temporal dimension that is considered by Wojahn (2001) or Burghouwt and de

---

4 The literature considers as hubs airports where one or several airlines propose a large number of flights, and passengers can transfer between flights to get to their intended destination.
Wit (2005). The second component represents the airline’s ability to mitigate disturbances by providing alternative flights within its network that we will denote as NetWeave. NetWeave does not measure the airline’s quality in terms of service provided to customers under network disturbances, but it measures the airline’s richness of alternative routings. The last component, NetSize, measures the network size in terms of flight segments or nonstop routes served. We illustrate the position of US carriers according to the three indicators, NetHub, NetWeave and NetSize, emphasizing the differences between legacy carriers and LCCs. Empirical studies by Jarach, Zerbini, and Miniero (2009), Aydemir (2012), Klophaus, Dr. Conrady, and Fichert (2012) and Bitzan and Peoples (2016) show the convergence in business models between the two categories of operators. However, we observe that some differences remain in terms of network structure: for instance, NetWeave remains higher for low-cost carriers.

We confirm these preliminary observations by analyzing the evolution of the network structures on the US domestic market over a ten-year period. We estimate the three indicators by a simultaneous system of equations. The estimated model highlights the differences and similarities in network evolution among legacies and LCCs. We also emphasize the influence of economic conditions and air market characteristics on the structure of airline networks, introducing jet fuel price and output gap as proxies for economic growth, as well as some controls for important air market shocks. We show that the impacts not only differ according to the type of airline but that their magnitude is specific to the indicator at stake.

The remainder of this paper is organized as follows. The next section presents a literature review. Section 3 describes the methodology for constructing indicators, combining the most important topological measures provided in graph theory, that will be presented in Section 4, through a principal component analysis. In Section 5 we apply this methodology to the US domestic market. We extract three principal components or network indicators and interpret them. Based on the three indicators, a graphical representation of the airline networks is provided. We put an emphasis on the differences between legacies and LCCs. Section 6 is dedicated to the estimation of the evolution of the three indicators over the period of analysis. We distinguish the evolution between legacies and LCCs controlling for some economic conditions of the market. Last, we conclude and suggest further possible applications of the indicators.

2. Literature Review

We can disentangle two branches within the literature on airline network structures: a graph theory branch with its focus on network mathematical properties and an economic branch traditionally comparing perfect hub-and-spokes and point-to-point to networks. In this section, we summarize the main results obtained in each area. Our objective is to link these two branches building simple continuous indicators based on graph theory that represent the complex reality of airline network organization and will enable us to study the economic impact for airlines of their network choices.

Air, road and rail transportation are industries organized in networks connecting separate locations. Graph theory provides powerful analytical tools to investigate network structures and their evolution. Several studies in transportation try to identify network characteristics such as the network robustness to disturbances, the transmission capabilities in the dissemination of information or in spreading communicable diseases, and more generally the evolution of the transport systems. The difficulty arises from the large number of measures available. In general, the literature refers to particular graph theory measures and justifies their relevance for the study of specific objectives. Ház Nagy et al. (2015) use graph theoretical centrality measures and global characteristics such as network diameter, average path length, degree distribution and community structure to understand and compare network characteristics of public transportation systems in several Hungarian cities. Examples of resilience studies can be found in the work of Liu and Tan (2013) for Wuhan City subway networks, Angeloudis and Fisk (2006) for the world subway system, or Chatterjee, Manohar, and Ramadurai (2016) for bus network in India. Zhu and Luo (2016) calculate average values of degree, clustering coefficient, betweenness, shortest path and some others, to characterize Guangzhou’s subway network and analyze the evolution of these parameters for the future development of the subway system.
In the airline sector, most of the studies focus on hub properties. Martín and Voltes-Dorta (2008) propose a hubbing concentration index that captures the number of passengers who make some onward connection. Burghouwt and Redondi (2013) present a compilation of connectivity indicators for airports with most of them coming from graph theory. Other studies analyze different network characteristics at the world, country or regional levels. For instance, Da Rocha (2009), Gautreau, Barrat, and Barthélemy (2009), Wang et al. (2011), Wandelt and Sun (2015), Dunn and Wilkinson (2016) and Du et al. (2016) examine different network properties at the country level. Lordan and Sallan (2017) investigate the robustness of the European airport network in case of airport isolation caused by random or targeted attacks, while Sun, Gollnick, and Wandelt (2017) study the robustness of the worldwide airport network. Diverse applications of the complex network theory to air transportation are summarized in Zanin and Lillo (2013) and discussed in Cook et al. (2015). Few works are dedicated to the network analysis at the airline level. Reggiani et al. (2009) and Reggiani, Nijkamp, and Cento (2011) study Lufthansa’s network and Cento (2008) analyzes several European carriers’ networks.

Although the network structure is a key determinant of airline profitability, costs or delays, the literature is sparse and as stated in the introduction, in most of the cases perfect hub-and-spoke systems are compared to complete point-to-point networks. We believe that the absence of continuous network indicators justifies the limitation of the analysis to these two distinct network structures. Some studies, however, have tried to surpass this barrier: Borenstein (1992), Reynolds-Feighan (1998, 2001) and Burghouwt, Hakfoort, and Ritsema van Eck (2003) study hubbing by airlines. They use measures such as the Gini index, the Theil entropy measures, the Herfindahl Hirschman index, the coefficient of variation, or modifications of these measures, depending on the available sources of information, to see whether airlines are close to a hub-and-spoke network. Such measures have been applied to study the impact of hubs on the airline’s cost structure (Pels, Nijkamp, and Rietveld 2000; Ryerson and Kim 2014), the level of competition (Hendricks, Piccione, and Tan 1997), prices (Tan and Samuel 2016), the level of congestion and delays (Mayer and Sinai 2003; Brueckner 2002), or to study several of these characteristics combined (Bilotkach, Mueller, and Németh 2013; Brueckner and Zhang 2001).

Nevertheless, hubness is only one network property whereas graph theory measures can characterize other network attributes. One of the challenges faced while applying graph theory is that among many graph-theoretical measures there can be several highly correlated concepts. This complicates the choice of the most appropriate measure and hampers the simultaneous use of similar variables in econometric analysis. For these reasons, we propose to apply a PCA analysis on the most common graph theory measures used in the literature to build tractable indicators.

3. Methodology

The question we address is the following: can we characterize the structure of an airline network so that it could be possible to analyze its evolution, predict airlines’ decisions in terms of hub and route creation or evaluate the network optimality? This section presents a methodology to construct such network indicators that can be relevant to describe the network structure and evolution.

The section begins defining airlines networks with a mathematical concept: graphs. First, we specify the main assumptions to represent networks as graphs. Then we explain the methodological foundation to construct the network indicators.

3.1. Network Model and Assumptions

The air transport network is a complex system of flight connections between cities. Such a system can be described by a graph and its structure can be studied using techniques developed in graph theory. A graph is an abstract representation of interconnected objects. It is defined as a set of nodes that are joined
by a collection of edges (Diestel 2006). Edges represent relations between the nodes of a graph. We consider the air transport network at the airline level and describe an airline network as a graph. Cities served by the airline are the graph nodes, and flight segments operated between cities are the edges.

In order to apply graph theory tools to study an airline network, a preliminary step devoted to flight data representation is required. The objective of this step is to obtain the data in the form of a connection or adjacency matrix. For an airline, such a matrix describes the existence or absence of an edge between each pair of nodes within the network at a given date. At this stage, we make several assumptions:

**Assumption 1:** airlines make network decisions at the city level rather than at the airport level.

Although an airline may serve several airports in a metropolitan area, these airports are considered as a single airport for that airline. Under this assumption, airlines using several airports in the same city take network decisions based on all the flights proposed from all the airports in the city. This is consistent with the airport grouping suggested by Brueckner, Lee, and Singer (2013).

**Assumption 2:** a graph associated with an airline network is undirected and unweighted.

Our study focuses on structural, or topological, properties of airline networks. This means that we are interested in the network structure in terms of connections between city nodes, no matter the direction, nor any edge characteristics such as flight frequencies or seats. Under assumption 2, we define a flight segment as a dichotomous variable taking value one if a nonstop connection between two cities exists, no matter the direction or the frequencies and taking value zero otherwise.

The last assumption is related to the relationship between major airlines and their feeders. Airlines can decide to operate all the flights they propose to their customers or to outsource the operations to regional carriers or feeder companies. These flights have to be coordinated between both carriers:

**Assumption 3:** major airlines and feeders coordinate their network decisions.

The choice of subcontracting or integrating routes in a network is crucial in air transport industry as suggested by Forbes and Lederman (2009), Levine (2011) and Gillen, Hasheminia, and Jiang (2015). We assume that network choices for the feeders should be done in perfect coordination with their majors and, therefore, we recode regional/feeder airlines to their major partners.

### 3.2. Graph theory and PCA

Under the previous assumptions an airline network can be represented with an adjacency matrix. If a network evolves over time, its matrix changes accordingly. Each adjacency matrix allows to compute various graph theory measures. In the literature, several of these measures are considered to study the airline networks, however some of them are correlated and in fact reflect in different ways the same network property. This complicates the selection and interpretation of the most appropriate measures to implement studies on airline networks. Moreover, this flaws the comparison among existing studies. It is therefore essential to identify the most meaningful variables, or to build some combinations of them, which can reflect accurately the network structure observed from the data. In this article, we propose to follow the latter: we reduce a sample of selected graph theory measures into a small set of indicators built from linear combinations of the measures. The main challenge is then to be able to interpret the indicators in line with the airline network structure.

---

6 For instance, John F. Kennedy International Airport (JFK) and Newark Airport (EWR) are recoded as NYC for American Airlines, if it serves both airports.

7 An edge or a node in a graph can be given a specific quantitative characteristic, or weight. If no weights are assigned to edges or nodes, the graph is said to be unweighted.

8 The use of undirected graphs implies that nonstop flights between New York and Los Angeles and nonstop flights between Los Angeles and New York are considered as a single flight segment or route.

9 For instance, flights operated by Ravn Alaska (7H) are recoded as Alaska Airlines (AS). A full list of the recoded carriers and flights can be provided upon request.
Reducing the number of variables to a few interpretable linear combinations can be performed by one of the two most widely used dimension-reduction techniques: principal component analysis (PCA) or factor analysis (FA). Although they share the same objective, to simplify a set of variables, PCA and FA differ analytically. PCA aims to explain most of the total variance observed in the dataset by a smaller set of new components, called principal components. The goal of FA is to understand which factors underlie the covariances between the original variables; it defines the original variables as linear combinations of these factors.

Our purpose is to obtain a reduced set of meaningful variables to describe airline networks, through a combination of graph measures, so that the newly constructed variables can effectively identify the network structure. Therefore, we focus on the principal component analysis as it allows such a transformation.

Once the PCA is applied and a set of principal components is selected, these components must be interpreted to describe the network structure, to be able to compare airlines’ networks and to understand their evolution. Interpretability of the principal components depends both on the dataset and on the selection of the original variables, this is one of the drawbacks while using PCA. The selection of graph measures to be included in the analysis is crucial to guarantee this interpretability.

We consider various graph theory measures, some of which are commonly used in the transportation literature. A description for each of them will be provided in the following section. For the sake of convenience, we split the graph theory measures into two groups: overall and node-based measures. Overall measures relate to the whole network. In this case a single measure need to be computed for the whole network. We argue that all the overall graph theory measures are robust to changes in the sample and should be used to build airline network indicators with a PCA. Node measures assess how nodes (or cities) are related to each other within a network: node-based measures are studied at the node level. Then, in order to obtain measures for the whole network, we need to summarize node measures through centralization indexes and descriptive statistics (maximum, minimum, average, standard deviation, percentiles, etc.). Given this large variety of statistics for node-based measures, their final selection for the implementation of the PCA should be driven by the market characteristics and the possibility to accurately interpret the obtained principal components. Therefore, there is no general rule for the selection of node measures and their contribution to the principal components might vary slightly with sample changes.

The principal components obtained from PCA are linear combinations of the selected graph measures. We argue that they can be considered as indicators for airline network structure. That is, at each period of time, their respective values allow to draw the network. These indicators could then be used as inputs in models predicting, e.g., network evolution, profitability or prices.

4. Graph Theory Measures

We now describe all the network measures used in this study and provide their interpretation in terms of airline networks. We distinguish overall network measures that characterize the whole network, from node-based network measures that are calculated at the node level and express the relative connectivity of a node compared to the others in the network. Their respective formulas and some illustrative examples are presented in Table A.2. For each measure, the corresponding variable name will be indicated.

4.1. Overall Network Measures

The elementary network characteristics, i.e., the number of cities (nbCity) and the number of flight segments (nbFS) convey what is called the order of a network and its size, respectively. The number of

---

10 See Fodor (2002) for a survey of dimension-reduction techniques.

11 A detailed description of the technique can be found in Abdi and Williams (2010) and in appendix B for our application to the US domestic market.

12 Note that the network size is measured in terms of segments and not markets. Indeed, airlines can serve markets between two cities via connecting flights; however, that information is not available in our database.
flight segments measures the total number of existing nonstop connections between all the cities in the network. The size of the network can be one of the criteria used for network selection: a too small size could lead to the exclusion of a network. The threshold must be determined according to the sample of networks analyzed.

Another concept of network size is the diameter (\text{diamG}). It represents, in the case of airlines, the minimum number of nonstop flights needed to connect the two most remote cities in the network. The diameter is another criterion that can be used for network selection. It allows to identify particular networks, associated with a low network size as we will see in the next section. \text{diamG} is not included in the PCA analysis as some airlines present disconnected networks\textsuperscript{13} and \text{diamG} cannot be measured in the case of disconnected networks.

The network density (\text{DensG}), or graph density, is the number of existing edges with respect to all the possible edges. A complete graph, the graph where any pair of nodes has a direct connection, contains all possible connections and thus achieves the maximum density, 1, as can be seen from Table A.2. For any airline network, the higher the value of network density the more nonstop flights the network offers, i.e., most of the cities can be reached directly from any city. Typically, a high value of density indicates that the network is close to a point-to-point organization. We argue that this measure has to be included into PCA analysis.\textsuperscript{14}

\section*{Node-based Network Measures: Transitivity and Centrality Measures}

Centrality measures are intended to determine the structural relevance of a single node in a network, quantifying in different ways the importance of a node among the other nodes.

The degree centrality of a node (\text{Cdeg}) is a purely local measure. It is the number of direct connections the node has with respect to the number of all possible direct connections the node may have. The degree centrality of a node reveals how locally well-connected each node is. In an airline network, the more non-stop flights connect a city with other cities, the higher the degree centrality of this city. For a given network there are as many values of degree centrality as nodes.

A subtler measure of a node’s importance is the closeness centrality, which is based on geodesic distance.\textsuperscript{15} However, closeness centrality becomes useless for disconnected graphs since the distance between two nodes belonging to different components is infinite by convention. Da Rocha (2009) extended closeness centrality to disconnected graphs by defining the harmonic centrality index (\text{Char}). In an airline network, the harmonic centrality of a city indicates how fast every other city in the network can be reached from this city. A fast trip between two cities means a low number of flight segments required to form the trip. Table A.2 shows that disconnected networks generally have low values of harmonic centrality.

Betweenness centrality (\text{Cbet}) is also derived from geodesics. It quantifies the number of times a node occurs on all the graph geodesics. In an airline network, betweenness centrality gives a high value to the city that may occur most often as an intermediate stop on the routes between any pair of other cities containing the least number of stops. In this case a city with a hub position will exhibit a high value of betweenness centrality. For instance, the largest value of betweenness centrality is achieved in a star graph by the central node of the star, and equals 1, whereas all other nodes within the star have betweenness centrality equal to 0. Note that a high value of betweenness centrality may also refer to cities that play the role of a bridge between two distinct parts of a network as shown in Table A.2.

\textsuperscript{13} If there are nodes in a graph that cannot be reached from others, the graph is disconnected. In other words, a disconnected graph splits into several connected components, whereas a graph that is in one piece is connected. In Table A.2, all the graphs are connected except the last two columns that illustrate disconnected graphs.

\textsuperscript{14} Given the construction of \text{DensG} presented in table A.2, \text{densG}, \text{nbCity} and \text{nbFS} present high correlation levels and can be collinear under some scenarios. To avoid collinearity issues, we decide to exclude \text{nbCity} from the PCA analysis.

\textsuperscript{15} A geodesic distance between two nodes is the minimum number of non-repeated edges that connect the two nodes, i.e., the number of edges (flights) in a shortest path connecting the nodes (cities).
Eigenvector centrality (\textbf{Ceig}) is based on the concept that connections to high-scoring nodes contribute more to a node score than connections to low-scoring nodes.\textsuperscript{16} A high value of eigenvector centrality for a city characterizes how well this city is connected to other well-connected cities. When a city has a high eigenvector centrality, this may indicate that the city is a hub with many direct connections or it is linked to several highly connected cities.

Every centrality measure can be applied to the whole network rather than only to a node, and in that case, we address ourselves to descriptive statistics or \textit{centralization} indexes. Among the descriptive statistics we can consider the maximum, minimum, average, standard deviation, or different percentiles. The network centralization gives an answer to the question of how central the most central node is in relation to how central all the other nodes are. The centralization index belongs to $[0,1]$. It reaches its highest value for a star graph in terms of degree and betweenness centralization (Freeman 1978). Similarly, the maximum harmonic centralization index is obtained for a star graph as shown in Table A.2. In terms of eigenvector centralization, the most centralized structure is the graph with a single edge (and potentially many isolated nodes). For connected graphs, the value is high if the graph has a single star topology (as illustrated in the first column of Table A.2). Each centralization index is named concatenating ‘G’ and the original name of the index, so that the centralization indexes for \textit{Cdeg, Char, Cbet,} and \textbf{Ceig} become \textit{Gdeg, GChar, Gbet,} and \textbf{GCeig}.

Finally, a node and a pair of its neighbors\textsuperscript{17} form a \textit{triplet} that can either be open (when the three nodes are connected by two edges) or closed (when the nodes are fully connected). The \textit{global clustering coefficient}, or graph \textit{transitivity}, is the number of closed triplets over the total number of triplets (both open and closed). The transitivity coefficient (\textit{transG}) of an airline network can be understood as a measure of the network connectivity. A hub-and-spoke structure (a star network) presents a zero value for transitivity, however as seen in Table A.2 low values of transitivity can also represent other types of structures. Instead, high values of network transitivity, imply denser groups of nodes. Transitivity is a measure of particular importance for the network characterization. For instance, Trapote-Barreira, Deutschmann, and Robusté (2016) use network transitivity, associated with network density to study delay propagation.

5. Application to the US Domestic Market

The methodology proposed in the previous section to construct network indicators is now applied to the case of the US domestic market. After describing the data source, we will move on to the results of PCA and to the network characterization.

5.1. Data

Our dataset is built from the Official Airline Guide (OAG), which provides information on worldwide scheduled traffic such as route characteristics (departure and destination airports or distance) and flight characteristics (time of departure and arrival, duration, available seats or operating airlines). We consider one-way direct passenger flights between 2005 and 2015 for all the operating carriers within the US domestic market.\textsuperscript{18} As is usual in the literature, daily data has been extracted for the third quarter, i.e., July 1\textsuperscript{st} to September 30\textsuperscript{th}, since the third quarter presents the highest annual traffic. Moreover, the restriction to a single quarter across years avoids a seasonality treatment that would add no significant information but would be computationally expensive.

\textsuperscript{16} The score of a node is proportional to the sum of its neighbors’ scores. Hence a node can have a high eigenvector centrality either because it has many neighbors or because its few neighbors are important, or both. A node with high eigenvector centrality will not necessarily be highly connected, and a node with many direct connections will not necessarily have a high eigenvector centrality.

\textsuperscript{17} Two nodes are said to be \textit{neighbors} if they are connected by an edge.

\textsuperscript{18} The choice of the US domestic market is motivated by the large literature existing on that market, and the possibility of extending our analysis with other public databases such as the DB1B (which provides information on prices).
As stated in assumption 3, we group regional/feeder airlines and their major partners. Additionally, we delete from the database non-US operating carriers. These records, representing less than 1% of the database, are considered as mistakes since cabotage is forbidden. We exclude small airlines that operate networks with extremely short routes (none exceeding 200 miles) or that exclusively operate aircrafts with a capacity never exceeding 10 seats. We also remove airlines that operate for less than 4 years during the period 2005-2015, considering them as non-relevant cases for studying the airlines’ network evolution. These two steps in the database cleaning combined represent less than 2% of the observations.

We use the diameter to remove small airlines with a specific network type for which the diameter equals one. This corresponds to 6 airlines. Half of the airlines in our database have networks that are disconnected. Diameter is not defined for a disconnected graph and consequently this measure is discarded in the subsequent analysis.

The final dataset contains monthly route level information for each operating carrier, for the third quarter of 2005-2015. We have 124,682 monthly-route observations. The number of operating carriers per year ranges from 19 to 22. The number of cities varies from 400 to 516.

5.2. Result: Three Indicators

The graph measures described in the preceding section are calculated monthly for each operating carrier for the third quarter of 2005-2015. We calculate overall measures and node-based centralization and centrality measures. Among the latter, and in order to describe the whole network, the usual descriptive statistics are computed. Therefore, the minimum (min) and maximum (max) over all nodes in the network as well as its mean (mean), median (med), 5th and 95th percentiles (p5 and p95) are computed for degree, harmonic, betweenness and eigenvector centrality.

However, as discussed in the methodology section, not all the graph measures presented above are equally relevant to describe the network structure and obtain interpretable indicators. We include in the PCA analysis the number of flight segments, network density and network transitivity. Regarding degree, harmonic and betweenness, we include their centralization and maximum centrality measures (GCdeg, maxCdeg, GChar, maxChar, GCbet, maxCbet). Indeed, centralization is higher if the network contains very central nodes as well as very peripheral nodes. Hub-and-spoke networks will have high centralization measures and the most central city will have a high centrality score (i.e., the maximum values of degree, harmonic and betweenness centrality are high). Table A.3 presents the average value and standard deviation for each measure and airline considered for PCA over the studied period. As can be seen in this table, the betweenness centrality for Frontier Airlines (F9) hub, Denver, has a value of 0.94. In point-to-point networks, cities have roughly the same importance within the network, therefore the network will present low centralization measures while the maximum of degree, harmonic and betweenness centrality measures can be high or low.

Finally, we include eigenvector centralization and the mean of eigenvector centrality (GCeig and meanCeig). Eigenvector centrality shows if a city is well-connected to other well-connected cities. Large eigenvector centrality characterizes a city that either has many non-stop connections or one that is connected to many important cities. A high mean value of eigenvector centrality indicates that the cities in the network are on average well connected. Moreover, a value of eigenvector centralization close to zero indicates that most of the cities have roughly the same importance, i.e., all are roughly equally well connected either having the same number of non-stop connections or being connected to important parts of the network, which suggests a point-to-point structure. As we can see in Table A.3, Frontier Airlines’ (F9) network has the highest eigenvector centralization index, while Southwest (WN) has the lowest value. This is consistent

---

19 We eliminated 6 airlines that operate very small networks, with no more than 3 flight segments. Their IATA codes are: 1X, 3E, CH, P1, U5, V2.
20 The full list of airlines is presented in Appendix A.
21 PCA have been tested for several combinations including different sets of node centrality statistics. Our final selection of the node centrality statistics is driven by the possibility of accurately interpreting the obtained principal components.
with the graph structure of these two airlines: star topology for Frontier airlines versus highly connected network for Southwest.

Ultimately, PCA analysis is performed for the data set made of the 11 selected graph-theoretical measures calculated for each airline over time: this corresponds to 629 observations for 11 variables. The first three principal components obtained from the analysis explain 94.69% of the sample variability. We choose to keep these three principal components corresponding to the largest eigenvalues of 5.0364, 3.4168 and 1.9627 respectively. This choice, as detailed in Appendix B, agrees with the commonly used eigenvalue one criterion suggested by F. Kaiser (1960) and with the Cattell (1966) scree test.

The first principal component presents high positive correlations with six variables (\textit{maxCdeg, GCdeg, maxChar, GChar, maxCbet, GCbet}) as shown in Table 1. These variables correspond to node centrality measures of a network and their centralization indexes. Thus, the first principal component can be interpreted as an indicator of the presence of hubs in the network, or as an indicator of a network topology ranging from point-to-point configuration to hub-and-spoke topology. The larger the value of such an indicator, the closer the network structure to a hub-and-spoke; and vice versa, low values of the indicator should imply that the network has a point-to-point structure.

The second principal component is strongly correlated with four variables. It increases with density (\textit{densG}), transitivity (\textit{transG}) and mean eigenvector centrality (\textit{meanCeig}) and decreases with eigenvalue centralization (\textit{GCeig}). This component can be viewed as a measure of network interlacing. A high value of the second principal component may reflect the network richness of alternative routings involving non-stop and one-stop flights, whereas its low value shows that the network does not contain alternative paths to attain a destination.

The third principal component is strongly correlated with only one of the original variables, flight segments (\textit{nbFS}). It increases with increasing \textit{nbFS}, i.e., the number of connections that are offered with nonstop flights.\footnote{For instance, Allegiant Air, G4, presents a NetSize of 242 flight segments in 2015. This means that 242 city pairs are proposed with nonstop flights. The use of undirected graphs implies that nonstop flights between New York and Los Angeles and nonstop flights between Los Angeles and New York are considered as a single flight segment or route.} We take this variable in its original form and we will denote it as \textit{NetSize}.

We denote the first principal component as \textit{NetHub} since the component proposes a hubness measure of an airline network. For the second principal component the notation \textit{NetWeave} will be assigned following its interpretation as a measure of network interlacing and the third principal component is \textit{NetSize}.

### 5.3. Network Airline Characterization

The three indicators presented in the previous subsection allow comparing the airlines’ networks based on objective network characteristics. We compute the scores on the principal components for each airline-
month of our dataset. We study July, August and September for the period 2005-2015. The way the components are constructed gives rough theoretical limits for the components, each one ranging between −11 and 11 (as explained in Appendix B). On our dataset NetHub ranges from −5.24 to 4.63, and NetWeave, from −2.93 to 7.54.

Based on the airline scores on these two indicators we can characterize the airline networks as follows:

1. An airline network can be classified as hub-and-spoke with one unique hub when NetHub is high, whatever the value of NetWeave;
2. When NetHub approaches zero and if NetWeave is also close to zero, the network structure remains a hub-and-spoke, but the number of hubs increases;
3. When NetHub becomes negative and if NetWeave is positive and high, the network has a point-to-point structure;
4. When NetHub becomes negative and if NetWeave is negative and low, the network should have a path or a circle structure, however no airline in our database meets this case.

Figure 1 displays this classification.

These indicators can shed new light on the airline classification between legacy carriers\textsuperscript{23} and low-cost carriers (LCCs).\textsuperscript{24} Figure 2 shows the airlines’ positions in September 2015. We observe that LCCs do not exhibit a common pattern in their network structure. Spirit Airlines (NK) and Southwest (WN) have low NetHub and high NetWeave. This position is characteristic of a point-to-point network. Interestingly, we observe that the other airlines classified into the LCC category are not located in the same area. Frontier Airlines (F9) is a perfect star network (with its hub in Denver) although this airline is nowadays developing

\begin{itemize}
\item[23] We include in this category airlines that were established before the Airline Deregulation Act of 1978, i.e., American Airlines (AA), Alaska Airlines (AS), Continental Airlines (CO), Delta Airlines (DL), Hawaiian Airlines (HA), Northwest Airlines (NW), United Airlines (UA) and US Airlines (US). Historically these airlines provided a higher service level to their passengers than LCCs although the differences have diminished over time. The full list of airlines and their classification is available in Table A.1.
\item[24] LCCs comprise Allegiant Air (G4), JetBlue (B6), Frontier Airline (F9), Spirit Airlines (NK), Virgin America (VX) and Southwest Airlines (WN).
\end{itemize}
several hubs (for instance, Orlando or Las Vegas). JetBlue (B6) has a hub-and-spoke network with several hubs.

Jarach, Zerbini, and Miniero (2009) and Bitzan and Peoples (2016) highlight that both type of airlines could be converging towards a hybrid model. Indeed, as seen from NetHub, there is no clear distinction between legacy carriers and LCCs. Nonetheless, we observe a difference between the two airline categories in terms of NetWeave. Most of the LCCs present positive values of NetWeave except Allegiant Air (G4), whose west coast and east coast bases share few common flights. Reversely most of the legacy carriers have NetWeave negative values. The only exception is Hawaiian Airlines (HA), whose network is highly conditioned by the geographical constraints of the islands. A similar result is observed by Lordan and Sallan (2017) who study the robustness of airline networks to airport isolation. They find that robustness, their equivalent to NetWeave in our study, is higher for LCCs than for full service carriers over a sample of airlines from Europe, North America and China.

![Figure 2](image)

Figure 2 Airline networks represented in a two-dimensional coordinate system of NetHub and NetWeave in September 2015. Points are labeled with airline IATA designators. The full airline list and their respective codes are provided in Table A.1.

We now analyze the interaction between NetHub and NetWeave indicators and the third indicator, NetSize. Figure 3 shows the relationship between NetSize and NetHub. We observe that when NetSize increases, NetHub decreases to some level between 0 and −3, both for LCCs and legacies. The relationship between NetSize and NetWeave is illustrated in Figure 4. When NetSize increases NetWeave seems to approach a level around −1. The picture is, however, somewhat different between LCCs and legacies. All legacies seem to be stable around this NetWeave level and have negative values, while no clear trend exists among LCCs.
Figure 3  *NetSize* vs *NetHub*. Each point corresponds to an airline for a particular year. Monthly indicators are averaged to obtain yearly data. Southwest (WN) evolution is highlighted by a line linking all its observations.

Figure 4  *NetSize* vs *NetWeave*. Each point corresponds to an airline for a particular year. Monthly indicators are averaged to obtain yearly data. Southwest (WN) evolution is highlighted by a line linking all its observations.
Given the relationship between *NetHub*, *NetWeave* and *NetSize*, an increase in *NetSize* suggests a tendency toward a multi-hub structure. Only Southwest (WN) is apart from this general tendency. Southwest, created in 1976, is traditionally quoted in the air transportation literature as the main example of the LCC business model. When analyzing the structure of its network, we observe in Figures 3 and 4 the continuous growth of its *NetSize*. The number of flight segments has been multiplied by more than 1.5 between 2005 and 2015. In 2015, Southwest’s *NetSize* was comparable to the post-merger entities of United Airlines (UA) and Continental Airline (CO). Moreover, Southwest’s network *NetWeave* indicator is among the highest in our airline sample, and remains stable since 2005. Meanwhile, Figure 3 shows that Southwest *NetHub* indicator remains negative and low. The scores of Southwest’s indicators suggest a large network with a point-to-point structure. We therefore highlight an important difference between Southwest’s strategy and the other airlines’ strategies in terms of network evolution. Given the timespan of our database we explore in section 6 the evolution of the indicators over the last decade.

5.4. Robustness

We have conducted some tests to highlight the robustness of our results. We have applied the same methodology to several randomly selected subsamples: in each draw, we extract 70% of the observed airlines. In all the cases, F. Kaiser's (1960) eigenvalue one criterion and Cattell’s (1966) scree test confirm the selection of three principal components. The robustness check shows some uncertainty about the contribution of mean eigenvector centrality (*meanCeig*). In 40% of the tests, *meanCeig* contributes most to the third principal component rather than to the second one. However, the interpretation of the principal components remains unchanged. This result could be expected since *meanCeig* is one of the descriptive statistics among the node-based network metrics and therefore its relevance depends on the considered sample.

If the sample selection is not random, PCA results are modified and depend on the selection process. For instance, we build a sample with the 7 biggest carriers: we select carriers with a number of flight segments higher than 300 on average over the period of analysis. Only two principal components remain. Although the initial variables’ contributions change, the two principal components can be interpreted as *NetHub* and *NetWeave*. Instead, *NetSize* is no longer a relevant indicator, which is consistent with the restricted sample where all carriers have relatively the same number of flight segments.

6. Trends in Network Indicators and Airline Mergers

Airline network choices are long-term decisions as flights must be scheduled several months before their departure. When airlines decide to open new routes or close existing ones, the three indicators are affected simultaneously. We have already noted the differences and similarities between LCCs and legacy carriers in terms of *NetHub*, *NetWeave* and *NetSize*. We test if these differences may remain, increase or disappear over time. For instance, if an ideal network configuration exists in terms of costs and profitability, most of the airlines should try to shift their network towards this ideal configuration. The drivers of the decision could be the intrinsic characteristics of airlines, but also air transport market evolution, or some exogenous shocks affecting the economy or air transport organization. In this section, we analyze the evolution of the above defined indicators. We present the model and its explanatory variables, and then we estimate it and interpret the results. We take into account the possible differences between legacies and LCCs.

6.1. Simultaneous system of equations

Thus, our objective is to study the three indicators evolution over several trends and factors, stressing the differences between the types of airlines. We estimate the system of three equations expressed as:
\[ Net_{gi} = \alpha_{gi} + \beta_{gi}t + X_i\gamma_{gi} + Z_i\delta_{gi} + \epsilon_{git}, \]

where Net stands for the network indicators and subscript \( g \) denotes the three equations of our indicators: \( g \in \{\text{Hub}, \text{Weave}, \text{Size}\} \). Subscript \( i \) indicates an airline and subscript \( t \) indicates the year. Subscript \( l \) indicates if an airline is classified as legacy, LCC or regional. We use yearly data obtained from the average of the monthly observations.

We include airline fixed effects, \( \alpha_{gi} \), using Southwest as the benchmark airline, allowing each airline to exhibit some initial or historical particularities in terms of network evolution.\(^25\) A time trend, \( t \), is included and we allow a distinction between the evolution of LCCs, legacies and regionals. These are the main variables of interest of the model. Some statistically significant differences between the estimated coefficients could confirm the previously observed differences among legacies, LCCs, and regional carriers in their network evolution.

\( X \) is a matrix including two economic variables, jet fuel prices and output gap.\(^26\) Jet fuel nowadays account for the main share of airline operating costs: 20% according to IATA (2016). During our period of analysis, its share of operating costs reached the historical level of 33% in 2008 when the price per gallon attained US$3. Obviously, the fuel price evolution affects the airlines’ strategies in terms of improvement in fuel efficiency, pass-through to customers, hedging or network restructuring. Given that network choices are taken in advance, it is not clear whether jet fuel price affects the network structure contemporaneously or with some lag. The impact of jet fuel prices may depend on several factors such as the airline’s fleet age or hedging strategies. Airlines applying hedging should see a small impact due to current variations of fuel prices and a limited effect of past prices. Airlines that do not apply hedging should see a higher impact at the current level of fuel prices. Whether this impact is positive or negative could be different across the indicators, \( Net_{\text{Hub}}, Net_{\text{Weave}}, \text{or } Net_{\text{Size}} \). We expect different effects on airlines depending on their classification and we separate the effect of jet fuel prices over the three airlines categories: legacies, LCC and regionals.

A similar analysis is applied to the variable output gap. The airline industry is usually considered as cyclical. For instance between 2008 and 2009 the US airline domestic operating revenue decreased globally by more than 15% (Bureau of Transportation Statistics). Still the impact of the cycles may differ over time and airlines. Cornia, Gerardi, and Shapiro (2012) show that price dispersion over a sample of US carriers is affected by the business cycle and that the impact is larger for legacy carriers than for LCCs. We follow their approach by differentiating the impact of output gap over the three airlines categories. The expected impact over the airline network structure is not obvious, although the relationship between economic conditions and network structure is addressed in the literature at the airport or city level. Wang et al. (2011) analyze the overall structure of the Chinese air transport network and the centrality of individual cities. They use centrality indexes such as degree, closeness and betweenness and find that these indexes are highly correlated with socio-economic indicators of the cities, in particular with the regional GDP. In the same way as jet fuel prices, the output gap could affect the network structure through different time lags. If network decisions are taken in the past based on accurate forecasts, the impact of the current output gap should be the relevant variable for our study. If instead airlines are myopic about the future or they face financial constraints due to their results on the previous years, past levels of output gap would be more relevant for our analysis. We test the fit of our model with output gap and jet fuel prices at different time lags and contemporaneously.

Finally, \( Z \) includes several dummies to control for events that could affect airline’s network strategic decisions. First, we introduce dummies for the four mergers that took place during the considered time frame. A merger may imply a radical change in the network structure depending on the pre-merger structure.

\(^25\) We use a fixed effect model as the Breusch Pagan Lagrange Multiplier test rejects the use of a pooled regression and the Hausman test rejects the null assumption of independence between the individual-fixed effects and the exogenous variables of the model. In case of rejection of the null assumption the model is specified as a fixed effect model.

\(^26\) The output gap is an economic measure of the difference between the actual output of an economy and its potential output. Data on the output gap is obtained from IMF. Data on jet fuel prices are collected from the U.S. Energy Information Administration (EIA). The jet fuel price and output gap evolutions can be found in Figure A.1, Appendix A.
of the airlines and the percentage of overlapping routes. Our dummy accounts for the exogenous impact of the merger on the structure of the merged network. The four mergers are:

- **DLNW Merger**: Delta Air Lines (DL) and Northwest Airlines (NW) merged between 2008 and 2010. From 2010 these two airlines are considered as one.
- **UACO Merger**: United Airlines (UA) and Continental Airlines (CO) merged between 2010 and 2012. From 2012 these two airlines are considered as one.
- **WNFL Merger**: Southwest Airlines (WN) acquired AirTran Airways (FL) between 2010 and 2014. In 2015 these two airlines are considered as one.
- **F9YX Merger**: Frontier Airlines (F9) and Midwest Airlines (YX) merged in 2010. From 2011 these two airlines are considered as one.

We also introduce dummies for the airlines which went into chapter 11 bankruptcy protection during our period of analysis. Under chapter 11 protection North American airlines are allowed to restructure their business, breaking their previously agreed labor union contracts or freeing up cash which may translate into a network expansion or shrinkage depending on the airlines economic viability. This protection has been used in particular by North-American airlines after the 2008 global financial crisis. Three airlines over our period of analysis benefited of the chapter 11 protection: Frontier Airlines from April 2008 to October 2009, Sun Country Airlines from October 2008 to February 2011, and American Airlines from November 2011 to December 2013.

### 6.2. Estimation and discussion of the results

We estimate the system of equations (1) on a panel data set where the 28 airlines are observed over ten years. We calculate the yearly average per airline of the three indicators which leads to an unbalanced panel with 211 airline-year observations. The same explanatory variables are included in the three equations. This model leads to the estimation of 16 parameters, plus airlines fixed effects parameters for each of the three equations.

Since the endogenous variables are principal components and not observed values of a variable, they might be subject to measurement errors. In linear models however, as long as the measurement error of the endogenous variable is independent of the error of the model, $\varepsilon_{git}$, the measurement error simply leads to an increase in these errors without impacting the consistency of the estimator. Instead, if the three indicators are used in further studies to explain the airline’s profits, cost or other characteristics this measurement error should be taken into account and if possible controlled by instrumental variables.

Our objective is to estimate simultaneously the three linear equations proposed in (1) which includes airline individual effects. The differences among airlines may result in panel heteroscedasticity, and the Wald test confirms its presence. Additionally, autocorrelation could be present given the panel structure of the data. This is confirmed by the Wooldridge test. Therefore we include an AR(1) error in our estimation using a Prais-Winsten regression and estimate the disturbance covariance matrix by FGLS.

---

27 A fifth merger took place in 2015, the AAUS Merger: American Airlines (AA) and US Airways (US) merged between 2013 and 2015. From October 2015, these two airlines should be considered as one, however our data set ends in September 2015 so that the merger is not considered.

28 Delta and Northwest went into chapter 11 protection from September 2005 to May and April 2007 respectively. We do not include a dummy for these two cases since this period corresponds to a single observation in 2006.

29 The Chi2 value for the Wald test with 66 degrees of freedom is $2.0 \times 10^{10}$ with a p-value < 0.000. The null hypothesis of homoscedasticity is rejected. The Wooldridge test for autocorrelation gives a value of $F(1, 65) = 13.671$ with a p-value = 0.0004. We reject the null hypothesis of no first-order autocorrelation.

30 We use the `xtpce` procedure from Stata.
Table 2 presents the results of the estimation split by indicator. We observe that LCCs decrease their NetHub over time as shows Trend LCC (column 1), while legacies do not exhibit a significant trend. At the same time, NetHub airline dummies are statistically different for legacies and LCCs. The latter are, on average, five times higher than the former. The average fixed effect for LCCs and legacies and their evolution over time are presented on figure 5. The estimated coefficients for legacies and LCCs are statistically different, implying that the NetHub gap is vanishing over time.

In terms of NetWeave (column 2), no significant trend is observed for LCCs while legacy carriers increase their NetWeave over time. The average LCCs fixed effects are four times higher than the legacy fixed effects. The NetWeave gap could also be vanishing over time according to the estimated results. However, the convergence won’t probably be attained in the near future as shows figure 5.

Finally, with respect to NetSize, legacies are on average bigger than LCCs. The average legacy NetSize is 212 flight segments bigger than the average LCC NetSize. However, the evolution over time is statistically different with LCCs growing while legacies NetSize remains constant.

The two first indicators, NetHub and NetWeave, are affected by the economic conditions of the market although NetSize is not. An increase of jet fuel price during a period leads to a decrease in both NetHub and NetWeave the following period for regional carriers. Legacies and LCC network structure are not impacted by the evolution of fuel price, probably due to hedging strategies used by these airlines to be protected against fuel price volatility. This result is consistent with the results of Carter, Rogers, and Simkins (2006), who find that hedging is a strategy used by the larger airlines with the least debt and highest credit rating.
The output gap controls for the economic performances of the overall American market. We introduce this control with a one period lag as we expect a delay between economic results and airlines’ strategic response. We find some positive and significant effects of the economic performances on NetHub for regional airlines and on NetWeave for legacies and regionals. These results are consistent with Wang et al. (2011) even though our approaches differs in two aspects. First their results are city oriented rather than airline oriented. Second, Wang et al. uses several graph theory measures by themselves while our contribution lies on building simple and continuous indicators from these graph theory measures.

We have constructed different variables to confirm the robustness of our results. We observe similar results when two lags instead of one lag variables are considered for output gap and jet fuel prices. Also, we estimate our system of equations replacing the lagged variables by contemporaneous variables. All parameters remain unchanged except the two economic variables whose significance change in some cases.

In particular, the contemporaneous output gap has non-significant effect over legacies and LCC while it has a significant impact for regionals over the three indicators. The lagged output gap is significant for NetWeave over legacy which could mean that legacies plan their decisions ahead of time. Our results are also robust to the use of contemporaneous or lagged jet fuel prices. Legacy and LCC carriers remain unaffected by contemporaneous variations of jet fuel prices; regional carriers are more affected.

Thus, we show that network evolution depends on intrinsic characteristics of airlines: while LCCs increase their NetSize and decrease their NetHub during the period of observation, legacy carriers increase their NetWeave. Still, legacies profoundly modify their networks through mergers. Three of the four recent mergers represent an increase in NetSize. Not surprisingly, mergers reduced NetHub in three of the mergers that are considered as the networks resulting after mergers exhibit a higher number of hub airports. Interestingly, two mergers affect NetWeave, the Frontier Midwest merger where only part of the Midwest network was integrated into Midwest while the rest was included into Republic Airline that operates as a feeder for several legacy carriers; and the Delta Northwest merger which involved two distinct airlines in terms of NetSize (Delta covers on average 40% more flight segments). To illustrate these changes, we display in Figure 5 and Figure 6 the DLNW and UACO mergers, respectively, since the two merging companies were legacies with comparable NetSize levels before the merger. Moreover, we can observe in the data a similar time-lapse before and after these two mergers.

The two figures show the evolution of the network indicators NetHub (horizontal axis) and NetWeave (vertical axis) before and after the mergers were completed. The change in NetSize is also displayed with a
bubble plot, where the disk size reflects $NetSize$. In both cases we can clearly observe an increase in $NetSize$ after the merger (periods after the merger are drawn in grey) and a $NetHub$ decrease. However, from the departing date of the mergers, 2010 for NW and DL, and 2012 for CO and UA, both groups move towards their pre-merger level of $NetHub$ certainly due to a network rationalization. The use of the indicators to measure the impact of a merger, instead of a dichotomous variable controlling for mergers shocks, is advantageous since it allows us to monitor the evolution of the merged entities. For instance, both mergers are followed by annual increases on $NetHub$ by more than 10% except 2015, the last year in our sample, where the change is smaller than 5%. This change in the trend could be interpreted as the end of the integration process between the airlines although more data is required to extract general conclusions.

![Bubble plot](image.png)

**Figure 6** Evolution of the indicators $NetHub$, $NetWeave$ and $NetSize$ for the DLNW merger.

---

31 More precisely, the $NetSize$ is taken to be the average number of flight segments of an airline network normalized by the maximum over all airlines’ networks.
Figure 7  Evolution of the indicators NetHub, NetWeave and NetSize for the UACO merger.

7. Conclusion

Previous studies propose several measures to analyze airline network structures however our literature review revealed the need for some continuous and interpretable instruments to measure airlines networks. In particular such measures should improve the existing studies which oppose hub-and-spoke to fully-connected network when addressing the dependence between network structure and airline economic performances. Our objective is to find a method that characterizes a network structure through continuous indicators. To fulfill this objective, we propose to implement the following methodology. First, an airline network should be described as a graph, so that the airline network is represented in the form of a connection, or adjacency, matrix. Such representation must contain information on the existence of nonstop flights provided by each airline between any city pair at a given date. Then the most relevant set of graph theory measures for the airline industry should be selected and calculated. Lastly, PCA must be performed to reduce the graph theory measures to a small interpretable set of new indicators.

We apply this methodology to the US domestic market and obtain three indicators, NetHub, NetWeave and NetSize, that accounts for 95% of the variability in the initial data. We provide interpretation for each indicator. NetHub measures the presence of hubs in a network; the higher NetHub value, the closer the network is to a hub-and-spoke structure. NetWeave indicates the network ability to provide alternative routes; the higher NetWeave, the larger number of route combinations available for an airline between any city pair. Finally, NetSize measures the number of flight segments supplied by the airline. The three indicators are continuous measures of the airlines’ network structure which improve the usual approach in the literature where traditionally dummies are used to capture changes in the networks.

We represent the airline positioning according to the three indicators and we estimate jointly their time evolution. We conclude that LCCs have a higher NetHub on average than legacies, although there seems to be a convergence since legacy carriers are increasing their NetHub over time, while LCCs NetHub remains constant. At the same time, LCCs are catching up with legacies in terms of NetSize. We also observe that LCCs exhibit a higher NetWeave on average than legacies. Here we show that network evolution participates to the process of airline business model convergence between legacies and LCCs. However, our results
stress that not all indicators are converging at the same path, we highlight some specificities across the different type of airlines. We show that airline network structure depends on the economic conditions of the market, measured through the jet fuel price and the output gap. The sense and magnitude of the dependence differ, according to the network indicator considered and the type of airline. Small regional airlines are more impacted by any increase in jet fuel price, while a positive economic cycle increases both NetHub and NetWeave for different airline types. Finally, we illustrate the impact on the indicators of the last main mergers in the US domestic market, Delta-Northwest and United-Continental. We show that in both cases the airlines see their NetHub level reduced the year of the merger, though afterwards it progressively recover its initial level. Although this is a known fact, the proposed methodology allows to measure these changes in a continuous manner which enables comparison of the integration level of the network after the merger.

Our methodology allows us to characterize airline networks and to measure their evolution in a simple manner through a reduced number of indicators. This provides new research opportunities. The new indicators can be used to measure the impact of the network structure on the airlines’ cost and profitability, or could be used in assessing the probability to enter a market. These analyses are extremely relevant for the study of airlines strategies. We argue that the obtained indicators should be considered jointly for a more appropriate analysis of these strategies. Regulators will as well benefit from such analyses, with for instance, the evaluation of the impact of networks structures on delays or pollution.

We focus on the US domestic market as information on delays, financial indicators and data on fares are regularly published by the US Department of Transportation for this market. The use of the network indicators associated with these datasets will allow to study the network optimal structure leading to the best airline performances. However, the methodology can be applied to other markets, to different sectors and databases. The indicators can be enhanced using weighted graphs, with weights associated to airports, following the approach of Gautreau, Barrat, and Barthélemy (2009) and Da Rocha (2009). Some weights could also be applied to the flight segments in terms of seats, frequencies, distance or combinations across these variables. The use of weighted graphs increases the difficulty in the calculation of the graph metrics, and is left for further research.
References


Appendix A: Graph measures and descriptive statistics

<table>
<thead>
<tr>
<th>Airline Name</th>
<th>IATA Code</th>
<th>Classification</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frontier Flying Service</td>
<td>2F</td>
<td>Regional</td>
</tr>
<tr>
<td>Silver Airways Corp</td>
<td>3M</td>
<td>Regional</td>
</tr>
<tr>
<td>American Airlines</td>
<td>AA</td>
<td>Legacy</td>
</tr>
<tr>
<td>Alaska Airlines</td>
<td>AS</td>
<td>Legacy</td>
</tr>
<tr>
<td>JetBlue Airways</td>
<td>B6</td>
<td>LCC</td>
</tr>
<tr>
<td>Continental Airlines</td>
<td>CO</td>
<td>Legacy</td>
</tr>
<tr>
<td>Delta Air Lines</td>
<td>DL</td>
<td>Legacy</td>
</tr>
<tr>
<td>Frontier Airlines</td>
<td>F9</td>
<td>LCC</td>
</tr>
<tr>
<td>Airtran Airways</td>
<td>FL</td>
<td>LCC</td>
</tr>
<tr>
<td>Allegiant Air LLC</td>
<td>G4</td>
<td>LCC</td>
</tr>
<tr>
<td>Hawaiian Airlines</td>
<td>HA</td>
<td>Legacy</td>
</tr>
<tr>
<td>Penair</td>
<td>KS</td>
<td>Regional</td>
</tr>
<tr>
<td>Spirit Airlines</td>
<td>NK</td>
<td>LCC</td>
</tr>
<tr>
<td>Northwest Airlines</td>
<td>NW</td>
<td>Legacy</td>
</tr>
<tr>
<td>Sun Country Airlines</td>
<td>SY</td>
<td>LCC</td>
</tr>
<tr>
<td>United Airlines</td>
<td>UA</td>
<td>Legacy</td>
</tr>
<tr>
<td>US Airways</td>
<td>US</td>
<td>Legacy</td>
</tr>
<tr>
<td>Virgin America</td>
<td>VX</td>
<td>LCC</td>
</tr>
<tr>
<td>Southwest Airlines</td>
<td>WN</td>
<td>LCC</td>
</tr>
<tr>
<td>Island Air</td>
<td>WP</td>
<td>Regional</td>
</tr>
<tr>
<td>Midwest Airlines</td>
<td>YX</td>
<td>Regional</td>
</tr>
<tr>
<td>Great Lakes Airlines</td>
<td>ZK</td>
<td>Regional</td>
</tr>
</tbody>
</table>
## Description of notation in Table A.2

<table>
<thead>
<tr>
<th>Notation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G$ or $G(V,E)$</td>
<td>The graph with set of nodes $V$ and edge set $E$.</td>
</tr>
<tr>
<td>$\mathcal{V}$</td>
<td>The set of nodes of a graph $G(V,E)$.</td>
</tr>
<tr>
<td>$E$</td>
<td>The set of edges of a graph $G(V,E)$.</td>
</tr>
<tr>
<td>$n_i$</td>
<td>An element of $\mathcal{V}$, i.e., node $i$ of the graph $G(V,E)$.</td>
</tr>
<tr>
<td>$e_{ij}$</td>
<td>An element of $E$, i.e., an edge connecting nodes $n_i$ and $n_j$.</td>
</tr>
<tr>
<td>$I_i$</td>
<td>The number of edges between neighbors of node $n_i$.</td>
</tr>
<tr>
<td>$k_i$</td>
<td>The number of neighbors of node $n_i$.</td>
</tr>
<tr>
<td>$c_\chi(n_i)$</td>
<td>One of the centrality measures of node $n_i$ in a graph.</td>
</tr>
<tr>
<td>$c_\chi(n^*)$</td>
<td>The largest value of $c_\chi(n_i)$ in a graph.</td>
</tr>
<tr>
<td>$\max \sum_{i=1}^{\left</td>
<td>\mathcal{V}\right</td>
</tr>
<tr>
<td>$d(n_i, n_j)$</td>
<td>The geodesic distance between two nodes $n_i$ and $n_j$, i.e., the minimum number of non-repeated edges that connect the two nodes.</td>
</tr>
<tr>
<td>$g_{jk}(i)$</td>
<td>The number of geodesics from node $n_j$ to node $n_k$ passing through node $n_i$.</td>
</tr>
<tr>
<td>$g_{jk}$</td>
<td>The total number of geodesics from $n_j$ to $n_k$.</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>A constant.</td>
</tr>
<tr>
<td>$a_{ij}$</td>
<td>An element of the adjacency matrix of a graph $G(V,E)$, i.e., of a square matrix $A = (a_{ij})$ with $a_{ij} = 1$ if nodes $n_i$ and $n_j$ are connected by an edge and 0 otherwise.</td>
</tr>
</tbody>
</table>
### Table A.2  Examples of graph measures

<table>
<thead>
<tr>
<th>Order (number of nodes),</th>
<th>nbCity</th>
<th>nbFS</th>
<th>DensG</th>
<th>DiamG</th>
<th>TransG</th>
<th>TransG</th>
<th>TransG</th>
<th>TransG</th>
<th>TransG</th>
<th>TransG</th>
<th>TransG</th>
<th>TransG</th>
<th>TransG</th>
</tr>
</thead>
<tbody>
<tr>
<td>Size (number of edges),</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$</td>
<td>E</td>
<td>$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Network Density, $D(G) = \frac{</td>
<td>E</td>
<td>}{</td>
<td>V</td>
<td></td>
<td>V</td>
<td>-1}$</td>
<td>0.1818</td>
<td>0.1818</td>
<td>0.1818</td>
<td>0.1818</td>
<td>0.1818</td>
<td>0.1818</td>
<td>1.0363</td>
</tr>
<tr>
<td>Diameter</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Transitivity, $C = \frac{\sum_{i=1}^{</td>
<td>V</td>
<td>} 2l_i}{\sum_{i=1}^{</td>
<td>V</td>
<td>} k_i(k_i - 1)}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Degree Centrality, $c_d(n_i) = \frac{\sum_{j=1}^{</td>
<td>V</td>
<td>} e_{ij}}{</td>
<td>V</td>
<td>- 1}$</td>
<td>max value, maxCdeg</td>
<td>1.0000</td>
<td>0.7000</td>
<td>0.6000</td>
<td>0.5000</td>
<td>1.0000</td>
<td>0.6000</td>
<td>0.6000</td>
<td>0.5000</td>
</tr>
<tr>
<td>Degree Centralization, $C_d = \frac{\sum_{i=1}^{</td>
<td>V</td>
<td>} (c_d(n^*) - c_d(n_i))}{</td>
<td>V</td>
<td>- 2}$</td>
<td>GChar</td>
<td>1.0000</td>
<td>0.6333</td>
<td>0.5111</td>
<td>0.3889</td>
<td>0.2889</td>
<td>0.1556</td>
<td>0.4111</td>
<td>0.0556</td>
</tr>
<tr>
<td>Harmonic Centrality, $c_h(n_i) = \frac{1}{</td>
<td>V</td>
<td>- 1} \sum_{j \neq i} \frac{1}{d(n_i, n_j)}$</td>
<td>max value, maxChar</td>
<td>1.0000</td>
<td>0.8167</td>
<td>0.8000</td>
<td>0.6833</td>
<td>1.0000</td>
<td>0.8000</td>
<td>0.8000</td>
<td>0.5000</td>
<td>0.5000</td>
<td></td>
</tr>
<tr>
<td>Harmonic Centralization, $C_h = \frac{2 \sum_{i=1}^{</td>
<td>V</td>
<td>} (c_h(n^*) - c_h(n_i))}{</td>
<td>V</td>
<td>- 2}$</td>
<td>GChar</td>
<td>1.0000</td>
<td>0.7444</td>
<td>0.6593</td>
<td>0.463</td>
<td>0.437</td>
<td>0.3037</td>
<td>0.4667</td>
<td>0.1111</td>
</tr>
<tr>
<td>Betweenness Centrality, $c_B(n_i) = \frac{2 \sum_{j \neq i \neq k} g_{jk}(i)}{</td>
<td>V</td>
<td>- 1}</td>
<td>(</td>
<td>V</td>
<td>- 2)$</td>
<td>max value, maxCbet</td>
<td>1.0000</td>
<td>0.8667</td>
<td>0.7778</td>
<td>0.6667</td>
<td>0.6667</td>
<td>0.5556</td>
<td>0.2222</td>
</tr>
<tr>
<td>Betweenness Centralization, $C_B = \frac{\sum_{i=1}^{</td>
<td>V</td>
<td>} (c_B(n^*) - c_B(n_i))}{</td>
<td>V</td>
<td>- 1}$</td>
<td>GCbet</td>
<td>1.0000</td>
<td>0.7822</td>
<td>0.7111</td>
<td>0.5444</td>
<td>0.6111</td>
<td>0.5022</td>
<td>0.2089</td>
<td>0</td>
</tr>
<tr>
<td>Eigenvector Centrality, $c_\text{EV}(n_i) = \frac{1}{\lambda} \sum_{j \in G} a_{ij} c_\text{EV}(n_j)$</td>
<td>mean value, meanCeig</td>
<td>0.2676</td>
<td>0.2635</td>
<td>0.2566</td>
<td>0.2694</td>
<td>0.2765</td>
<td>0.3015</td>
<td>0.2375</td>
<td>0.2548</td>
<td>0.2080</td>
<td>0.2227</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Eigenvector Centralization, $C_\text{EV} = \frac{\sum_{i=1}^{</td>
<td>V</td>
<td>} (c_\text{EV}(n^*) - c_\text{EV}(n_i))}{(</td>
<td>V</td>
<td>- 2)/\sqrt{2}}$</td>
<td>GCbet</td>
<td>0.07597</td>
<td>0.7373</td>
<td>0.7459</td>
<td>0.5741</td>
<td>0.3864</td>
<td>0.3146</td>
<td>0.2846</td>
<td>0.8627</td>
</tr>
</tbody>
</table>
### Table A.3
Descriptive statistics: means and standard deviations (in parentheses)

<table>
<thead>
<tr>
<th></th>
<th>2F</th>
<th>3M</th>
<th>AA</th>
<th>AS</th>
<th>B6</th>
<th>CO</th>
<th>DL</th>
<th>F9</th>
<th>FL</th>
<th>G4</th>
<th>HA</th>
<th>KS</th>
<th>NK</th>
<th>NW</th>
<th>SY</th>
<th>UA</th>
<th>US</th>
<th>VX</th>
<th>WN</th>
<th>WP</th>
<th>YX</th>
<th>ZK</th>
</tr>
</thead>
<tbody>
<tr>
<td>nbCity</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>maxCdeg</td>
<td>(21.00)</td>
<td>(8.40)</td>
<td>(54.00)</td>
<td>(7.90)</td>
<td>(24.00)</td>
<td>(14.00)</td>
<td>(8.60)</td>
<td>(10.00)</td>
<td>(22.00)</td>
<td>(9.60)</td>
<td>(9.70)</td>
<td>(6.40)</td>
<td>(12.00)</td>
<td>(13.00)</td>
<td>(26.00)</td>
<td>(16.00)</td>
<td>(3.70)</td>
<td>(8.40)</td>
<td>(4.00)</td>
<td>(14.00)</td>
<td>(12.00)</td>
<td></td>
</tr>
<tr>
<td>compG</td>
<td>0.06</td>
<td>0.19</td>
<td>0.03</td>
<td>0.03</td>
<td>0.11</td>
<td>0.03</td>
<td>0.02</td>
<td>0.05</td>
<td>0.09</td>
<td>0.06</td>
<td>0.18</td>
<td>0.09</td>
<td>0.27</td>
<td>0.02</td>
<td>0.11</td>
<td>0.03</td>
<td>0.03</td>
<td>0.32</td>
<td>0.19</td>
<td>0.35</td>
<td>0.06</td>
<td>0.06</td>
</tr>
<tr>
<td>diamG</td>
<td>7.40</td>
<td>4.80</td>
<td>3.80</td>
<td>6.20</td>
<td>3.40</td>
<td>4.50</td>
<td>3.80</td>
<td>3.10</td>
<td>2.90</td>
<td>5.50</td>
<td>2.20</td>
<td>4.70</td>
<td>3.20</td>
<td>5.30</td>
<td>3.50</td>
<td>4.20</td>
<td>4.20</td>
<td>2.30</td>
<td>3.00</td>
<td>3.00</td>
<td>4.00</td>
<td>7.80</td>
</tr>
<tr>
<td>transG</td>
<td>0.21</td>
<td>0.30</td>
<td>0.08</td>
<td>0.10</td>
<td>0.15</td>
<td>0.07</td>
<td>0.09</td>
<td>0.03</td>
<td>0.13</td>
<td>0.01</td>
<td>0.18</td>
<td>0.14</td>
<td>0.36</td>
<td>0.05</td>
<td>0.02</td>
<td>0.10</td>
<td>0.11</td>
<td>0.30</td>
<td>0.39</td>
<td>0.42</td>
<td>0.05</td>
<td>0.06</td>
</tr>
<tr>
<td>maxCh</td>
<td>0.25</td>
<td>0.47</td>
<td>0.79</td>
<td>0.45</td>
<td>0.88</td>
<td>0.70</td>
<td>0.68</td>
<td>0.95</td>
<td>0.92</td>
<td>0.65</td>
<td>0.95</td>
<td>0.45</td>
<td>0.82</td>
<td>0.63</td>
<td>0.81</td>
<td>0.60</td>
<td>0.56</td>
<td>0.97</td>
<td>0.81</td>
<td>0.71</td>
<td>0.77</td>
<td>0.55</td>
</tr>
<tr>
<td>GCdeg</td>
<td>0.20</td>
<td>0.33</td>
<td>0.77</td>
<td>0.43</td>
<td>0.82</td>
<td>0.68</td>
<td>0.67</td>
<td>0.93</td>
<td>0.87</td>
<td>0.61</td>
<td>0.88</td>
<td>0.39</td>
<td>0.62</td>
<td>0.62</td>
<td>0.78</td>
<td>0.57</td>
<td>0.54</td>
<td>0.80</td>
<td>0.64</td>
<td>0.48</td>
<td>0.74</td>
<td>0.52</td>
</tr>
<tr>
<td>maxChar</td>
<td>0.55</td>
<td>0.56</td>
<td>0.89</td>
<td>0.68</td>
<td>0.94</td>
<td>0.85</td>
<td>0.84</td>
<td>0.97</td>
<td>0.96</td>
<td>0.78</td>
<td>0.98</td>
<td>0.65</td>
<td>0.91</td>
<td>0.81</td>
<td>0.89</td>
<td>0.80</td>
<td>0.78</td>
<td>0.98</td>
<td>0.91</td>
<td>0.77</td>
<td>0.87</td>
<td>0.71</td>
</tr>
<tr>
<td>GChar</td>
<td>0.43</td>
<td>0.43</td>
<td>0.84</td>
<td>0.65</td>
<td>0.85</td>
<td>0.80</td>
<td>0.77</td>
<td>0.96</td>
<td>0.89</td>
<td>0.70</td>
<td>0.90</td>
<td>0.61</td>
<td>0.66</td>
<td>0.75</td>
<td>0.82</td>
<td>0.70</td>
<td>0.68</td>
<td>0.80</td>
<td>0.67</td>
<td>0.52</td>
<td>0.84</td>
<td>0.74</td>
</tr>
<tr>
<td>maxCeig</td>
<td>0.65</td>
<td>0.36</td>
<td>0.62</td>
<td>0.62</td>
<td>0.63</td>
<td>0.60</td>
<td>0.47</td>
<td>0.94</td>
<td>0.71</td>
<td>0.61</td>
<td>0.64</td>
<td>0.82</td>
<td>0.53</td>
<td>0.43</td>
<td>0.52</td>
<td>0.81</td>
<td>0.38</td>
<td>0.36</td>
<td>0.61</td>
<td>0.20</td>
<td>0.36</td>
<td>0.85</td>
</tr>
<tr>
<td>GCeig</td>
<td>0.60</td>
<td>0.33</td>
<td>0.61</td>
<td>0.61</td>
<td>0.62</td>
<td>0.59</td>
<td>0.46</td>
<td>0.93</td>
<td>0.71</td>
<td>0.62</td>
<td>0.81</td>
<td>0.50</td>
<td>0.40</td>
<td>0.51</td>
<td>0.79</td>
<td>0.38</td>
<td>0.35</td>
<td>0.58</td>
<td>0.19</td>
<td>0.32</td>
<td>0.84</td>
<td>0.77</td>
</tr>
<tr>
<td>meanCeig</td>
<td>0.11</td>
<td>0.18</td>
<td>0.05</td>
<td>0.06</td>
<td>0.13</td>
<td>0.06</td>
<td>0.05</td>
<td>0.11</td>
<td>0.12</td>
<td>0.10</td>
<td>0.21</td>
<td>0.14</td>
<td>0.20</td>
<td>0.05</td>
<td>0.18</td>
<td>0.05</td>
<td>0.05</td>
<td>0.28</td>
<td>0.10</td>
<td>0.28</td>
<td>0.12</td>
<td>0.11</td>
</tr>
<tr>
<td>GVecig</td>
<td>0.53</td>
<td>0.55</td>
<td>0.57</td>
<td>0.72</td>
<td>0.56</td>
<td>0.58</td>
<td>0.50</td>
<td>0.81</td>
<td>0.61</td>
<td>0.61</td>
<td>0.63</td>
<td>0.39</td>
<td>0.58</td>
<td>0.70</td>
<td>0.49</td>
<td>0.49</td>
<td>0.48</td>
<td>0.28</td>
<td>0.48</td>
<td>0.77</td>
<td>0.83</td>
<td></td>
</tr>
</tbody>
</table>
Figure A.1. U.S. Jet Fuel Price and Output Gap. Note: data on the output gap is obtained from the International Monetary Fund; data on jet fuel prices are collected from the U.S. Energy Information Administration (EIA).
Appendix B: PCA analysis

PCA constructs principal components as linear combinations of observed variables so that the resulting components are uncorrelated and preserve the maximum amount of the variance of the original data. The weights for the linear combinations are calculated with the constraint that their squares have to sum to 1 (Jolliffe 2002). This is a sequence of constrained maximization problems, such that the first step in the sequence is aimed at finding optimal weights of linear combination for a principal component that accounts for a maximum variance. Each subsequent step gives the next principal component by solving a similar optimization problem with an additional constraint that requires the principal components to be uncorrelated. PCA solves this set of maximization problems, and thus finds all optimal weights for the linear transformation of the original variables to the principal components, due to the eigenvalue decomposition of the correlation matrix. Namely, each principal component is related to an eigenvalue that reflects the variance the component accounts for; the corresponding eigenvector stores the weights of linear combination for the principal component.

Table B.1 displays 11 eigenvalues computed on the basis of the correlation matrix. They are ranked in decreasing order according to the amount of variance they account for. Since the correlation matrix provides a standardized measure across all variables, the total variance is equal to the number of variables, i.e., 11. The first component explains most of the variance (45.79%), as shown in Table B.1. The second component accounts for a maximal amount of variance that is not accounted for by the first component (31.06%) and it is uncorrelated with it. Each subsequent component explains less variance until all the variance has been accounted for. The eigenvalues thus add up to the total variance in the data. Since the total variance is equal to the number of variables, the scale of eigenvalues can be interpreted in terms of the number of variables with the highest contribution to the variance.

The eigenvalues obtained (Table B.1) show that the first three principal components explain 94.69% of the sample variability. We choose to keep these three principal components corresponding to the largest eigenvalues of 5.0364, 3.4168 and 1.9627. This choice agrees with commonly used eigenvalue one criterion, suggested by F. Kaiser (1960) and Cattell’s (1966) scree test. In PCA the total amount of variance is equal to the number of variables. Kaiser’s criterion shows that the three first eigenvalues obtained correspond to the principal components each capturing most significant amount of data variability greater than one. The remaining components with eigenvalues under 1 are less informative accounting for less variance than is generated by one variable. The scree plot (Figure B.1) confirms the choice. It is a graph of the eigenvalues of all the components. The substantial drop in the magnitude of eigenvalues starts after the third component, thus indicating that the three-component solution would be appropriate to retain for describing the data.
Table B.1  Eigenvalues of the correlation matrix

<table>
<thead>
<tr>
<th>Eigenvalue</th>
<th>Difference</th>
<th>Proportion</th>
<th>Cumulative</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5.0364</td>
<td>1.6196</td>
<td>0.4579</td>
</tr>
<tr>
<td>2</td>
<td>3.4168</td>
<td>1.4541</td>
<td>0.3106</td>
</tr>
<tr>
<td>3</td>
<td>1.9627</td>
<td>1.7379</td>
<td>0.1784</td>
</tr>
<tr>
<td>4</td>
<td>0.2248</td>
<td>0.0811</td>
<td>0.0204</td>
</tr>
<tr>
<td>5</td>
<td>0.1437</td>
<td>0.0034</td>
<td>0.0131</td>
</tr>
<tr>
<td>6</td>
<td>0.1403</td>
<td>0.1071</td>
<td>0.0128</td>
</tr>
<tr>
<td>7</td>
<td>0.0332</td>
<td>0.0021</td>
<td>0.0030</td>
</tr>
<tr>
<td>8</td>
<td>0.0311</td>
<td>0.0214</td>
<td>0.0028</td>
</tr>
<tr>
<td>9</td>
<td>0.0097</td>
<td>0.0088</td>
<td>0.0009</td>
</tr>
<tr>
<td>10</td>
<td>0.0009</td>
<td>0.0007</td>
<td>0.0001</td>
</tr>
<tr>
<td>11</td>
<td>0.0003</td>
<td></td>
<td>0.0000</td>
</tr>
</tbody>
</table>

Figure B.1. Scree plot and variance explained by the principal components.

The eigenvectors presented in Table B.2 show contributions, or “loadings”, of the observed variables to each principal component.
Table B.2: Eigenvectors

<table>
<thead>
<tr>
<th></th>
<th>PC1</th>
<th>PC2</th>
<th>PC3</th>
<th>PC4</th>
<th>PC5</th>
<th>PC6</th>
<th>PC7</th>
<th>PC8</th>
<th>PC9</th>
<th>PC10</th>
<th>PC11</th>
</tr>
</thead>
<tbody>
<tr>
<td>nbFS</td>
<td>-0.1800</td>
<td>-0.1322</td>
<td>0.5921</td>
<td>0.2754</td>
<td>0.7045</td>
<td>-0.0264</td>
<td>0.1436</td>
<td>0.0213</td>
<td>0.0601</td>
<td>-0.0419</td>
<td>-0.0192</td>
</tr>
<tr>
<td>densG</td>
<td>0.0242</td>
<td>0.5154</td>
<td>-0.1809</td>
<td>0.0604</td>
<td>0.3204</td>
<td>-0.0980</td>
<td>-0.1736</td>
<td>0.3565</td>
<td>-0.5862</td>
<td>0.2158</td>
<td>0.1943</td>
</tr>
<tr>
<td>transG</td>
<td>-0.1653</td>
<td>0.4733</td>
<td>-0.0384</td>
<td>0.0036</td>
<td>0.0905</td>
<td>0.8044</td>
<td>0.0405</td>
<td>-0.2255</td>
<td>0.1975</td>
<td>0.0157</td>
<td>0.0132</td>
</tr>
<tr>
<td>maxCdeg</td>
<td>0.3555</td>
<td>0.2679</td>
<td>0.2211</td>
<td>-0.2555</td>
<td>-0.0535</td>
<td>0.0468</td>
<td>0.3549</td>
<td>0.2702</td>
<td>-0.1569</td>
<td>-0.5349</td>
<td>-0.4160</td>
</tr>
<tr>
<td>GCdeg</td>
<td>0.3996</td>
<td>0.0960</td>
<td>0.2662</td>
<td>-0.2625</td>
<td>-0.0694</td>
<td>-0.0365</td>
<td>0.4977</td>
<td>-0.0684</td>
<td>0.0935</td>
<td>0.4726</td>
<td>0.4480</td>
</tr>
<tr>
<td>maxChar</td>
<td>0.3428</td>
<td>0.2302</td>
<td>0.3201</td>
<td>0.1703</td>
<td>-0.1413</td>
<td>-0.0232</td>
<td>-0.5465</td>
<td>0.3711</td>
<td>0.4823</td>
<td>0.0923</td>
<td>-0.0034</td>
</tr>
<tr>
<td>GChar</td>
<td>0.4111</td>
<td>-0.0364</td>
<td>0.2448</td>
<td>-0.1448</td>
<td>0.0860</td>
<td>0.0414</td>
<td>-0.4447</td>
<td>-0.6388</td>
<td>-0.3573</td>
<td>-0.0751</td>
<td>-0.0224</td>
</tr>
<tr>
<td>maxCbet</td>
<td>0.3829</td>
<td>-0.1669</td>
<td>-0.2334</td>
<td>0.4921</td>
<td>0.0422</td>
<td>0.1697</td>
<td>0.1039</td>
<td>0.0433</td>
<td>-0.0145</td>
<td>-0.4611</td>
<td>0.5223</td>
</tr>
<tr>
<td>GCbet</td>
<td>0.3892</td>
<td>-0.1765</td>
<td>-0.2023</td>
<td>0.4432</td>
<td>0.0556</td>
<td>0.1817</td>
<td>0.1672</td>
<td>0.0038</td>
<td>-0.0936</td>
<td>0.4530</td>
<td>-0.5496</td>
</tr>
<tr>
<td>meanCeig</td>
<td>0.1315</td>
<td>0.4232</td>
<td>-0.3549</td>
<td>0.0604</td>
<td>0.3142</td>
<td>-0.4818</td>
<td>0.1092</td>
<td>-0.3704</td>
<td>0.4144</td>
<td>-0.0988</td>
<td>-0.1122</td>
</tr>
<tr>
<td>GCeig</td>
<td>0.2274</td>
<td>-0.3457</td>
<td>-0.3343</td>
<td>-0.5423</td>
<td>0.5042</td>
<td>0.2071</td>
<td>0.0158</td>
<td>0.1986</td>
<td>0.0158</td>
<td>0.0082</td>
<td></td>
</tr>
</tbody>
</table>

Since PCA was performed based on the correlation matrix, to obtain a value for the principal component, first we need to standardize the raw data; that is, for each variable we need to subtract its mean value and divide the difference by the standard deviation. Then we multiply the row-vector of standardized variables by the eigenvector corresponding to the component. If we denote by \( Z_i \) a row-vector of standardized data for observation \( i \) and by \( e_k \) the \( k \)th eigenvector extracted from PCA, the principal component \( P_k(i) \) for the observation \( i \), is computed as follows:

\[
P_k(i) = Z_i e_k = (Z_{ij}, ..., Z_{i11}) \begin{pmatrix} e_{1k} \\ \vdots \\ e_{11k} \end{pmatrix} = Z_{i1} e_{1k} + ... + Z_{i11} e_{11k} = \sum_{j=1}^{11} Z_{ij} e_{jk} ;
\]

or using the expression for \( Z_{ij} = \frac{X_{ij} - \overline{X}_j}{\sigma_j} \),

\[
P_k(i) = \sum_{j=1}^{11} \frac{X_{ij} - \overline{X}_j}{\sigma_j},
\]

where for observation \( i \), \( X_{ij} \) is the value of \( j \)th variable, and \( \overline{X}_j \) and \( \sigma_j \) are respectively the mean and standard deviation of the variable.

The number of elements in each vector, \( Z_i \) and \( e_k \), is equal to the number of initial variables, i.e., 11. Since \( Z_i \) is a vector of standardized data and \( e_k \) is an eigenvector obtained for the correlation matrix, that in turn contains standardized data, then the maximum absolute value of the vector-product \( Z_i e_k \) is 11. This means that each principal component \( P_k(i) = Z_i e_k \) is between \(-11 \) and 11.