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ABSTRACT

In this paper, we present a multiple hypothesis based approach to test the performance of the receiver autonomous integrity monitoring (RAIM) fault detection and the corresponding integrity bound against multiple failures. In particular, we investigate the resistance of RAIM for the combined constellation of global positioning system (GPS) and Russian global orbiting navigation satellite system (GLONASS) against multiple faults, based on the proposed method. This paper evaluates the exact probability of missed detection (PMD) under each failure, and we compare this PMD to the requirement for single failure (i.e., 10^{-4}), as in lines with the proposed GPS/GLONASS MOPS test procedures. This is to check if the test procedures outlined in the MOPS could guarantee the required level of integrity even for multiple failures. It is shown that the maximum PMD for a pair of faults is approximately 10^{-2} when the test procedures and the corresponding assumptions from the MOPS are applied. Also, we compute total probability of hazardous misleading information (PHMI) by accounting for several fault hypotheses, including GLONASS constellation fault, and compare it with the integrity risk requirement of 10^{-7}. The maximum PHMI for multiple satellite failures goes up to approximately $2 \cdot 10^{-8}$ when the newly proposed methodology is applied.

1 INTRODUCTION

Global navigation satellite system (GNSS) measurements are vulnerable to rarely occurring faults such as satellite failures, which can lead to potential integrity threats for users. A Fault-detection algorithm, receiver autonomous integrity monitoring (RAIM) has been developed to mitigate the impact of those risks. The U.S. global positioning system (GPS) with RAIM has been used to support aircraft navigation since the mid-1990s [1]. Today's RAIM is used for supplemental navigation in the en route and terminal area phases of flight, and also supports lateral guidance during the approach phase of flight [2]. RAIM take advantage of redundant ranging measurements to perform self-contained fault monitoring at the user receiver level [3]. With the full deployment of the Russia's global orbiting navigation satellite system (GLONASS), an increased number of redundant GNSS measurements are available, which has recently drawn interest in the feasibility of a single frequency based GPS/GLONASS RAIM.

Accordingly, the design of a rigorous integrity test methodology for GPS/GLONASS receiver has been needed in developing the GPS/GLONASS Minimum Operational Performance Standards (MOPS) for GPS/GLONASS L1-only airborne equipment. These standards and test procedures must be validated in order to show that they protect the user concerning the higher level requirements of integrity and continuity relating to safety.

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Thus, this paper presents a multiple hypothesis based approach to test the performance of GPS/GLONASS RAIM fault detection (FD) and the associated integrity bound against multiple failures. Based on the newly proposed method, we investigate the resistance of GPS/GLONASS RAIM against multiple failures, including GLONASS constellation failure.

We first determine the worst case fault, which is the most difficult to detect whilst leading to a potential positioning failure. This corresponds, for each failure, to identifying both the worst fault direction and magnitude. Next, we evaluate the exact probability of missed detection (PMD) under each fault by using numerically fast and efficient integration method. In this procedure, we consider bi-normally distributed position error ellipse whose major axis is aligned with the worst fault direction. Also, previously proposed RAIM protection bounds for multiple faults are considered to evaluate the PMD. We then compare this PMD to the requirement for single failure (i.e., 10^{-4}), as in lines with the proposed GPS/GLONASS MOPS test procedures. This is to check if the proposed safety bounds could protect the users against multiple failures. We also compute total probability of hazardous misleading information (PHMI) by accounting for several failure modes: a single GPS fault and a single GLONASS fault, GPS double faults, GLONASS double faults, GLONASS constellation fault, and a combination of a single failure and GLONASS constellation failure. Also, we compare it with the integrity risk requirement of 10^{-7}.

This paper is comprised of five sections. Following the introduction, section 2.0 gives a brief description of GNSS based position solution and the current RAIM fault detection. In Section 3.0, the methodology of integrity risk evaluation for RAIM with multiple failures is presented. Section 4.0 discusses the results of PMD and PHMI simulation performed using the proposed method. This study is concluded in Section 5.0 with remarks for future work.

2 WEIGHTED POSITION SOLUTION AND RAIM

A typical linearized measurement equation is as follows:

$$\mathbf{y} = \mathbf{G}\mathbf{x} + \boldsymbol{\varepsilon} \tag{1}$$

where

- $\mathbf{x}$ is the five dimensional state vector which includes north, east and up position offsets from a linearization point, and two clock errors for GPS and GLONASS
- $\mathbf{y}$ is an n dimensional measurement vector
- $\mathbf{G}$ is the observation matrix which has dimension n x 5
- $\boldsymbol{\varepsilon}$ is measurement noise vector whose components are assumed to be independent and Gaussian distributed.

The weighted least squares solution for $\mathbf{x}$ and the corresponding estimate error covariance can be respectively calculated by

$$\hat{\mathbf{x}} = (\mathbf{G}^{\mathrm{T}}\mathbf{W}\mathbf{G})^{-1}\mathbf{G}^{\mathrm{T}}\mathbf{W}\mathbf{y} \tag{2}$$

$$\mathrm{Cov} = (\mathbf{G}^{\mathrm{T}}\mathbf{W}\mathbf{G})^{-1} \tag{3}$$

where

- $\mathbf{W}$ is the inverse of the measurement noise covariance matrix under nominal conditions.

The standard RAIM utilises redundant ranging measurements to check if there is either excessive noise or biases due to the possible faults in the measurement vector [4]. For this purpose, RAIM fault detection is performed based on the following threshold test:

$$\mathbf{y}^{\mathrm{T}}\mathbf{S}\mathbf{y} \; > \; \chi^2_{1-\alpha}(n-5) \tag{4}$$

where $\chi^2_{1-\alpha}(n-5)$ indicates the 1-α quantile of the central chi-squared distribution with n-5 degrees of freedom, and α represents the specified probability of false alarm. The matrix $\mathbf{S}$ in Equation (5) can be found by

$$\mathbf{S} = \mathbf{W}(\mathbf{I} - \mathbf{G}(\mathbf{G}^{\mathrm{T}}\mathbf{W}\mathbf{G})^{-1}\mathbf{G}^{\mathrm{T}}\mathbf{W}). \tag{5}$$

If there is no detection, RAIM produces protection bounds such as horizontal protection levels (HPLs). More details on HPL computation will be described in Section 3.

3 INTEGRITY RISK EVALUATION METHODOLOGY

This paper presents a multiple hypothesis based approach to test the performance of the current RAIM fault detection and the corresponding integrity bound against multiple failures. In this section, we provide a step-by-step description of the proposed method.

3.1 Multiple hypothesis approach

In the multiple hypothesis approach, the total integrity risk, also called probability of hazardous misleading information (PHMI), is defined as follows [5]:

$$\sum_{i=0}^{h} P(|\varepsilon_0| > l, |q| < T | H_i)\, P_{H_i} \tag{6}$$

where

- ε_0 is the error in the position estimate
- l is a specified alert limit that identifies hazardous conditions
- q is the detection test statistic and here χ^2 test statistic is used for conventional RAIM fault detection
- T is the detection threshold
- H_i for $i = 0,1,\cdots,h$ is the possible fault hypothesis associated with fault on measurement subset 'i'
- P_{H_i} is the prior probability of H_i occurrence.

Since fault distributions which the aircraft may experience are unknown, a bound on the probability of HMI under H_i need to be evaluated for the worst case fault. In Equation (7), $\boldsymbol{f}_i$ is a $n_i \times 1$ fault vector, and n_i indicates the number of measurements given H_i. If the components of fault vector are ordered to correspond with a fixed satellite order, the fault vector have non-zero components in the position for which faults are present in the corresponding satellites, and have zeros elsewhere.

$$\sum_{i=0}^{h}\left(\max_{\boldsymbol{f}_i} P\big(|\varepsilon_0| > l, |q| < T \,\big|H_i, \boldsymbol{f}_i\big)\, P(H_i)\right) \quad (7)$$

The worst case fault vector $\boldsymbol{f}_i$ for multiple satellite failures which maximizes the integrity risk under H_i is mathematically derived in [4][6]. More specifically, the analytic formulation for the worst case fault direction is given in [4][6], whereas the worst case magnitude can be found based on a straightforward line search algorithm. For the fault-free case (i.e., $i = 0$), the fault vector $\boldsymbol{f}_0$ should be $n_0 \times 1$ zero vector. If the evaluated integrity risk is lower than the requirement, the RAIM based navigation is considered available. This paper takes account of the integrity requirement of 10^{-7}.

We conduct not only PHMI analysis also probability of missed detection (PMD) analysis based on the proposed GPLS/GLONASS MOPS integrity test procedure [7]. PMD corresponding to each fault hypothesis can be defined as each conditional risk probability within Equation (7):

$$P_{MD,i} = \max_{\boldsymbol{f}_i} P\big(|\varepsilon_0| > l, |q| < T \,\big|H_i, \boldsymbol{f}_i\big) \quad (8)$$

More details on the computation of the probability in Equation (8) will be described in the following subsections.

3.2 Calculation of HPL against multiple failures

To examine the resistance of GPS/GLONASS RAIM against multiple faults, we use horizontal protection level (HPL) for the alert limit in Equation (7-8). In this paper, two type of HPL computations for fault detection (HPL_{FD}) are accounted for: one based on the previously proposed multi-bias RAIM protection method [4] and the other one based on the preliminary FD algorithm which has been discussed during GPS/GLONASS MOPS elaboration. Note that although the current MOPS [7] does not specify a formula for HPL calculation, the authors assume the HPL discussed during the elaboration. In [4], the HPL_{FD} computation for a fault mode i is performed as follows:

$$HPL_{FD} = \max_i\left(\|\mathbf{b}_{H,i}\| + k_{H,i}\sigma_H\right) \quad (9)$$

where

$\mathbf{b}_{H,i}$ is an error vector whose components are bias-induced errors in user horizontal position given fault vector $\boldsymbol{f}_i$

$k_{H,i}$ is the number of standard deviation for horizontal position error which corresponds to the specified probability of missed detection under fault condition i

σ_H is the standard deviation for horizontal position error which is calculated based on the error covariance matrix in Equation (3).

Here the first term is the protection for biases and the second term is the protection for measurement noise. In particular, more details on the mathematical derivation of $\mathbf{b}_{H,i}$ can be found [4]. At each epoch, the maximum value over possible fault conditions is taken as the resulting HPL_{FD} (see Equation (9)).

We also consider HPL_{FD} that has been discussed during the elaboration for GPS/GLONASS MOPS (see Equation (10)). We will take this HPL_{FD} as the alert limit for PMD computation in Equation (8). Unlike the HPL_{FD} in Equation (9), the proposed HPL_{FD} only considers the protection for the fault induced bias. At each epoch, the maximum value over possible fault conditions is taken as the resulting HPL_{FD}.

$$HPL_{FD} = \max_i \|\mathbf{b}_{H,i}\| \quad (10)$$

We will investigate how those two different user protection bounds can affect integrity risk assessment for the GPS/GLONASS RAIM in Section 4.

3.3 Computation of integrity risk probability

This section describes the calculation of the conditional joint probability in Equation (7-8). Since the position error, ε_0, and the RAIM fault detection test statistics, q, are statistically independent, the joint probability in Equation (7-8) can be expressed as [5]:

$$P\big(|\varepsilon_0| > l, |q| < T \,\big|H_i, \boldsymbol{f}_i\big) = P\big(|\varepsilon_0| > l \,\big|H_i, \boldsymbol{f}_i\big)\, P\big(|q| < T \,\big|H_i, \boldsymbol{f}_i\big) \quad (11)$$

To evaluate the first probability, we need to characterise the position error distribution under fault hypothesis i, which is shown in the following simple diagram.

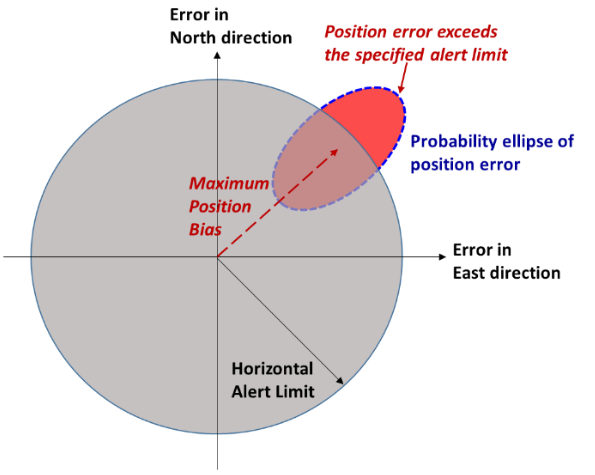

Figure 1 – Position error probability distribution and protection region

Here the maximum position bias is induced by the worst fault vector, $\boldsymbol{f}_i$, as in Equation (7-8). Also, the orientation and shape of the position error ellipse can be characterized by the error covariance matrix in Equation (3). In particular, the orientation of the distribution is normally not aligned with the direction of the maximum bias. However, we assume the worst error distribution such as the major axis of the error ellipse (or distribution) is aligned with the maximum bias direction, as shown in the Figure 1. The red shaded area outside the circle with a radius of the alert limit indicates that the position error exceeds the horizontal alert limit. Therefore, based on the magnitude of the maximum bias and the error covariance matrix, we can evaluate the positioning failure probability by numerically integrating the position error distribution over the shaded area. For this purpose, we have introduced a computationally efficient integration method in [8]. In addition, the second probability can be easily calculated from the known non-central chi-squared distribution (see Equation (4)).

4 SIMULATION RESULTS

So far we have described the method to evaluate integrity risk probability. In this section, we focus on the GPS/GLONASS RAIM performance analysis, and we carry out two separate investigations. Firstly, we conduct PMD performance analysis based on the PMD evaluation method proposed in this paper and GPS/GLONASS MOPS integrity test procedures and fundamental assumptions and parameters [7]. Secondly, we perform PHMI analysis based on the newly proposed methodology. In particular, PHMI analysis is carried out for three intended cases. Those three cases are different depending on which protection bound is taken as the alert limit (l) in Equation (7-8) (See Table 1).

Table 1 – Different simulation scenarios

Case	Alert limit (l)
1	Equation (10)
2	Equation (9)
3	556m (alert limit for RNP 0.3)

Underlying assumptions and integrity parameters specified in [7] are used to perform the simulations. Single-frequency 24 GPS/ GLONASS baseline constellation with formal parameters [7][9] is used. Also, all simulations were performed based on a single day period. Table 2 shows some key simulation parameters.

Table 2 – Simulation parameters

Parameters	Values
Integrity requirement	10^{-7}
PMD requirement	GPS: 10^{-4} / GLO: 10^{-4}
P_{sat}	GPS: 10^{-5} / GLO: 10^{-4}
P_{const}	GLO: 10^{-4}
σ_{URA}	GPS: 2.5m / GLO: 18m
Mask Angle	5 deg.
Time Step	10 min.
Simulation Duration	24 hours

4.1 Probability of missed detection performance analysis

Integrity monitoring test procedures outlined in the GPS/GLONASS MOPS [7] suggests verification of whether or not the user equipment can satisfy the missed alert requirement. According to the proposed MOPS' instruction [7], we conducted PMD simulations for dual satellite failures below:

- two independent single GLONASS satellite faults
- only GPS satellite failure and single GLONASS satellite fault

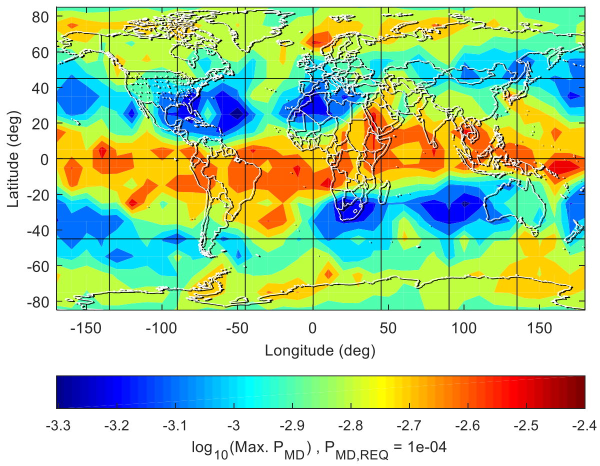

Figure 2 - Maximum PMD against dual failures obtained when the GPS/GLONASS MOPS test method is applied to PMD evaluation. The highest PMD is represented in log scale.

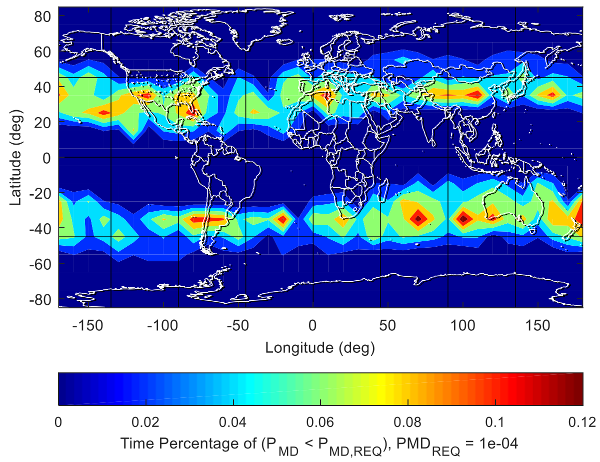

Figure 3 - Percentage of time that the evaluated PMD is lower than the associated probability of missed detection requirement of 10^{-4}.

At each user location and time epoch, we compute PMDs for possible dual failures (see Equation (8)), and then take the maximum PMD among evaluated PMDs. Afterwards, we compare it with the corresponding PMD requirement of 10^{-4}, as in lines with GLPS/GLONASS MOPS integrity test procedures.

In Figure 2, the maximum PMD is represented in log scale, and it ranges from approximately 10^{-3} to 10^{-2}, which is much higher than the proposed missed detection requirement of 10^{-4} [7]. Figure 3 shows the percentage of time that the evaluated PMD at each epoch and user location is lower than or equal to the given requirement. As shown in the figure, there are no regions where the proposed requirement is met. Thus, if we apply the protection bound (see Equation (10)) to the future GPS/GLONASS RAIM, the fault monitoring performance could be poor for multiple failures. Thus, integrity test procedures, including FDE algorithm, proposed by the GPS/GLONASS MOPS appears to be incomplete for various satellite failures.

4.2 PHMI performance analysis

Since previous PMD report is limited to verification for double satellite faults, we need to thoroughly investigate the resistance of GPS/GLONASS RAIM against multiple failures. Therefore, in this section, we carried out multiple hypotheses based PHMI analysis by accounting for several failure modes:

- a single GPS satellite fault
- a single GLONASS satellite fault
- GLONASS constellation fault
- two independent single GLONASS satellite faults
- two separate single GPS satellite faults
- only GPS satellite failure and single GLONASS satellite fault

In this paper, we assume the prior probability of GPS constellation fault as zero according to the underlying assumptions in the draft MOPS [7]. However, it is worthwhile to examine how GPS constellation failure can affect the integrity risk to fully understand the system performance, and this issue will be further investigated in our future work. All simulations were conducted using the same satellite constellation and simulation parameters (see Table 2), and PHMI performance was assessed for the different three cases (see Table 1).

The first case determines the integrity risk for GPS/GLONASS RAIM when user HPL is generated based on the proposed FD algorithm (see Equation (10)). Figure 4 shows the simulation result of PHMI in log scale. The maximum PHMI ranges from approximately 10^{-5} to 10^{-6} that are much higher than the integrity requirement of 10^{-7}. Figure 5 illustrates the percentage of time that the evaluated PHMI is lower than the associated requirement of 10^{-7}. There are no regions where the integrity requirement is satisfied. Therefore, as in the case of PMD evaluation, currently proposed fault monitoring method and the protection for the bias could lead to frequent integrity fails of the future GPS/GLONASS RAIM under multiple error conditions.

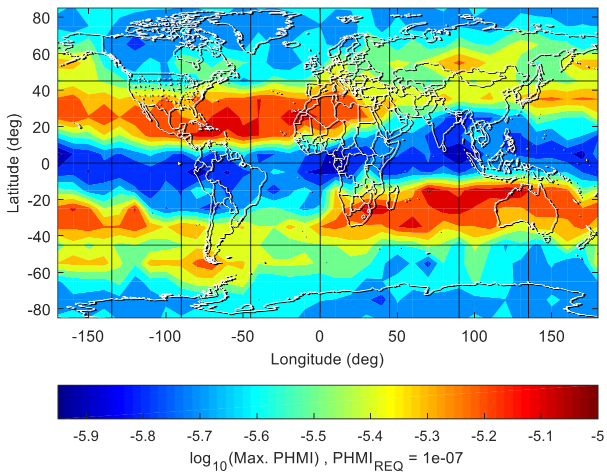

Figure 4 - Maximum PHMI under multiple failure modes when the horizontal user protection bound based on the proposed MOPS test procedure is applied. Each maximum value is expressed in log scale.

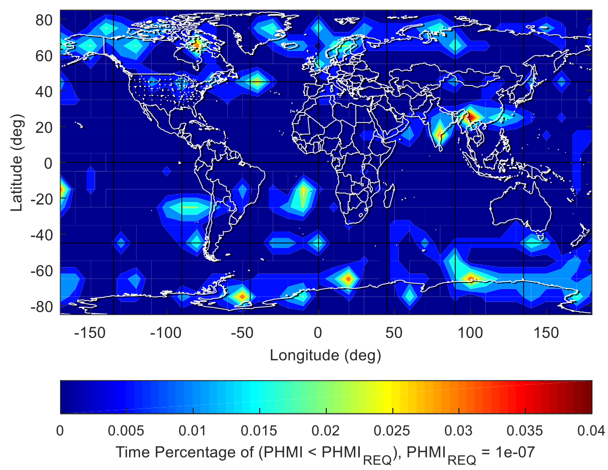

Figure 5 - Percentage of time that the evaluated PHMI is lower than the associated requirement of 10^{-7}.

We also investigate whether or not a more conservative safety bound in Equation (9) can protect the users against multiple satellite failures. Figure 6 shows the maximum PHMI in log scale that is obtained using the conservative user protection bound. The maximum PHMI goes up to approximately 10^{-8} and thus the integrity requirement can be satisfied in all regions in the world. Therefore, the GPS/GLONASS RAIM protection outlined in this work (i.e., Equation (9)) appears to be safe even for multiple failures. Accordingly, derivation of an optimised user protection bound for the future GPS/GLONASS RAIM should be one of the essential components to be considered for the design of the new integrity test procedures and the related FDE algorithm for GPS/GLONASS MOPS.

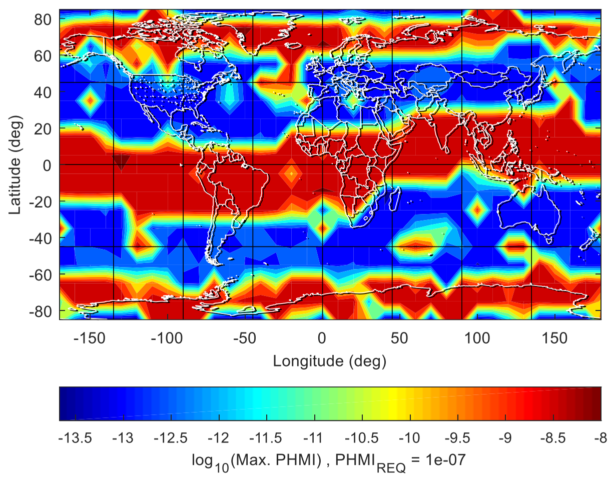

Figure 6 - Maximum PHMI in log scale obtained using the multi-bias RAIM protection bound with the measurement noise protection. The maximum goes up to approximately $2{\cdot}10^{-8}$.

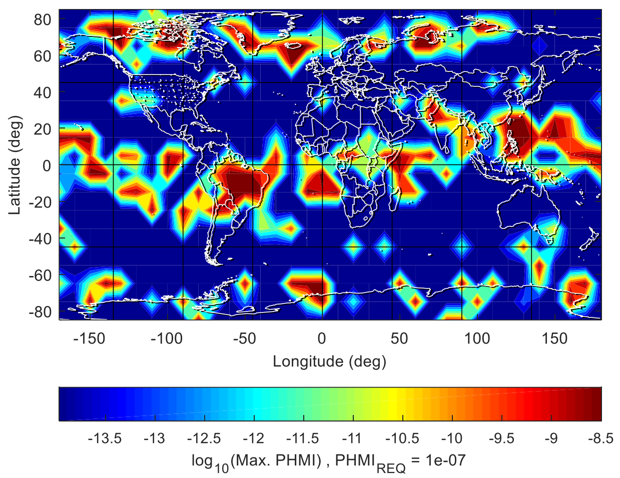

Figure 7 - Maximum PHMI in log scale based on the alert limit for RNP 0.3 operation (i.e., 556m).

Next, we further examine if the GPS/GLONASS RAIM can guarantee horizontal services supported by today's RAIM such as RNP 0.3 operation even under multiple failure conditions. The integrity risk evaluation result based on the alert limit for RNP 0.3 operation (i.e., 556m) was obtained using the same simulation parameters and the baseline constellation as the previous two cases. As we expected, integrity requirement (i.e., 10^{-7}) can be met in all regions of the world for such huge position limit.

In this section, we demonstrated the benefits of implementing the proposed methodology for integrity risk evaluation and the different protection bound from one which has been discussed for the GPS/GLONASS MOPS. However, the maximum integrity risk which is lower than 10^{-8} may have been observed merely due to chance, because all possible satellite geometries were not investigated in this work. Thus, our further study would examine more satellite geometries which could make the performance worse by accounting for a multiple of the periods of GPS and GLONASS constellation.

5 CONCLUSION

In this paper, we present an integrity test methodology for RAIM with multiple faults based on the multiple hypothesis approach. We also examine the performance of the future GPS/GLONASS RAIM based on the proposed method. As a result, we found that the GPS/GLONASS RAIM protection could not be safe for multiple failures when the proposed MOPS test procedure and the underlying assumptions are applied to the user equipment. Also, we conducted preliminary integrity risk assessment for GPS/GLONASS RAIM with multiple faults based on the newly proposed method. We demonstrated that the different user protection bound from the one that has been proposed for GPS/GLONASS MOPS is needed. However, this work is concentrated on possible fault modes and the related error bounding for GPS/GLONASS RAIM.

Future work should therefore include follow-up work designed to thoroughly evaluate integrity test procedures, including fault injection test, proposed by the current MOPS. Also, the simulation with expanded period concerning more satellite geometries and GPS constellation fault needs to be performed. This work would help to design integrity monitoring and the associated error bounding for GPS/GLONASS L1-only airborne equipment.

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