

# A Riemannian framework for curves with velocity information: Application to detection of bad runway conditions

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**Abstract.** Curves can be made amenable to functional data statistics by representing them as points on the so-called shape manifold, that is formally defined as a quotient of the manifold of immersions. It is not a Hilbert space, but it can be provided with Riemannian metrics that allow geodesic distance computation, although some care must be taken in order to avoid degeneracy of the metric. Furthermore, the manifold constructed that way is generally not complete, so that geodesics may exist only in a neighborhood of a given curve. The purpose is to introduce a new kind of Riemannian metric that is especially adapted to the study of curves where the velocity is a discriminating feature. In such a case, the original shape space approach cannot be used since the parametrization invariance will wipe out the velocity information. Motivated by a real use case where one wants to assess runway adherence condition by observing only the radar tracks of landing aircraft, a partial parametrization invariance is introduced, yielding a bundle shape space model on which a relevant metric can be defined. The design of the metric was based on the equations of motion and reflects the internal structure of the data. The equation of geodesics will be given, along with a practical computation algorithm based on a shooting method. The performance of the approach for low adherence detection will be assessed on a set of simulated trajectories and compared with competing algorithms.

**Keywords:** functional data analysis, similarity measure, shape manifold, curve clustering, airport safety