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GPU based Computational Simulation of Aircraft Evacuation: Temporal and Spatial Analysis

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Abstract

The effectiveness of Aircraft Emergency Evacuation plays a vital role in the safety of the passengers on board an Aircraft, in case of Emergency landing. In this paper, the implementation and development of a Cellular Automata (CA) based simulator which can be used to simulate the Aircraft Evacuation process is presented. Given the seat-map of the Aircraft, number of passengers, passenger feature distribution and number of functional exits, the simulator can calculate the approximate time of Evacuation. For computational implementation, a bi-dimensional as well as uni-dimensional grid which represents the 2D Aircraft seat-map and an agent which control the passenger movement during Evacuation of the Aircraft is used. Each agent represents a passenger and is characterized by the properties of a human being. An agent has properties like age, sex, walking speed, response time, position and status. The CUDA framework for the parallel implementation of our algorithm/code which can be executed on GPUs (graphics processing unit) has been used and thereby speeding up the simulation process. Several test cases are performed and the results on Aircraft Evacuation times have been compared with existing data collected by Aircraft manufacturers. Detailed investigations reveal useful information on the relationship between Evacuation time and important attributes such as passenger age and sex, number of gates open, passenger distribution etc. The simulator facilitates the investigation of spatial and temporal movement of the passengers, as well as the visualization of pattern formation and collective behaviour.

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Keywords: Aviation Safety; Aircraft Evacuation; Cellular Automata; GPU Computing

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1. Introduction

Demand for air travel has increased steadily over the last decades and the aviation Industry has forecast that-further substantial growth, nearly doubling of the air traffic, into the next coming decades [1].These forecasts have led Aircraft manufacturers to design and produce airframes capable of carrying as much as nine hundred passengers as well as newer generation airframes made of composites. One of the important aspects from the beginning of the aviation history is that the passenger safety has always been taken with high priority. Henceforth, substantial improvement in the safety standard of the aviation from design to better operations and maintenance procedures are implemented in the last decades.

Though the rate of accidents has decreased drastically in the last decades, the percentage of passengers surviving after the accident/incident has not decreased in comparison to the improvements achieved in other areas. A survey performed by the European Transport Safety Council [2] assesses that 40 percent out of the 1500 persons who die every year in aircraft accidents, around 600 passengers die in technically "survivable" accidents. This study shows that more than half of them die from the direct result of the impact, and the others die because of fire, smoke, or problems that arise during the Emergency Evacuation process.

Due to these reasons, not only the issues concerned with the prevention of the occurrence of accidents are tackled with great care but also issues contributing to improving the survival rate in the event of an accident/incident. Accidents can be classified either as fatal (non-survivable), non-fatal (survivable) or technically survivable. There are two ways to prevent fatalities in air travel: by preventing accidents and by protecting aircraft occupants when accidents occur.

In order to increase the survivability of passengers in case of an accident, one major area that needs immediate attention is Cabin safety. Cabin safety cannot be defined precisely as it covers a very diverse responsibility and interests, which mainly includes crashworthiness, operations, human factors, psychology, and biodynamics. However, it can be classified in three major areas, interacting with each other namely: **a**. Impact protection, **b**. Fire survivability and **c**. Emergency evacuation [3].

1.1. Emergency Evacuation

Emergency evacuation is an event, which seldom occurs at the scale of an airlines and that is extremely rare at the level of individuals. To further enhance the standard of Aviation safety, enhanced cabin improvements at the design stage plus more efficient training including coordination between Cabin crew to manage the Emergency Evacuation is necessary.

Major factors affecting the emergency evacuation process are:

- Configurational aspects related to a/c, Environmental aspects (fire, toxic bas…), Procedural (Role and responsibility of Cabin Crew during evacuation), Behavioural aspects of passengers and Cabin Crew (Fear, anxiety, Cultural and language, Competitive behaviours amongst passenger, Queuing Behaviours, Herding Behaviours, Dual roles of Cabin Crew..) [3]

1.2. Emergency Evacuation demonstration for Certification Requirement

Beginning in 1965, the Federal Aviation Administration (FAA) required each air carrier operating under Part 121 of the Federal Aviation Regulations to perform full scale Evacuation demonstrations under simulated Emergency conditions prior to receiving operating certification for new aircraft or seating configurations.

The air carrier demonstration was designed to evaluate crew training and the adequacy of Evacuation procedures. FAA initially imposed a 120-second egress time limit for Evacuating all passengers and crew. FAA attributed a 1967 change in maximum egress time to 90 seconds due to improvement in slide technology that had occurred since the initial standard was released. In 1982, after study of actual and demonstrated Emergency Evacuations, FAA allowed Type Certificate holders, under specified conditions, to use the results of a successful demonstration conducted by either the manufacturer or another airline rather than to conduct a new test. The stated goal of requiring the full-scale demonstrations is to provide a benchmark by which FAA can consistently evaluate Evacuation capability using various seating and exit configurations. FAA claims that a consistent measure of success is achieved by requiring all manufacturers to strive for the same 90-second limit. According to FAA, the demonstration is not an acceptable Evacuation performance standard. That is, manufacturers must also comply with specific equipment and minimum configuration requirements in addition to successfully demonstrating complete Evacuation within 90 seconds. Performance standards, on the other hand, are expressed using objective performance goals alone-no specific design or operating criteria are established. In addition to the 90-second time limit, FAA full-scale Evacuation demonstration criteria include the following [4] [5].

- The demonstration must be conducted during the dark of night or with the dark of night simulated the airplane's emergency lighting system can provide the only illumination of exit paths and slides;
- A specified mix of passengers "in normal health" must be used-for example, at least 30 percent must be females and at least 5 percent must be over 60 years of age;
- The passengers may not have participated in a demonstration in the previous 6 months and
- Not more than 50 percent of the Emergency exits may be used.

In a 1989 advisory circular, FAA provided guidance to manufacturers, on how to determine whether analysis and tests might be used in place of full-scale demonstration. The AC (Advisory Circular) also provided guidelines for setup and conduct of the demonstration. Among other things, the AC identified two equivalent age sex distributions. Under the 1989 FAA guidelines, manufacturers may replace participants in the highest age category (i.e., the one most susceptible to injury) with greater numbers of persons aged 51 to 60 years and need not use minors.

1.3. Limitation of Full scale Demonstration

Full-scale certification demonstrations are both hazardous and costly. Intended to serve as a benchmark for functional ability of emergency equipment and procedures, the test is not useful for system optimization. The Emergency Evacuation scenario used in full-scale demonstrations does not represent most accidents conditions, where impact forces and fire effects frequently impair passengers. Participants - in demonstration knows they face no such danger in their efforts to quickly exit the aircraft, so panic is not present. However, the test still exposes participants to a range of injuries, from bumps and bruises to serious, permanent injury. Furthermore, during seven full-scale demonstrations conducted by manufacturers between 1972 and 1980, 166 of 2,571 total participants received injuries, or 6.5 percent. Of the 3,761 participants in 12 demonstrations conducted between 1981 and 1991, 212 received injuries (5.6 percent). The cost of conducting full-scale Evacuation demonstrations, including test set-up, payments to volunteers, analysis, and so forth, reaches upward of $2 million for wide body transports.

One potential problem with the test procedure is that the mix of test participants required by FAA is often not representative of the flying public on a given flight. In general, passenger demographics vary from region to region and seasonally. Tests conducted using passenger loads with higher percentages of women and elderly persons, or with children and persons with disabilities, would likely generate longer average evacuation times.

An unrealistic passenger mix, combined with the absence of surprise, trauma, fright, and panic, produces optimistic indications of an aircraft's evacuation safety capability [6] [7]. However, industry and many others are understandably loathe to subject demonstration participants to the presence of fire, smoke, and additional debris, for fear of increasing the likelihood of injury. On the other hand, any changes to the certification process designed to reduce the risk of injury require analysis of the comparability of results.

Additionally, without the benefit of repeated trials, one cannot be confident that a single certification test result truly represents an Aircraft Evacuation system's capability. Neither a margin of error or confidence level can be determined. By comparison, use of anthropomorphic dummies allows auto manufacturers to conduct realistic crash response tests repeatedly and with high validity, without threat to human safety, and to determine performance relative to government standards. EASA, FAA and the Aviation community struggle to achieve agreement on whether the value of but one full-scale Evacuation demonstration for certification warrants the risk.

Though the cost of Evacuation demonstration is insignificant compared to overall Aircraft program development and airplane construction: among all factors it is the hazard of serious injury, not test costs, plus limitation of the single Evacuation trial generated Aviation actors interest in modifying the existing certification criteria and developing alternative testing and assessment methods.

The full scale Evacuation for A380 certification (as shown in Fig. 1) was performed as per the EASA/FAA rules, but the passenger were volunteers (from Airbus and public) who knows it is not life threatening scenario, selected Cabin crew were highly skilled, all medical team were available in case of injuries, whole family participating in the demonstration were missing. Though the Evacuation criteria as per Authority rules were fulfilled, do this really represent the real Emergency Evacuation scenario? An alternative methodology, replicating the real Emergency Evacuation scenario is needed to enhance the Aviation standard.

Fig. 1. A380 - 90 seconds Evacuation Demonstrations: 873 occupants(853 Passengers + 20 Crews) [3]

1.4. Utilities of simulation tool

The simulation tools provide Aviation stakeholders, alternative solutions, which can be more reliable than the full scale evacuation trial. The simulation tool provides more reliable data, as it provides Aircraft manufactures to run several trials, different strategies as well as visualize scenarios in which each passenger has a different state of mind. Parameters like the number of operational gates, the passenger distribution in the aircraft, behavioral characteristics of passengers, location of sensors and crew members etc. can be varied and helps in getting results which leads to better understanding of the evacuation process. As several parameters can be altered, it provides much more realistic results compared to Evacuation drills, where volunteers are already aware that it is not real life threatening scenario.

2. Computational Model and simulation tool

The numbers of simulation techniques for investing aircraft evacuation problem have increased in recent years. Evacuation simulation can be done using a macroscopic model such as fluid dynamics based model, or microscopic models such as cellular automata, particle models, agent based simulation etc. The simulation model proposed in this paper uses cellular automata model implemented via CUDA on GPUs. The movement of individual passenger as well as the interaction among passengers and obstacles during evacuation process can be efficiently analyzed using the proposed model.

2.1. Cellular Automata

Cellular automata are artificial mathematical models of dynamical systems, discrete in space and in time, whose behavior is completely specified in terms of some local law. A cellular automaton can be considered as a stylized universe where, space is represented by a uniform grid, with each grid cell containing some data related to the evolution of the system such as its state; time advances in discrete steps, the laws of the universe are used at each step through which each cell computes its new state from that of its previous state and close neighbors [8, 9]. The first cellular automaton used by Von Neumann was qualified as a universal computer while the Game of Life of J.H. Conway had only two states per cell, either filled or empty, ‘alive’ or ‘dead. Essentially cellular automata allow adopting a cell-based approach to model processes in a two–dimensional space. A typical cellular automata system is composed of four components: cells, states, neighborhood and rules. Cells are the smallest units of the system having adjoining neighbors; they are characterized by discrete states. The state of a cell can change only based on transition rules, which are defined in terms of neighborhood functions. The transition rules are the real engines of change in cellular automata and the system adapts according to these well-defined rules. These rules control the transformation of a cell state to another cell state over a specific period of time depending on the neighborhood of the cells. The notion of neighborhood is central to the cellular automata paradigm. Thus, the system's laws are local and uniform [10, 11]. Several research works involving crowd behavior using CA based models using different attributes have been reported in the literature [12, 13].

In this paper, we have developed a CA based simulation tool using simple rules to investigate the emergency evacuation problem. The simulation process is computationally expensive because it involves spatial and temporal evolution of large number of passengers, where each individual is like a discrete entity being able to react to their surrounding environment based on certain well defined rules and attributes. In this simulation the global evolution of the system (evacuation process) is a sum total of the collective behavior arising from individual passenger interaction based on certain local rules [14]. The accuracy of simulation of this non-linear complex system depends on the number of parameters considered for modeling the evolution process and the accuracy of the rules defining the mutual interaction among these parameters. The results presented in this paper are from the first version of the tool where several approximations have been used to keep the model simple. The goal is to estimate the approximate time required for passenger evacuation in an airplane with real design, dimensions and seat-map. The CA based analysis helps us in understanding the effect of the most important parameters on evacuation time and at the same time facilitates high degree of parallelism during computation. The simulation tools allow changes in important variables such as number of exits, passengers with different movement speeds based on age and gender.

2.2. Implementation and Computational Tool development

Four components of a cellular automata based model: cell, state, neighbourhood, and rules in this case represents the movement space inside the aircraft, passenger dependent state, occupied or unoccupied space and rules which governs the movement of the passengers respectively. The cell is the smallest unit of space and its state is either occupied or unoccupied. The state of a cell depends on the movement of the object across cells, which primarily depends on the state of the neighbouring cells. The rules specify the limiting conditions for the cell's state to change depending upon its neighbouring states. A simple transition rule in this cellular automaton model is given by Li and Yeh as follows, where S(t+1) represents the future state which is a function of present state and neighbourhood: $S(t + 1) = F(N, S(t))$.

The first approach using cellular automata involved space discretized into cells which could be either occupied or unoccupied by a passenger. The person could move in any of the four directions depending upon several conditions such as whether the adjacent cells are occupied or unoccupied by other passengers, has obstacle or no obstacle etc. while always aiming to move towards the exit. The situation is described in Fig. 2 below. Here P is the passenger, L represents the people in the left side, R in right side and B represent passengers behind. The passenger P then analyses his neighbourhood and then moves towards the exit door where the passage is empty. Apart from space, time is also discretized in the model. We have used a time step (delta t) of 178 milliseconds. Each cell of the grid signifies a capacity, which can be occupied by a passenger. Thus there are two possible states that a cell can be in while the system is in progress i.e. occupied or unoccupied (0 or 1). At every time step (delta t) a passenger scans its neighbourhood and analyzes its movement towards the destination. If the front cell is unoccupied then the passenger takes a step forward but if the cell is occupied it waits for the cell to get unoccupied, this process is repeated iteratively and hence the passenger moves towards the exit door. If two or more passengers try to occupy the same cell at the same given point of time then the issue is resolved by randomly choosing a passenger. At the time of evacuation the priority of the passenger is to take the shortest best possible path to the exit. Every passenger chooses to take the path, which can take him to the exit door with the shortest distance travelled. In this model we assume that half of the doors are working. For our simulations presented in this paper, we have taken the seat-map of an Aircraft (A320) as shown in Fig. 3.

In our model we take 3 inputs at the start of the simulation, firstly the name of the aircraft; according to the name we import the structural dimensional values from a file. Our second input is number of passengers and the third is to identify the doors which we intend to open in that particular simulation. For example if the third input is 1 1 1 1 0 0

0 0, then the first 2 main exit doors and the first two emergency exits doors are open and rest are closed. The tool consists of 1 bi-dimensional and 9 uni-dimensional grid denoting seating arrangement, aisle, 4 main exit grids and 4 middle emergency exits. Though, a good framework has been established for developing the tool, several factors stills needs to be considered and integrated in the tool: mainly dual role and responsibility of Cabin crew (do passengers obey the leadership of cabin crew during the evacuation process), behavioural aspects of individuals (family members - wife, children travelling together..), passenger age group (young people, old persons, disable person) violent or non-violent nature of the passengers, fire in the cabin, blockage of several exists in case of emergency situation. More research needs to be performed in these areas and integrated in the tool. At present, our computational implementation uses 10 matrices/arrays, one of the matrixes has dimension of the seat arrangement, i.e if the aircraft has 30 rows and 6 seats in each row then the dimension would be 30X6. There is a matrix representing aisle where cell size (delta x or delta y) is 1 inch, thus if the aisle is of 1040 inches as in the case of A320 aircraft aisle, matrix will be of 1040X1 dimension. Similarly we have 4 matrices for emergency exits with cell size (delta x) as 1 inch of 54X1 and 4 matrices of 54X1 for the main exits.

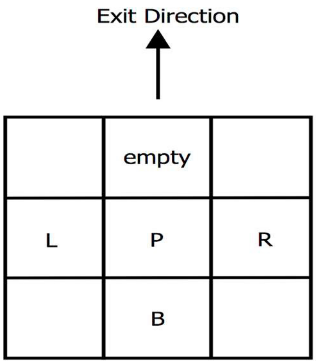

Figure 2: Possible Movement of Passenger

We have taken 5 possible states that the passenger can be in: 0 if the passenger is in his seat, 1 if passenger is in seat aisle, 2 if in the exit aisle, 3 if in the middle exit aisle, 4 if the passenger is out of the plane. At every iteration, the passenger makes a movement based on available options in the neighbourhood and exit location. Each passenger is assigned a unique ID at the start of the simulation, so at any given time the state of passenger can be represented by 3 dynamics variables x , y location and status. Now at each time the passenger makes a move, then temporal and spatial data along with the current state is stored in an output file which can be analyzed after the end of the simulation. We also create two more output files, one to save the time of evacuation of every passenger coming out of the aircraft and another one to save the distance travelled by each passenger from his/her seat to the exit door.

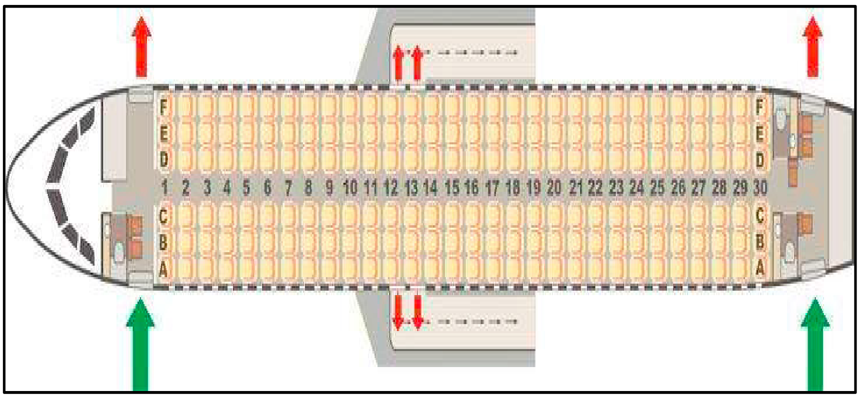

Figure 3: Seat map of the simulated aircraft

2.3. GPU based simulator

GPU-accelerated computing is the use of a graphics processing unit (GPU) together with a CPU to accelerate scientific and engineering applications. The computational model proposed above is inherently parallel and is easily adaptable to the single program multiple data (SPMD) philosophy used in extracting parallel speedup using GPUs. Use of GPU computing makes our simulation much faster compared to serial execution. We have used CUDA parallel programming framework for our implementation. The thread organization of CUDA framework and memory hierarchy of the nVIDIA GPU architecture has been efficiently harnessed to get the optimal performance [15, 16].

3. Simulation Process

In the simulation we have given different speed to passengers depending upon their age and sex. For males: if the age is in the range 20-40 years, their speed is taken as 2m/s, if it is in the range 40-50 years then 1.5 m/s and if the range is 50-70 years then 1m/s, similarly for females it is 2m/s, 1m/s and 0.5m/s respectively. Now, the aircraft design which we have considered for our simulation is Airbus-A320; it has 30 rows of seats, each row consists of 6 seats and an aisle between them. The dimensions of the seat are 18 inch in width and the seat pitch is 30 inches; the width of the aisle is 19 inches. There are 8 exits in the plane and the passenger try to choose one of those exits. These exits might be open or closed during the simulation; we have kept only 4 out of 8 exits open for our present analysis.

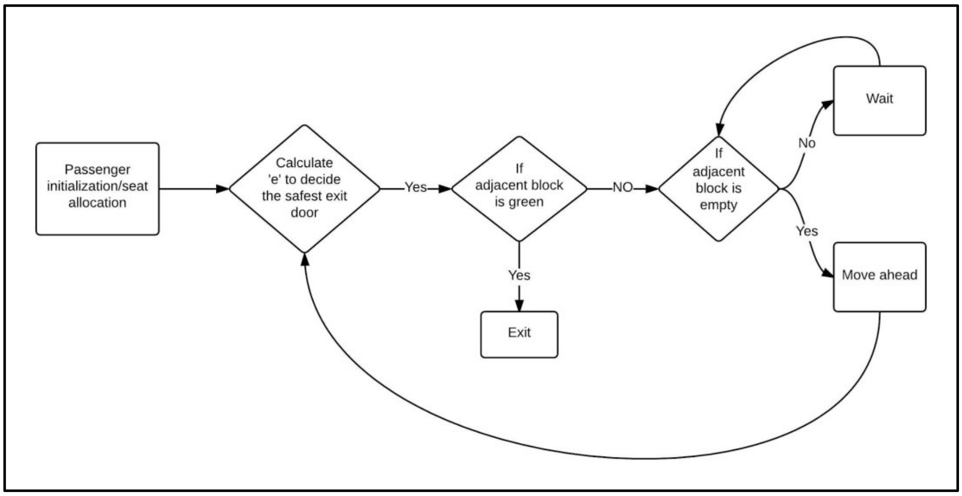

Figure 4: Flowchart of our Computational Algorithm

The main steps of our algorithm represented in Fig. 4 are as follows:

Step 1: Passenger initialization (number of passengers) and seat allotment is done i.e. the passengers are allocated seats randomly. Also taking into account the dimensions of the person, required number of grid points (cells) is filled with the passenger and these cells are assigned proper state. Number of open exits is decided for the simulation.

Step 2: Once the evacuation simulation starts, the passenger checks if the adjacent sides are empty or not, if it is empty it moves towards it else waits for it to be empty. In this algorithm the passenger checks the status of the adjacent cells in every iteration and continues to check until it is empty and passenger reaches the exit.

Step 3: The passenger in the seat aisle always attempts to move towards the central aisle according to its present seat/isle location.

Step 4: When the passenger is in the middle passage i.e. central aisle it calculates the nearest open exit using the distance formula. Then it either moves upwards or downwards depending on nearest exit.
Step 5: When the passenger comes in the row in which the exits are present, it moves towards the exit which is available and if both the exits are available it chooses the door randomly.
Step 6: The passenger has evacuated.

A screenshot of our simulator is shown in Fig. 5 which represents the 2D view of the seat-map.

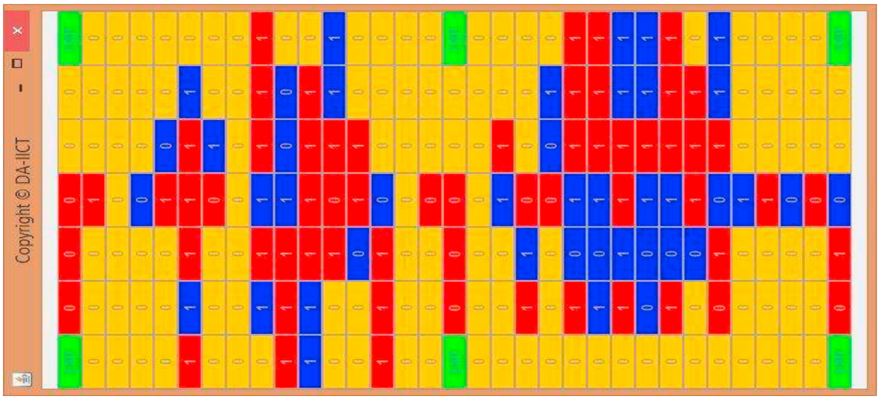

Figure 5: Screenshot of the simulator. 2D seat map of Aircraft.

4. Data Analysis and Results

We have a global matrix which we store into a file in all iterations of the main loop. The global matrix is a convolution of all the matrices which we use during our simulation. The global matrix has all the information collected from bi-dimensional and uni-dimensional grids and we get a matrix of 1's and 0's of dimension 1040X130. After generating all the output files at each time step (delta t) we visualize the data using MATLAB. The 1's represent the presence of a passenger in the grid and 0's represent absence of the passenger. We have calculated the data for time of evacuation by changing the number of passengers. We have considered three scenarios for this study, firstly when there is a mixed population of passengers (both male and female), secondly when there are only male passengers and finally when all passengers are female. The data showing the change in the time is shown in the Table 1. Time of evacuation is calculated by using a mean of at least 3 simulations.

Table 1:Evacuation Time Data

Number of Passengers	Time(ALL) seconds	Time(Male) seconds	Time(Female) seconds
50	30.609	26.122	31.684
80	38.916	27.503	31.928
100	42.535	30.345	34.892
120	43.661	32.984	42.2
150	47.965	33.998	42.864
180	52.472	38.667	43.946

In Fig. 6 we can also see how time to evacuate increases with the increase of number of passengers. Fig. 7 shows the evacuation time as a function of number of open exits. Increasing the number of open exits drastically decreases the total evacuation time, however 3 open exits gives the optimal results. Number of passengers (50, 80, 100, 120, 150, 180) have been varied for all the cases (case 1 to case 8). In addition to these case studies, we have also studied average distance travelled by each passenger during evacuation and how it varies with initial passenger distribution as well number of open exits. More studies are required to make a conclusion on this issue and the associate spatial pattern. We also see that, average evacuation time per passenger decreases as the number of passengers increase when two or more gates are open.

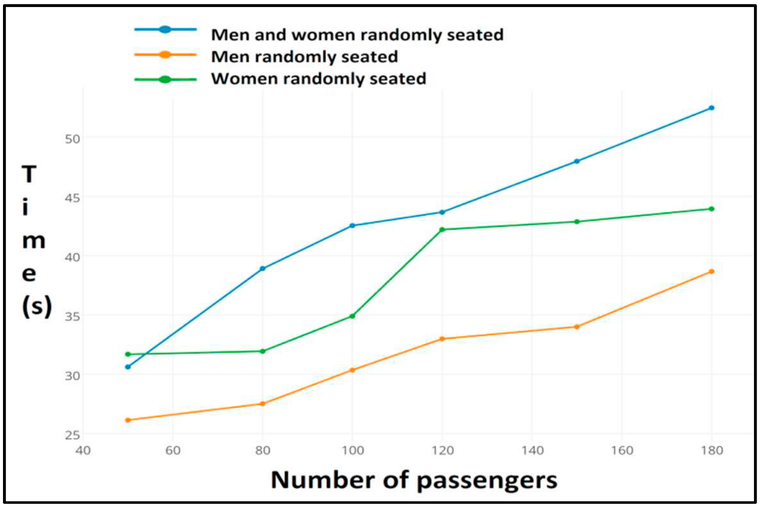

Figure 6: Number of Passengers Vs Time taken to evacuate

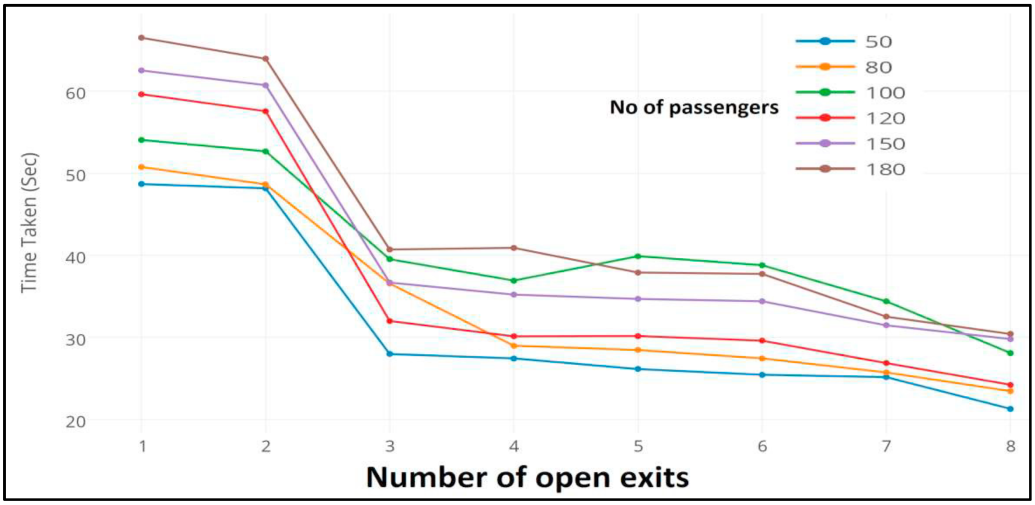

Figure 7: Number of Gates open vs time taken to evacuate

5. Conclusions

The cellular automata model has been considered to develop an initial framework for replicating Aircraft Evacuation real case scenario. In this model, several parameters: age, sex and agility as the passengers attributes, male and female distribution, speed time to evacuate are taken into account but width of passengers are represented uniformly.The CUDA framework for the parallel implementation of our algorithm (code) which can be executed on GPUs (graphics processing unit) has been used. Future improvements are required to integrate passenger behaviours (fear, anxiety, panic, group dynamics, family members, disabled persons, speed of each person), role and responsibility of Cabin crew, and Impact during evacuation (fire propagation, toxics gases). Aircraft Evacuation times have been compared with existing data collected by Aircraft manufacturers. Detailed investigations reveal useful information on the relationship between Evacuation time and important attributes such as passenger age and sex, number of operational exit gates, passenger distribution etc. The simulator facilitates the investigation of spatial and temporal movement of the passengers, as well as the visualization of pattern formation and collective behavior. The data analyzed through the simulation has provided promising evidence that with integration of additional parameters (such as Passenger behaviour, Role and responsibility of Cabin crew and Impact during emergency situation: fire, toxic gases) a robust, fast and accurate computational tool of Aircraft evacuation process can be developed.

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