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BIOGRAPHIES

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Yoan Gregoire is radionavigation and radiolocation engineer in the navigation/location signals and equipment department in CNES, the French Space Agency. His GNSS activities cover signal processing and performance evaluation of GNSS systems. He is in charge of the development of a GNSS Signal Observatory which goal is to study signal deformation and their impact on user accuracy.

ABSTRACT

The objective of this paper is to investigate the impact of nominal biases that affect code pseudorange measurements on GNSS signals. This impact is looked at position level, in a civil aviation context, considering that a least square algorithm is used to estimate the position from pseudorange measurements.

As an input to this work, it is assumed that the pseudorange nominal biases can be written as the sum of three components following the proposition made in [1]:
- Delays induced by the satellite antenna.
- Delays induced by the receiver antenna.

- Distortions induced by the satellite payload, the satellite antenna and the receiver antenna.

One important feature of those models is that they are dependent upon:
- the satellite antenna nadir (angle between the satellite/centre of the Earth line and the satellite/user line) and/or
- the satellite elevation and azimuth with respect to the user.

By consequence a pseudorange nominal bias is dependent upon the relative position between the user and the satellite.

The two main contributions of this publication are:
- The proposition of three models that are able to characterize the three code pseudorange nominal bias components. These models are based on a wide review of the state-of-the-art regarding each bias source reported on GPS L1 C/A signals.
- The estimation of the impact on the position of code pseudorange biases considering proposed nominal bias models. To take into account that pseudorange nominal biases are dependent upon the relative position between the user and the satellites, this estimation is made at different locations around the world and at different epochs using realistic orbital parameters to reproduce constellation geometries. A worst case is provided for each location, and corresponds to the maximum absolute position error obtained during a period of 24 hours (position biases are assessed every 2 minutes).

1 INTRODUCTION

To estimate its PVT (Position, Velocity, Time), a GNSS receiver uses pseudorange measurements representative of the distance between itself and the visible GNSS satellites and of the synchronization of its clock with GNSS satellite clocks.

Even in fault free conditions (also called nominal case) these measurements are stained by several errors. The quality of the PVT depends on the quality of pseudorange

measurements and on errors that affect these measurements.

In some applications that require high performance (in terms of accuracy, integrity, continuity and availability), such as in civil aviation, these errors can be classified in two categories:
- Random errors that can usually be overbounded by zero-mean Gaussian distributions. The summed effect of these errors is assumed to be random with a distribution overbounded by a Gaussian distribution with zero mean and a standard deviation termed UERE (User Equivalent Range Error) [1].
- Biases that have a long-term variation that might not be reflected by the UERE. These biases are not necessarily absorbed in the receiver clock bias state when computing the PVT since they can have different magnitudes for different pseudorange measurements [1].

As underlined in [2], the distinction between noise and bias is important. For example, in civil aviation, it affects the way the error sources are incorporated into the aircraft position protection level calculations, an important tool related to integrity. The objective of this paper is to investigate the impact of nominal pseudorange biases on the position in a civil aviation context.

In the first section the code pseudorange model and the nominal bias are defined for the targeted civil aviation case: dual-frequency receivers using CSP (Constellation Service Provider) navigation message that permits to correct satellite clock (constant pseudorange nominal bias) and T_{gd} (time group delay) errors. The general code pseudorange bias model defined in [1] is presented. The concept is to divide the pseudorange nominal bias in three components:
- delays induced by the satellite antenna,
- delays induced by the receiver antenna,
- distortions induced by the satellite payload, the satellite antenna and the receiver antenna.

In the second section, three models are proposed to characterize the pseudorange nominal biases mentioned above for a GPS L1 C/A user. These models are based on a wide review of the state-of-the-art regarding each bias source. Due to the lack of information, nominal bias models are not investigated for other signals in this article. One important feature of these models is that they are dependent upon:
- the satellite antenna nadir and/or
- the satellite elevation and azimuth with respect to the user.

The three components of the nominal bias model can then be added to the ideal code pseudorange measurements to reproduce nominally biased pseudoranges. As biases are dependent upon the relative orientation between the satellite antenna and the receiver antenna, the total pseudorange nominal biases have to be assessed assuming:
- a given constellation geometry,
- a given receiver antenna location,
- a given receiver antenna orientation.

In the third section, the impact of code pseudorange nominal biases on the GNSS receiver estimated position is assessed. Because pseudorange measurements biases are dependent upon the constellation geometry, the receiver antenna location and the receiver antenna orientation, the nominal position error is also dependent upon these three parameters. Then simulations were designed to take into account these three parameters:
- Regarding the impact of constellation geometry, a GPS YUMA file is used to reproduce constellation geometries every 2 minutes during 24 hours, thus representing 720 epochs.
- For the impact of the receiver antenna location, 10 005 locations around the world are tested.
- For the impact of the receiver antenna orientation, the receiver antenna is always vertical but different azimuth angles (rotations along the vertical axis) are considered.

For these different locations, the worst absolute position nominal errors are provided in the vertical direction and in the horizontal plane. The worst case is obtained for each location, and corresponds to the maximum absolute position error among the 720 different epochs and the different tested antenna orientations. The worst impact of the three bias components on the position is looked at separately and then considering that the three components are added together.

In the last section, a conclusion summarizes the different results provided in the document and makes recommendations for future works.

To conclude, this publication proposes new results about the impact of code pseudorange nominal biases on the GNSS user position estimation. These new results rely on:
- The selection of models used to characterize code pseudorange biases in a single frequency case.
- The implementation of a Matlab® program that is able to evaluate position biases obtained from biased code pseudorange measurements in different scenarios.

2 CODE PSEUDORANGE BIAS DEFINITION

2.1 Code pseudorange definition

The iono-free GNSS code pseudorange measurements in a multi-constellation GPS/GALILEO dual-frequency receiver can be modeled as [1]:

$$P^{L1-L5i} = \frac{f_{L1}^2}{f_{L1}^2 - f_{L5}^2} P^{L1i} + \frac{f_{L5}^2}{f_{L5}^2 - f_{L1}^2} P^{L5i} \qquad (1)$$

$$P^{E1-E5j} = \frac{f_{E1}^2}{f_{E1}^2 - f_{E5a}^2} P^{E1j} + \frac{f_{E5a}^2}{f_{E5a}^2 - f_{E1}^2} P^{E5j} \qquad (2)$$

where
- P^{Xi} is the pseudorange measurement from signals sent by the ith satellite at frequency X ('E' is for Galileo, 'L' is for GPS, 'E1-E5' is for Galileo iono-free and 'L1-L5' is for GPS iono-free).
- f_X is the frequency X.

and

$$\frac{f_{L1}^2}{f_{L1}^2 - f_{L5}^2} = \frac{f_{E1}^2}{f_{E1}^2 - f_{E5a}^2} \approx 2.261 \qquad (3)$$

$$\frac{f_{E5a}^2}{f_{E5a}^2 - f_{E1}^2} = \frac{f_{L5}^2}{f_{L5}^2 - f_{L1}^2} \approx -1.261 \qquad (4)$$

The iono-free measurements could therefore be modeled as:

$$
\begin{aligned}
P^{L1-L5i} &= \sqrt{(x-x^i)^2 + (y-y^i)^2 + (z-z^i)^2} \\
&+ \delta\tau^i + clock\&eph^{L1-L5i} + mult_\rho^i + n_\rho^i \\
&+ b_\rho^{L1-L5} + b_\rho^{L1-L5i}
\end{aligned}
\qquad (5)
$$

$$
\begin{aligned}
P^{E1-E5j} &= \sqrt{(x-x^j)^2 + (y-y^j)^2 + (z-z^j)^2} \\
&+ \delta\tau^j + clock\&eph^{E1-E5j} + mult_\rho^j + n_\rho^j \\
&+ b_\rho^{E1-E5} + b_\rho^{E1-E5j}
\end{aligned}
\qquad (6)
$$

where
- x^i, y^i, z^i are the satellite positions computed by the user receiver using the CSP navigation message.
- $clock\&eph^{Xi}$ is the sum of residual range errors due to ephemeris error and satellite clock error with respect to the CSP reference frame and CSP clock reference. Note that in the case of GPS L1/L5 iono-free measurement, this residual error also includes the error affecting the broadcast $T_{GDL1/L5}$ (group delay between L1 and L5 signals) as the GPS ground segment monitors the L1/L2 iono-free measurements. In this document, this residual error is assumed to be random (as it is assumed in ARAIM for example [1]), with a distribution overbounded by a Gaussian distribution with zero mean and standard deviation termed URE (User Range Error). This residual error may include a long-term bias of a few hours reflecting the rhythm of the ODTS (Orbit Determination & Time Synchronisation) output, assumed to be included in the URE.

- $\delta\tau^i$, $mult_\rho^i$ and n_ρ^i are the residual tropospheric and multipath plus noise errors that affect the code pseudorange. They are assumed by the receiver algorithm to be random errors with a distribution overbounded by a zero-mean Gaussian error with variance modeled as σ_{tropo}^2, $\sigma_{mult\_\rho}^2$ and $\sigma_{noise\_\rho}^2$. This assumption is considered for example in the ARAIM application.
- b_ρ^{L1-L5}, b_ρ^{E1-E5} are the receiver clock offsets with respect to GPS reference time and Galileo reference time estimated from code pseudorange measurements. These clock offsets should include all propagation delays common to all satellites of the same constellation from user antenna reference center point to signal processing module. This offset represents the error term which is identical to all measurements of the same constellation. Note that, from this definition, the receiver clock offset may include payload, plus ephemeris or SV clock delays identical to all used satellites from the same constellation. By consequence it may vary depending on the set of satellites used to compute the PVT.
- b_ρ^{L1-L5i}, b_ρ^{E1-E5j} are the code pseudorange iono-free nominal biases for GPS satellite i, and Galileo satellite j. Each quantity is a bias with long-term variation that is not reflected in the UERE (the standard deviation of the summed effect of all code pseudorange errors). This term can also be called inter-PRN bias. It may include an average component identical to all measurements of the same constellation (if several constellations are used). This identical component, affecting all measurements of a constellation, may also be included in the receiver clock offset.

In the following, the notation P^{Xi} will be used when referring to both constellations (as well as $b_\rho^X, clock\&eph_\rho^{Xi}, b_\rho^{Xi}$) for the sake of simplicity.

2.2 Nominal bias definition

In this document, the only error component of interest is b_ρ^{Xi}. Indeed residual range errors due to troposphere, multipath, noise and ephemeris errors and satellite clock error are assumed to be random errors with a distribution overbounded by a zero-mean Gaussian error and are included in the UERE. On the contrary, the concept of nominal bias b_ρ^{Xi} is different, as it varies slowly and according to characterizable features.

Depending on the application, the definition of the nominal bias is different:
- Standalone receiver: in this case, the nominal bias is defined as a bias on a code pseudorange measurement that does not affect all

measurements in the same way and then is not absorbed in the receiver clock bias.

- Differentially corrected receiver: in this case, the nominal bias is defined as a bias on a code pseudorange measurement that is not affecting all measurements of the user receiver in the same way, minus the bias on the pseudorange measurement (that is not affecting all measurements in the same way) estimated from the same signal and affecting the ground segment receivers (called in the following the reference station receivers).

2.3 Nominal biases origin

A question arises: how different biases can be induced on the different signals processed by the receiver? To answer this question, it is necessary to understand which parameters can induce such different biases depending on the processed signal.

Any antenna and electronic device can be characterized by their group and phase delays, among other things. This means that the signal going through these elements will be delayed and phase shifted (if the group and phase delay is constant over the bandwidth of the signal) or delayed, phase-shifted and distorted (if the group and phase delay varies over the bandwidth of the signal). The latter is the most common when considering the entire emitter/receiver chain [3]. Note that the group and phase delay of an antenna is generally dependent upon the incidence angle of the signal. This means that the bias and distortion of the signal might be dependent upon the angle between the antenna direction and the signal of interest direction.

By consequence, the nominal bias can be split in two components that affect differently the different received signals:

- nominal bias due to nominal GNSS signal distortions,
- nominal bias due to nominal group delays.

Even if the shape of the signal is not distorted by pure group delays, the term signal deformation can be used to refer to as signal distortions as well as signal delays. By consequence, in the following the term deformation is used and encompasses the two nominal bias components.

Nominal GNSS signal distortions as well as nominal delays have several sources.

- The satellite generation and transmission chain. Indeed, payload electronic components do not permit to generate an ideal signal. In addition, the satellite antenna also introduces, deformations due to the variation of its group with the pointing angle towards the used.
- The receiver antenna. The receiver antenna introduces a deformation due to its group delay variations with azimuth and elevation depending

on the direction of arrival of the satellite signal [1].

- Filtering effects at receiver processing chain level that can distort the received signals. The particularity of effects induced by the receiver processing chain is that they affect in the same way all received signals. Assuming that all received signals are identical, after the receiver processing chain the errors obtained on pseudorange measurements are the same for all signals. The problem is that, if received signals are different (as it is the case in reality) the receiver processing chain induces an additional distortion that is dependent on the input received signal initial distortion. This filtering can generally be represented as an equivalent RF filter which main characteristics are:
  - Bandwidth,
  - Filter technology,
  - Differential group delay.

2.4 General nominal bias model

It was seen above that components of nominal biases are:

- Signal distortions that can be generated by the satellite and the receiver antennas and electronic devices.
- Signal delays that can be generated by the satellite antenna and electronic device and the receiver antenna.

The two components of the nominal bias are dependent upon the relative angles between the satellite antenna and the receiver antenna even if it is usually assumed that the signal distortion component is not dependent upon these relative angles.

Only these two kinds of signal deformations are considered as inducing a nominal bias because:

- they affect in a different way the different received signals,
- they can be a-priori assessed and cannot be characterized by a zero-mean Gaussian distribution without decreasing drastically integrity and accuracy.

By consequence, the nominal bias can be written as (model already introduced in [1]):

$$b^{Xi}(N, el, az) = b_{dist}^{Xi}(N, el, az) + b_{delay}^{Xi}(N, el, az) \quad (7)$$

where

- $b_{dist}^{Xi}(N, el, az)$ is the bias component due to the distortion which is dependent upon N, el, az and the receiver,
- $b_{delay}^{Xi}(N, el, az)$ is the bias component due to the group delays which is dependent upon N, el and az,
- N is the transmitted signal nadir angle in the satellite antenna frame,

- el is the incoming signal elevation in the receiver antenna frame,
- az is the incoming signal azimuth in the receiver antenna frame.

From results provided by the state-of-the-art, $b_{delay}^{Xi}(N, el, az)$ can be assessed by splitting this component into two terms as follows:

$$b_{delay}^{Xi}(N, el, az) = b_{SV}^{Xi}(N) + b_{ant}^{Xi}(el, az) \quad (8)$$

where

- $b_{SV}^{Xi}(N)$ is the nominal bias induced by the satellite antenna group delay variation ([4], [5], [6], [7]),
- $b_{ant}^{Xi}(el, az)$ is the nominal bias induced by the receiver antenna group delay variation ([8], [9], [10], [1], [2],...).

3 PROPOSED MODELS TO CHARACTERIZE THE THREE COMPONENTS OF THE CODE PSEUDORANGE NOMINAL BIAS

3.1 Model of the bias induced by the satellite antenna delay

Results from [5], estimated in L1/L2 iono-free conditions are used to estimate the bias induced by the satellite antenna delay for each GPS satellite at different nadir angles. More precisely $b_{SV}^{L1-L2i}(N)$ values given in centimeter in Figure 1 are considered. A hypothesis proposed in [1] is assumed in this article: $\left| b_{SV}^{L1-L5i}(N) \right| \leq \left| b_{SV}^{L1-L2i}(N) \right|$. The model proposed regarding b_{SV}^{L1-L2i} is valid for iono-free measurements. It is proposed in this article to also apply this model for b_{SV}^{L1i}. Indeed, it can be deduced from results provided in [7] that this method is conservative. Nevertheless additional studies are required to estimate precisely $b_{SV}^{L1i}(N)$.
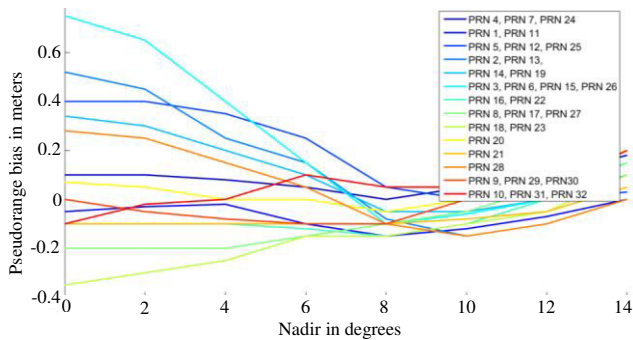

Figure 1 – Pseudorange biases induced by the satellite antenna function of the nadir for different satellites.

3.2 Model of the bias induced by the receiver antenna delay

In this section two simple models are introduced to characterize pseudorange biases induced by the receiver

antenna delay function of the elevation and the azimuth of the satellite in view in the antenna coordinates system. These models are assumed valid for L1 processed signals.

The first model, referred to as model1 in this paper, is the model introduced in [11]:

$$b_{ant}^{L1i}(el, az) = \beta_i(el) \times \sin(2az) \quad (9)$$

Civil aviation requirements on the maximum differential group delay induced by the receiver antenna limit the values of $\beta_i(el)$ [12]:

$$\beta_i(el) = \begin{cases} \left(2.5 - 0.04625(el - 5°)\right) \times \dfrac{c}{10^{-9}} \\ \qquad if\ 5° \leq el < 45° \\ 0.65 \times \dfrac{c}{10^{-9}} \\ \qquad if\ el \geq 45° \end{cases} \quad (10)$$

Where

- c is the speed of the light,
- el is the satellite elevation in degree.

The second model, referred to as model2 in this paper, is the model used in [1] and which characterizes the bias entailed by a typical civil aviation receiver antenna. This model is based on a typical antenna response represented with Bessel functions. To be at the limit of the civil aviation requirements, a factor (equal to 1.8) is applied on the overall $b_{ant}^{L1i}(el, az)$ values compared to [1].

Figure 2 shows the values (in meter) of the pseudorange biases induced by the receiver antenna as a function of the elevation and the azimuth. On the left, $b_{ant}^{L1i}(el, az)$ is obtained with model1 and on the right, $b_{ant}^{L1i}(el, az)$ is obtained with model2.
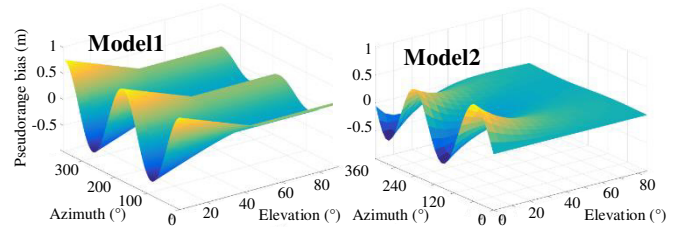

Figure 2 – Pseudorange biases (in meter) induced by the receiver antenna function of the azimuth and the elevation angles. On the left for model1, on the right for model2.

To provide more insights on these 2 models, the value of $b_{ant}^{L1i}(el, az)$ is shown in Figure 3 only as a function of the elevation (the worst case among all azimuths is kept) for the model1 in red and for the model2 in blue. As expected, the red plot matches the civil aviation requirement on the maximum differential group delay defined in (10). It is noticeable that the model2 meets this requirement, indeed the blue plot is below the red plot.
The value of $b_{ant}^{L1i}(el, az)$ in nanosecond is shown in Figure 4 (for the model 2) only as a function of the

azimuth. Different curves correspond to different elevations. It can be seen that the model2 is close to observation made on real civil aviation antennas [8].

A model which is able to characterize the bias induced by a L5 receiver antenna ($b_{ant}^{L5i}(el, az)$) or a L1/L5 antenna could permit to estimate the bias induced by the receiver antenna(s) in a dual frequency context using the following equality:

$$b_{ant}^{L1-L5i} = \frac{f_{L1}^2}{f_{L1}^2 - f_{L5}^2} b_{ant}^{L1i} + \frac{f_{L5}^2}{f_{L5}^2 - f_{L1}^2} b_{ant}^{L5i} \quad (11)$$

Such models are not investigated in this article due to the lack of data. It is noteworthy that pseudorange biases values provided in this section for GPS L1 C/A signals are induced by the receiver antenna considering an incoming signal at L1 frequency. It is assumed that the signal spectrum does not have any influence on results presented in this section. As Galileo E1 and GPS L1 C/A signals have the same central frequency and as same antennas are used to process both signals, results from this section are also valid for Galileo E1 signals.
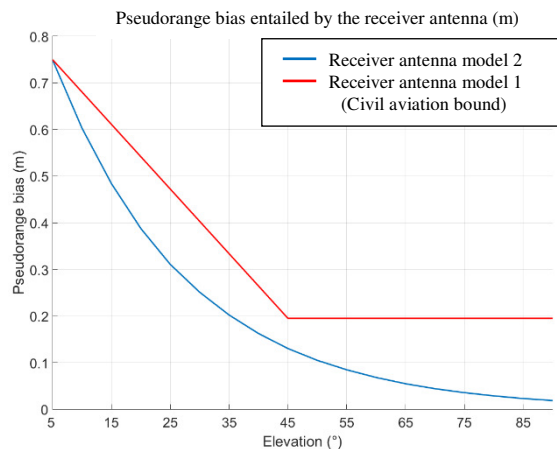

Figure 3 – Pseudorange biases induced by the receiver antenna function of the elevation in the case of model2 in blue and in the case of model1 in red.
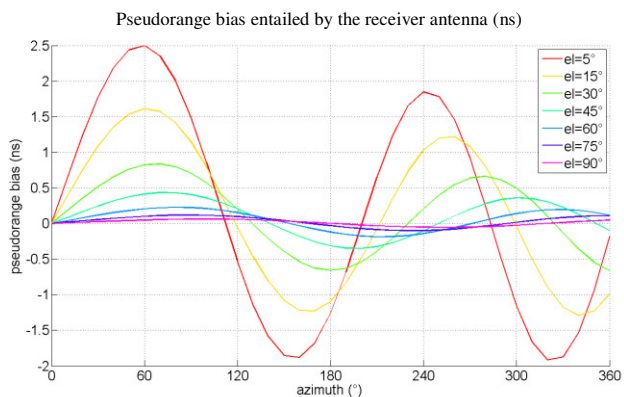

Figure 4 – Pseudorange biases induced by the receiver antenna function of the azimuth in the case of model2.

3.3 Model of the bias induced by a signal distortion

No satisfying model was found in the literature to characterize the pseudorange biases induced by distortions.

For L1 single-frequency, conservative values of $b_{dist}^{L1i}(N, el, az)$ were estimated between -50 cm and +50 cm based on [3], [13], [14], [15], [16]. These extreme values were obtained considering that a change of elevation, nadir and azimuth has an impact on the distortion. Further investigations are necessary to estimate values of b_{dist}^{Xi} in dual frequency and for different signals. For example, the value of b_{dist}^{L5i} could permit to estimate b_{dist}^{L1-L5i} knowing b_{dist}^{L1i} and using:

$$b_{dist}^{L1-L5i} = \frac{f_{L1}^2}{f_{L1}^2 - f_{L5}^2} b_{dist}^{L1i} + \frac{f_{L5}^2}{f_{L5}^2 - f_{L1}^2} b_{dist}^{L5i} \quad (12)$$

To estimate the impact of pseudorange bias on the position when a least square algorithm is considered, a theoretical concept can be developed. This concept relies on the fact that the position biases and pseudorange biases are linked by the following equality as demonstrated in [17]:

$$\begin{bmatrix} x_{bias}(k) \\ y_{bias}(k) \\ z_{bias}(k) \\ b_{bias}(k) \end{bmatrix} = S(k) \times \begin{bmatrix} b_\rho^{X1} \\ \vdots \\ b_\rho^{XN} \end{bmatrix} \quad (13)$$

with

$$S(k) = [H_{loc}^t H_{loc}]^{-1} H_{loc}^t(k)$$
$$= \begin{bmatrix} S_{1,1}(k) & \dots & S_{1,N}(k) \\ S_{2,1}(k) & \dots & S_{2,N}(k) \\ S_{3,1}(k) & \dots & S_{3,N}(k) \\ S_{4,1}(k) & \dots & S_{4,N}(k) \end{bmatrix} \quad (14)$$

$$H_{loc} = -\begin{bmatrix} CE_1 CA_1 & CE_1 SA_1 & SE_i & -1 \\ \vdots & \vdots & \vdots & \vdots \\ CE_N CA_N & CE_N SA_N & SE_N & -1 \end{bmatrix} \quad (15)$$

and

$$CE_i = cos(el_i(k))$$
$$CA_i = cos(az_i(k))$$
$$SE_i = sin(el_i(k))$$
$$SA_i = sin(az_i(k))$$

where
- el_i is the elevation in radian of the satellite transmitting the ith processed signal in the receiver antenna coordinates system.
- az_i is the azimuth in radian of the satellite transmitting the ith processed signal in the receiver antenna coordinates system.
- H_{loc} is the geometry matrix H given in the local receiver antenna coordinates system. This definition of H_{loc} is given for example in [18].

- N is the number of signals used to compute the PVT.
- b_ρ^{Xi} is pseudorange bias on the ith processed signal.
- k is the index that refers to the epoch.
- $x_{bias}(k), y_{bias}(k), z_{bias}(k)$, are position biases along the East, the North and the up directions in the receiver antenna coordinates system.
- $b_{bias}(k)$ is the receiver clock bias.

(13) can also be written as:

$$\begin{bmatrix} x_{bias}(k) \\ y_{bias}(k) \\ z_{bias}(k) \\ b_{bias}(k) \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^{N} S_{1,i}(k)\, b_\rho^{Xi} \\ \sum_{i=1}^{N} S_{2,i}(k)\, b_\rho^{Xi} \\ \sum_{i=1}^{N} S_{3,i}(k)\, b_\rho^{Xi} \\ \sum_{i=1}^{N} S_{4,i}(k)\, b_\rho^{Xi} \end{bmatrix} \qquad (16)$$

Assuming that all measurements can be affected by a bias with an absolute amplitude b_{max}:
- the maximum position error along the horizontal plane is equal to:

$$b_{max} S_{tot_H}(k)$$
$$= b_{max} \sqrt{\left(\sum_{i=1}^{N} |S_{1,i}(k)| \right)^2 + \left(\sum_{i=1}^{N} |S_{2,i}(k)| \right)^2} \qquad (17)$$

- the maximum position error along the vertical is equal to:

$$b_{max} S_{tot_V}(k) = b_{max} \sum_{i=1}^{N} |S_{3,i}(k)| \qquad (18)$$

As a consequence, to estimate the worst impact of pseudorange biases induced by distortions, (17) and (18) can be used (simulations are required to represent all possible values of S).

One important remark is that the worst case can be reached in one dimension for all satellites geometries whereas in more than one dimension some satellites geometries do not permit to reach the worst case. The consequence is that results provided using this strategy are more conservative looking at the impact of pseudorange biases in a 3D position or in a 2D position than looking at the impact of pseudorange biases in a 1D position. As an example, the worst case is obtained along the z-axis when choosing b_ρ^{Xi} such as:

$$b_\rho^{Xi} = b_{max} \times sign(S_{3,i}) \quad \forall i \qquad (19)$$

or

$$b_\rho^{Xi} = -b_{max} \times sign(S_{3,i}) \quad \forall i \qquad (20)$$

whereas in more than one dimension, the worst case may not be reached if:

$$sign(S_{X1,i}) = sign(S_{X2,i}) \quad for\ at\ least\ one\ i \qquad (21)$$

and

$$sign(S_{X1,i}) = -sign(S_{X2,i}) \quad for\ at\ least\ one\ i \qquad (22)$$

where $X1$ represents the first dimension and $X2$ represents the second dimension.

The conclusion is that S_{tot_V} gives the maximum position error that can be reached in one dimension whereas in more than one dimension (for example considering S_{tot_H}) the maximum position error derived from $S_{tot_H} \times b_{max}$ may not be reached (if conditions from (21) and (22) are fulfilled together) and by consequence may be too conservative.

4 IMPACT OF CODE PSEUDORAGE NOMINAL BIASES ON THE POSITION

4.1 Simulation set-up

To estimate the position bias knowing pseudoranges bias, the concept developed in section 3.3 regarding biases entailed by distortions can be reused applying (13). The position bias is then dependent upon the matrix S and pseudorange measurements bias.

As it is the case for the matrix S, pseudoranges biases are dependent upon the constellation geometry, the receiver antenna location and the receiver antenna orientation. As a consequence, the nominal position error is also dependent upon these three parameters. Results provided in this article are then dependent upon the simulation set-up and the choice of these 3 parameters:
- Regarding the first parameter, the GPS YUMA file from the 04/02/2017 (and available in the celestrak website) is used to reproduce constellation geometries every 2 minutes during 24 hours (720 epochs). The constellation is based on 31 satellites, the satellite PRN 4 was not available.
- For the second parameter, 10 005 locations around the world are tested: 69 values in latitude (from $-85°$ to $85°$ every $2.5°$) and 145 values in longitude (from $-180°$ to $180°$ every $2.5°$).
- For the last parameter, the receiver antenna is always vertical but different azimuth angles (rotations along the vertical axis) are considered.

A least square algorithm (not weighted) is used to estimate the position from pseudoranges. The satellite mask angle is chosen equal to $5°$ (to know which satellites are used to estimate the PVT).

4.2 Position biases induced by the satellite antenna delay

Applying the model that is proposed in section 3.1 to characterize the pseudorange bias induced by the satellite, maximum (top) and average (bottom) absolute position biases per location are shown on Figure 5 for the horizontal plane and on Figure 6 for the vertical. Note that the scales are different on the different plots.
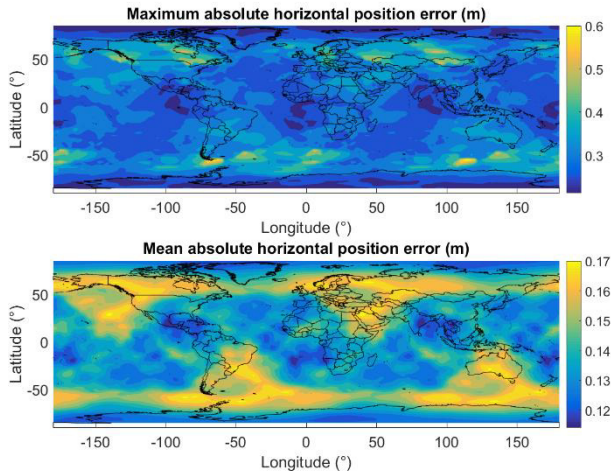

Figure 5 – Maximum (top) and mean (bottom) absolute position biases induced by the satellite antenna (horizontal).
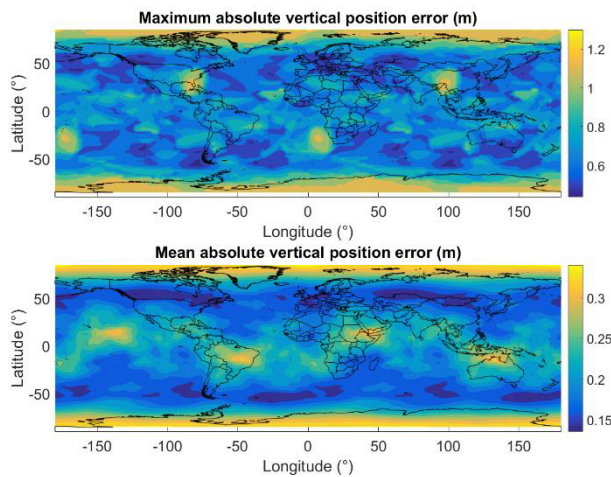

Figure 6 – Maximum (top) and mean (bottom) absolute position biases induced by the satellite antenna (vertical).

From the two figures above, some general behaviors are noteworthy:
- Horizontal position biases are small at the equator and at poles but can be high at mid-latitudes.
- Vertical position biases are small at the equator and at latitudes between 40° and 60° and -60° and -40° but are high at poles and at latitudes between 20° and 40° and -40° and -20°.
- The vertical position biases are in general higher than the horizontal position biases.

These behaviors are observed for the maximum as well as for the mean absolute values. The maximum of maximum bias values and the maximum of mean bias values are given in Table 1 for the horizontal, the vertical and the 3D positions. All values provided in the table are maxima along all longitudes and all latitudes

Table 1 – Maximum of the maximum and the mean position biases induced by the SV antenna delay in different cases.

	Maximum of maxima	Maximum of means
Horizontal position	0.6 m	0.2 m
Vertical position	1.3 m	0.4 m
3D position	1.4 m	0.4 m

4.3 Position biases induced by the receiver antenna

Figure 7 and Figure 8 show the maximum absolute position biases entailed by pseudorange biases induced by the receiver antenna delay. Figure 7 corresponds to results obtained from model1 and Figure 8 to results obtained from model2. On these two plots, the antenna orientation is the same: the satellite azimuth is equal to 0° (or equivalently 360°) when the satellite is at the North of the receiver antenna. The two main conclusions from these figures are that:
- Results are different depending on the model that is used. Even if the same general behavior is observed, results are lower with model2. This conclusion is logical since the amplitudes of pseudorange biases are higher with the model1 than with the model2. As a consequence, results obtained with the model1 are more conservative.
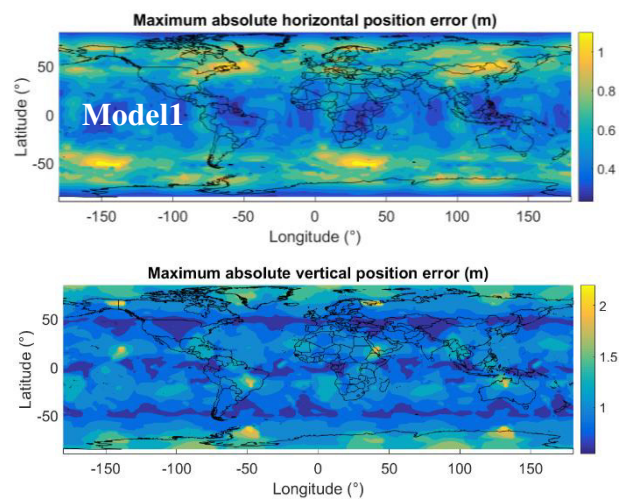- The maximum of all absolute position biases is not higher than 2.3 m with model1.

Figure 7 – Maximum absolute horizontal (top) and vertical (bottom) position biases induced by the receiver antenna using model1.
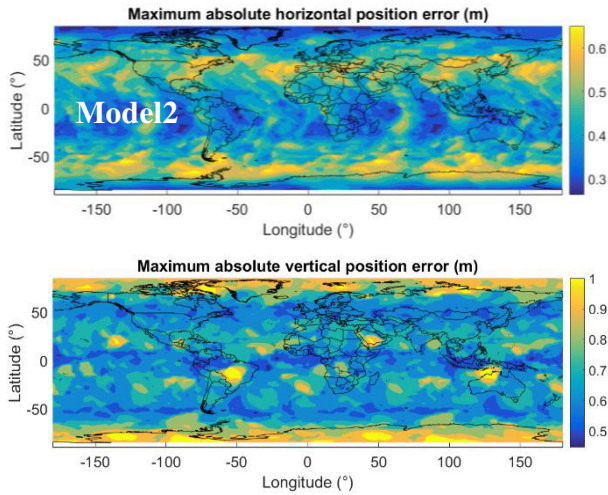
Figure 8 – Maximum absolute horizontal (top) and vertical (bottom) position biases induced by the receiver antenna using model2.

From results obtained in this section, it is decided to estimate the position bias induced by the receiver antenna delay from the model1 that is the most conservative and the simplest one. Because the antenna orientation has an impact on the position bias, a worst case is envisaged: the highest maximum absolute position bias among different azimuths is estimated. The advantage is that because of model1 symmetrical property, it is only necessary to test antenna rotations between 0° and 90°.

Figure 9 (horizontal position bias) and Figure 10 (vertical position bias) show:

- On the top the worst case (maximum absolute position bias) among 9 different antenna orientations (azimuth shifts equal to 0°, 10°, 20°… 80°) considering model1.
- On the bottom the average of absolute position biases among 9 different antenna orientations (azimuth shifts equal to 0°, 10°, 20°… 80°) considering model1.
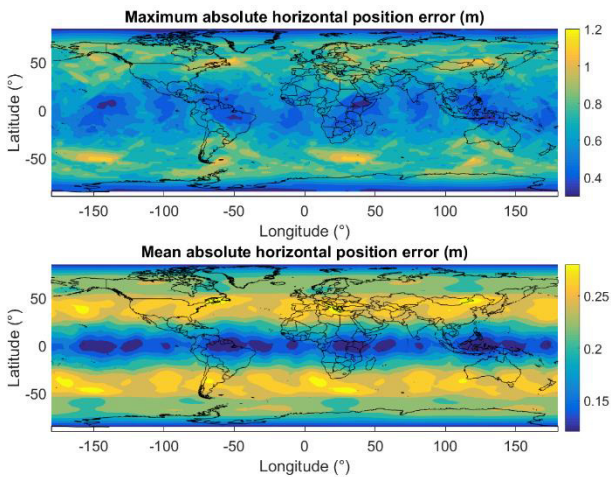

Figure 9 – Maximum (top) and mean (bottom) absolute horizontal position biases induced by the receiver antenna delay considering different antenna orientations (model1).
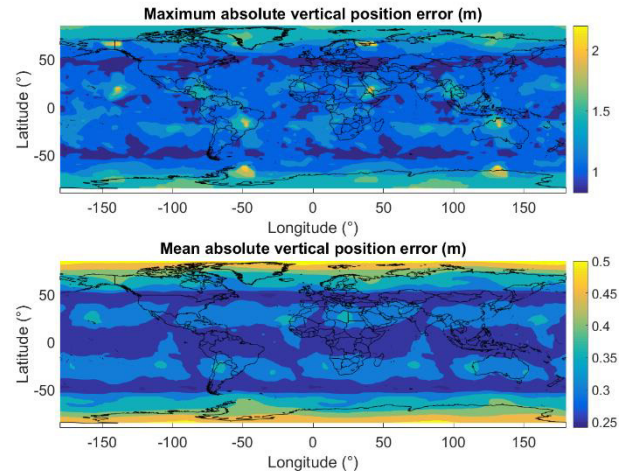

Figure 10 – Maximum (top) and mean (bottom) absolute vertical position biases induced by the receiver antenna delay considering different antenna orientations (model1).

The maximum of maximum bias values and the maximum of mean bias values are given in Table 2 for the horizontal, the vertical and the 3D positions. All values provided in the table are maxima over all locations.

Table 2 – Maximum of the maximum and the mean position biases induced by the receiver antenna delay in different cases.

	Maximum of maxima	Maximum of means
Horizontal position	1.2 m	0.3 m
Vertical position	2.3 m	0.5 m
3D position	2.3 m	0.6 m

4.3 Position biases induced by distortions

As it has been seen previously, S_{tot_H} and S_{tot_V} permit to estimate the impact of code pseudorange biases on the position via (17) and (18). A worst case can be reached considering that all pseudorange biases have their maximum value and push the position bias in a constructive way (conspiring biases). It was underlined in section 3.3 that additional conservatism is obtained when the impact of pseudorange biases on the position is assessed in more than one dimension. Figure 11 gives the maximum horizontal (top) and vertical (bottom) absolute position biases induced by distortions estimated based on the conservative concept described in section 3.3.

The maximum of position bias values are given in Table 3 for the horizontal, the vertical and the 3D positions. All values provided in the table are maxima over all locations.

Table 3 – Maximum of the position biases induced by signal distortions.

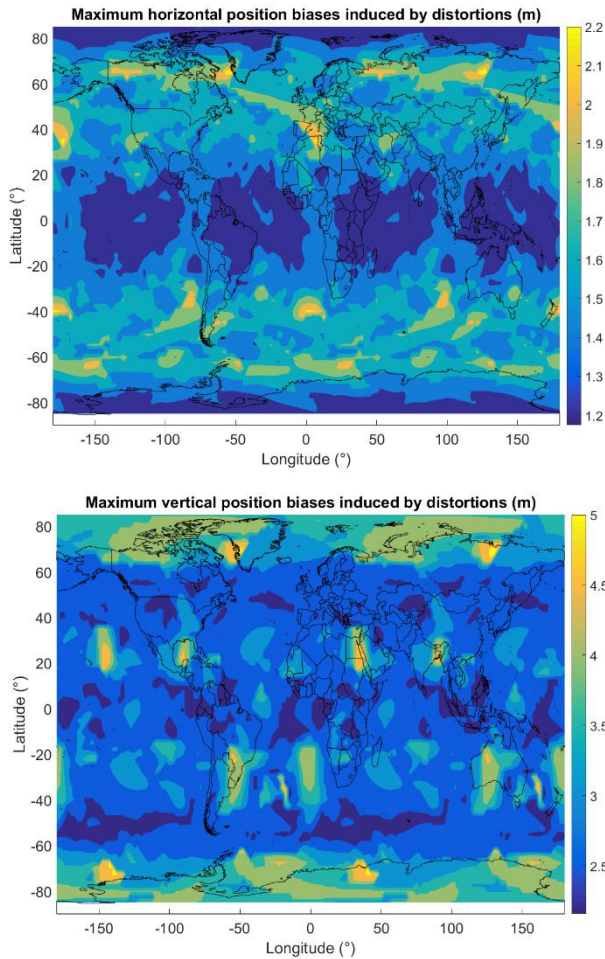|	Maximum
Horizontal position	2.4 m
Vertical position	5.1 m
3D position	5.5 m

Figure 11 – Maximum absolute horizontal (top) and vertical (bottom) position biases induced by distortions.

4.4 Position bias induced in nominal conditions on a standalone receiver

To estimate the total impact of nominal pseudorange biases on the position error the following strategy is used:

1) Add the two bias components due to the satellite antenna and the receiver antenna delays to each pseudorange. Estimate the vertical and the horizontal position biases caused by the combination of the two pseudorange biases. It is expected to have results close to results from Figure 5 (and Figure 6) plus results from Figure 9 (and Figure 10) with a slightly lower amplitude since the two biases are combined at pseudorange level. The maximum (top) and the mean (bottom) impacts on the horizontal positions of the two biases combination at pseudorange levels lead to Figure 12. The maximum (top) and the mean (bottom) impacts on the vertical positions of the two biases combination at pseudorange levels lead to Figure 13.

2) Add to the above obtained values the maximum position biases induced by signal distortions. The concept is then to:
   - add results from the top of Figure 11 to results from the top of Figure 12 to obtain the total horizontal position biases in nominal conditions shown on the top of Figure 14,
   - add results from the bottom of Figure 11 to results from the top of Figure 13 to obtain the total vertical position biases in nominal conditions shown on the bottom of Figure 14.
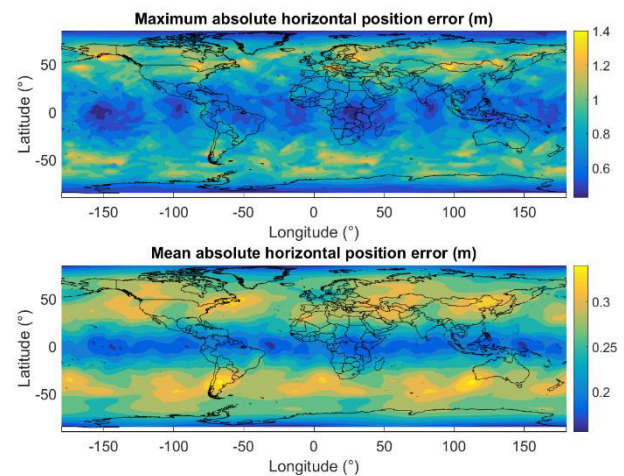

Figure 12 – Summed effect of pseudorange biases induced by the satellite and the receiver antenna group delays on the horizontal positions.
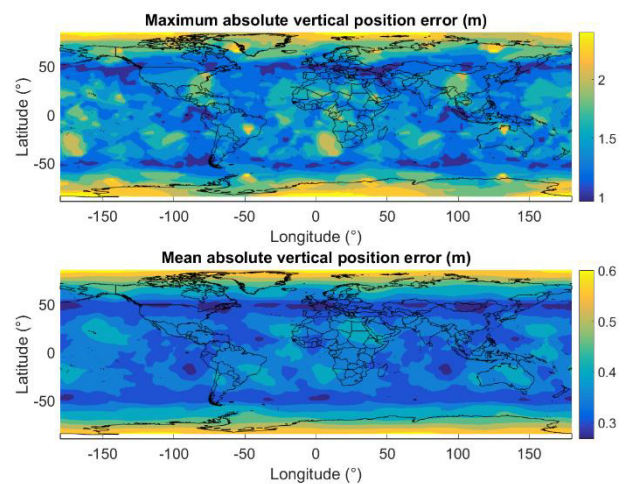

Figure 13 – Summed effect of pseudorange biases induced by the satellite and the receiver antenna group delays on the vertical positions.
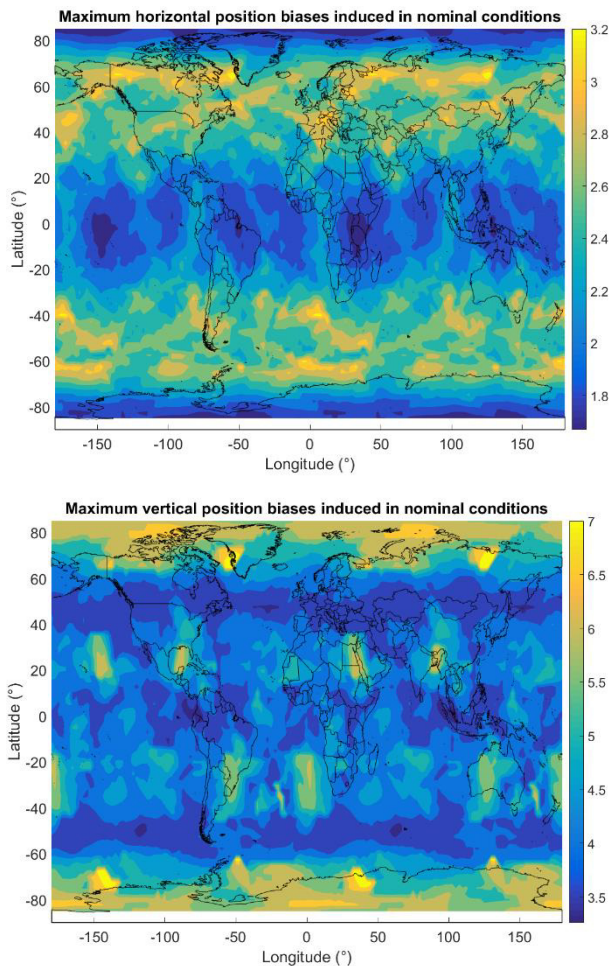
Figure 14 – Summed effect of the three components of nominal biases on the horizontal (top) and the vertical (bottom) positions.

Using this strategy, position biases caused by distortions are processed differently than position biases caused by the satellite antenna and the receiver antenna group delay variations. Indeed, pseudorange biases induced by distortions are not summed to other biases at pseudorange level but only at position level. This strategy adds conservatism to results but was retained to limit the computational burden.

The maximum of maximum position bias values over all locations are given in Table 4 for the horizontal, the vertical and the 3D positions. In green is given the maximum position bias induced by the satellite antenna and the receiver antenna group delay variations. In red is the maximum position bias induced by distortions. In black is the maximum position bias induced by nominal signal deformations (that includes the 3 bias components). The aggregate impact of the satellite and the user antenna delays on the position is lower than the impact of distortions on the position (2.6 m and 5.5 m respectively on the 3D position in the worst conditions). Values provided in Table 4, in nominal conditions, are relatively high and reach 8.1 m looking at the summed impact of all bias components on the 3D position. It can be justified by

the conservatism used to derive these worst values among the different epochs:

- Regarding the impact of the satellite antenna group delay: the iono-free model is applied conservatively to a single frequency case.
- Regarding the impact of the receiver antenna group delay: the conservative model1 is considered and the worst case is obtained among different antenna orientations.
- Regarding the impact of signal distortions: it is assumed that biases affect pseudorange measurements in a constructive way (red values going up to 5.5 m).
- Finally 3D position biases are high because of the strong impact of pseudorange biases on the vertical direction in the position domain.

Table 4 – Maximum of the maximum position biases in different cases.

	Maximum of maxima
Horizontal position	1.5 m ** 2.4 m ** **3.4 m**
Vertical position	2.6 m ** 5.1 m ** **7.5 m**
3D position	2.6 m ** 5.5 m ** **8.1 m**

5 CONCLUSIONS AND FUTURE WORKS

In this paper, the impact of nominal deformations on the user position was assessed by simulations in a civil aviation context. Based on different publications from the literature, biases that affect pseudorange measurements were assessed and modeled for GPS L1 C/A signals. More precisely, in section 3, three models are proposed to characterize the three components constitutive of the total pseudorange bias:

- bias induced by the satellite antenna delay variation,
- bias induced by the receiver antenna delay variation,
- bias induced by distortions.

From these pseudorange models and in conditions described in section 4.1, simulations were run to estimate the impact of the three different biases on the absolute position. Position biases given in section 4 provide results for a standalone user. Using proposed models to characterize the three different biases, signal distortions have the highest impact on the 3D user position bias (up to 5.5 m) while the aggregate impact of the satellite and the user antenna delays does not exceed 2.6 m. These nominal position bias values are relatively high and this because:

- Conservative models were retained to characterize biases induced by satellite antenna and receiver antenna delays.
- A conservative strategy was used to estimate the impact of pseudorange biases induced by distortions on the position. On the one hand, it is

assumed that b_{max} is equal to 50 cm (the value of 10 cm was used for example in [19]). On the second hand, it is assumed that b_{max} affects all pseudorange measurements in a constructive way leading to a high bias in the position domain.

Regarding future works, several recommendations are proposed:

- Different bias models could be tested. In particular, the parameter b_{max} used to derive the impact of signal distortions on the position estimate could be defined more precisely instead of considering only a worst case (conspiring biases). In addition b_{SV}^{L1i} could be modeled more precisely than using the b_{SV}^{L1-L2i} model.
- The different plots should be improved by testing different set-ups (for example a GPS constellation with 24 satellites or/and with Galileo satellites).
- Instead of only looking at the worst case, another observables could be studied. For example, world maps that give position biases that are obtained 95% of the time could be plotted.
- It could also be of interest to implement differential GNSS algorithms to assess the impact of pseudorange nominal biases on a differential user position. Although primary results were obtained, they could not be shown in this article due to the lack of space.
- Finally concepts developed in this article could be applied to other signals than GPS L1 C/A to estimate the impact of nominal pseudorange bias on Galileo signals users and/or dual frequency users.

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