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# Solving aircraft conflicts by continuous optimization and mixed-integer nonlinear programming

Sonia Cafieri<sup>1</sup>, Andrew Conn<sup>2</sup>, Marcel Mongeau<sup>1</sup>

<sup>1</sup> ENAC, Université de Toulouse  
{sonia.cafieri,marcel.mongeau}@enac.fr

<sup>2</sup> IBM Watson Research Center, NY  
arconn@us.ibm.com

**Mots-clés** : *Penalty function approach; Mixed-Integer Nonlinear Programming; Aircraft conflict avoidance; Continuous optimization*

## Introduction

Aircraft conflict avoidance for en-route flights involves ensuring some standard separation (typically set at 5 nautical miles) between aircraft. It is crucial to ensure flight safety and remains a challenging problem in Air Traffic Management (ATM). Taking into account the increasing air traffic on the world scale and its impact on air traffic controllers' workload, a higher level of automation in ATM urgently needs to be introduced. In the present work, we propose two novel optimization formulations, where the decision levers are both aircraft speed changes and heading angle changes. Both formulations exploit the removal of the infinite-dimensional feature of the separation constraint using a method introduced in [1], and rely on a linearization of angle-related nonlinear terms.

## 1 Optimization models

The first formulation is based on Mixed-Integer Nonlinear Programming (MINLP). MINLP enables the simultaneous consideration of continuous variables (aircraft speeds, heading angles, etc.) as well as integer ones (in particular, binary, to model logical choices), and to model the inherently combinatorial pairwise nonlinear (more precisely : non-convex quadratic) separation constraints.

The second formulation we propose is a purely continuous optimization model, where we introduce an exact  $l_1$ -penalty function, tailored to the problem at hand, to deal with the difficult aircraft separation constraints.

A variant of the introduced models, that aims at increasing the aircraft separation distance, results in robust solutions for air traffic controllers. Contrary to the above formulations, this variant does not attempt at minimizing the (complex nonlinear) deviation of the aircraft (with respect to its initial speed and heading angle), concentrating only on maximizing the (linear) separation distance between aircraft.

## 2 Results and perspectives

Numerical results on a set of problem instances validate the approaches while highlighting the versatility of the proposed models.

The first proposed model is solved to global optimality by the spatial branch-and-bound method implemented in COUENNE. The continuous  $l_1$ -penalty model is solved extremely rapidly to local optimality by the local-optimization nonlinear interior-point solver IPOPT. CPU times

for the robust, separation-maximization variant of the MINLP model are impressively better than when minimizing the deviation.

The overall proposed optimization approach is expected to serve to design a decision-aid tool for air-traffic controllers to solve conflicts at the *tactical level* (i.e., 20 minutes before conflicts). An original graphical view (2D + time space) of the solutions obtained will be presented at the conference.

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## Références

- [1] S. Cafieri and N. Durand. Aircraft deconfliction with speed regulation : New models from mixed-integer optimization. *Journal of Global Optimization*, 58(4) :613–629, 2014.