Optimization of military missions impact on civilian 4D trajectories

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Abstract: Flying is the fastest and one of the safest methods of transport. Air traffic is increasing constantly and according to statistics from ICAO and aviation industry, approximately every 15 years traffic is doubling and as the consequence the airspace becomes more and more congested. In order to cope with the increase and to improve safety, air traffic management system will rely on a so-called trajectory based operations concept that will increase air traffic capacity by reducing the controllers workload. In addition to the increase of traffic of civil aviation, we have to take into consideration requirements from military operations and their needs for missions in the airspace. In order to fulfill military mission requests, we shall minimize its impact into civil aviation whenever possible. This will be achieved by transferring tactical conflict detection and resolution tasks to the planning phase. In this future air traffic management paradigm context, this paper presents a methodology to address such military missions planning. The proposed methodology aims at minimizing the interaction of trajectories of military missions into civilian trajectories. Results that we gain through this methodology are by allocating alternative departure times, alternative horizontal flight paths, and alternative flight levels to the military missions involved in the interaction. This paper presents a mathematical formulation of this mission trajectory planning problem leading to a mixed-integer optimization problem, whose objective function relies on the new concept of interaction between trajectories. A computationally efficient algorithm to compute interaction between military missions trajectories with minimal impact into civilian trajectories for large-scale applications is presented. Resolution method based on simulated annealing algorithm have been developed to solve the above optimization problems. Finally, the methodology is implemented and tested with military missions planed to occur in real air traffic data taking into account uncertainty over the French airspace. Mission conflict-free and robust 4D trajectory planning are produced within computational time acceptable for the operation context, which shows the viability of the approach.

Keywords: Military area, optimization, mission trajectory, airspace.

1 Introduction

Historically, State agreements between military aviation units and Air Navigation Service Provider (ANSP) have focused on the needs of State defense, security and emergency procedures as well as military readiness and response requirements. There is now a clearly defined need to establish techniques that support the efficient integration of military and civil aviation in day-to-day operations. Aviation operations of all types contribute significantly to the economy of a State, and, as such, their growth needs to be protected and encouraged. The Flexible Use of Airspace (FUA) concept provides that airspace should no longer be designated as either military or civil airspace but should be considered as one continuum and used flexibly on a day-to-day basis, in which all user requirements are accommodated to the greatest possible extent. [1]

In light of this, a need for the real-time use of airspace allowing a safe separation between civil and military aircraft has been clearly identified. In this respect, European Commission has issued regulation in order to reiterate Flexible Use of the Airspace (FUA). To this end, Eurocontrol has provided Airspace Concept and Strategy for the ECAC Area, which provides targets for airspace developments. In this document, the concept of Dynamic Airspace Allocation (adapt-

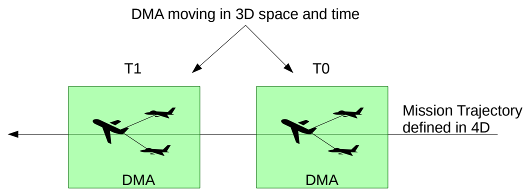

Figure 1 Example of DMA concept applied for tanker's mission trajectory.

able airspace allocation moving in time and place) is required to respond in real-time to changing situation in traffic flows and to short-term changes of users intentions.

The European air traffic management modernization program SESAR also proposes the concept of *Dynamic Mobile Area* (DMA) that is described by integral part of the *Mission Trajectory* (MT) described by a 4D data set. The DMA defines the volume of airspace that satisfies specific requirements from different Airspace Users. Example of the DMA for aerial refueling mission is illustrated in Fig. 1.

In this paper, we are addressing the optimization of a mission trajectory and its corresponding DMA, in order to minimize its interaction with the civilian trajectories.

Air traffic regulations impose that aircraft must always be separated by some prescribed distance. Aircraft are considered to be in *conflict* when these minimum separation requirements are violated. As the global air traffic demand keeps on increasing, congestion problem becomes more and more critical. One of the key solutions is to balance the air traffic demand and the overall capacity of the civil/military Air Traffic Management (ATM) system. In order to cope with the increasing demand, the future ATM system will rely on the trajectory based operations concept. In this concept, aircraft will be required to follow a negotiated conflict-free trajectory, accurately defined in 4 dimensions (3 spatial dimensions and time) in order to reduce the need of controller's intervention during the tactical phase. In this perspective, the key factor to improve the ATM capacity and to accommodate military needs is an efficient strategic 4D trajectory planning methodology to compute a conflict-free 4D mission trajectory for each aircraft.

In this work, we propose a methodology to address such a strategic planning of trajectories at national scale. The goal of the proposed method is to minimize interaction (in both the three dimensional space and in the time domain) between the MT and civil trajectories, by allocating an alternative horizontal flight path, departure time, and flight level to the MT. An interaction between trajectories occurs when two or more trajectories have an effect on each other; for instance, when trajectories occupy the same space at the same period of time. Therefore, contrary to the concept of conflict, the measurement of interaction does not only refer to the violation of minimum separation requirements. It also allows us to take into account other separation criteria, such as separation between the civil trajectory and the DMA.

The following sections of this paper is organized as follows. Section 2 presents mathematical model of the proposed methodology. Section 3 describes resolution algorithm based on simulated annealing. Implementation of the resolution algorithm to our proposed mission trajectory planning is presented in Section 4. Section 5 reports the results from computational experiments. Finally, conclusion is discussed in Section 6.

2 Mathematical Modeling

In order to develop a resolution algorithm, the first stage in the optimization process consists of modeling the real problem using a mathematical abstraction, which is as faithful as possible. Using this abstraction, it becomes possible to develop solution algorithms which can be executed on a computer. The modeling stage, then, consists of characterizing the state space and the objective space.

2.1 Data

Before establishing the mathematical model, one must first consider the given data associated to the problem. We consider a given airspace and a mission trajectory (MT) that have to be designed between two points A and B. The shape and the size of the dynamic mobile area (DMA) that has to be routed between A and B is also given(see Fig. 2). In the following, this area will be called *DMA*.

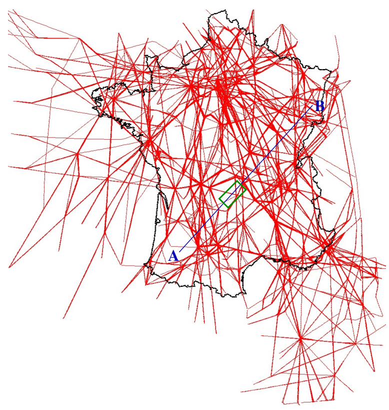

Figure 2 Initial mission trajectory connecting origin A to destination B in a given airspace.The moving area is represented by the green rectangle on the figure.

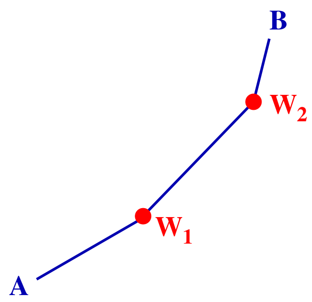

Figure 3 The shape of the mission trajectory is changed by using two waypoints (w_1,w_2).

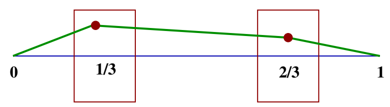

Figure 4 Waypoints are located into two rectangular areas. The center of the first one is located at $\frac{1}{3}$ and the second one $\frac{1}{3}$.

In a given airspace we consider a set of civilian aircraft trajectories (red traffic on Fig. 2) that will be noted Γ. One element of this set will be noted $\gamma_i(t)$ ($i \in [1, N]$ and $N = |\Gamma|$).

In this preliminary work, a direct route has been considered but the approach could easily extended to more complex mission trajectories. The current, the minimum and the maximum departure time of the mission will be noted ts_0, ts_{min}, ts_{max}. We will consider a constant speed for the mission (v_0). Similarly, the current, the minimum and the maximum flight level of the mission will be noted FL_0, FL_{min}, FL_{max}.

2.2 State Space

The mission trajectory will be noted $\gamma_{DMA}(t)$. We consider only en-route airspace and we propose to design this trajectory in terms of route (horizontal flight path) design, flight level and time of departure assignment. This represent the state space of our problem. In this problem, three decision variables will be considered :

1. Time of departure of the mission :
$$t_s \in [ts_{min}, ts_{max}].$$
2. Flight level of the mission :
$$FL \in [FL_{min}, FL_{max}].$$
3. Shape of the mission trajectory : γ_{DMA}.

Common practice conducted us to rely on a discretization of the departure time t_s and the flight level FL of the mission. Let δ_t be the departure-time-shift step size such that $t_s = ts_0 + \delta_t$. This yields $N_t = (ts_{max} - ts_{min} + 1)/\delta_t$ possible departure time of the mission. Similarly, let δ_{FL} be the flight level shift, this yields $N_{FL} = (FL_{max} - FL_{min} + 1)/\delta_{FL}$ possible flight level for the mission.

In our model, the shape of the mission trajectory is designed thanks to two waypoints (W_1,W_2) which are added along the route (see Fig. 3).

In order to locate such waypoints, we first build a unit segment (length = 1) for which we consider two rectangular areas (see figure 4). In each of those rectangles, we put one waypoint (w_1 and w_2) in order to design the initial shape. The size of the rectangle in the vertical dimension is fixed in order to limit the maximum deviation of the mission trajectory in terms of distance.

This initial shape is then scaled to the length of the segment $[A, B]$ and rotated by an angle θ corresponding to the direction of the segment $[A, B]$ (see figure 5).

For this rotation, we use the classical rotation matrix for computing the new coordinated of the waypoint in the airspace (W_1,W_2) :

$$\begin{bmatrix} x_{new} \\ y_{new} \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x_{old} \\ y_{old} \end{bmatrix}$$

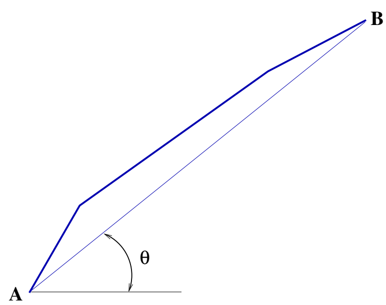

Figure 5 Computed shape of the mission after scaling and rotation.

2.3 Objective

The first objective of the mission planing is to minimize the impact of the military area (DMA) on the civilian traffic.

Let $\gamma_{DMA}(t)$ be the position of the center of the DMA at time t, one must consider the civilian trajectories in intersection with the such DMA at that time. We note $DMA(\gamma_{DMA}(t))$ the area for which the center is located at $\gamma_{DMA}(t)$. In this preliminary work, the DMA with is defined by a rectangular box with width (w_{DMA}), length (l_{DMA}), and height (h_{DMA}) to be set by the user.

If we note $y(t)$ as a number of civilian trajectories in intersection with the DMA area, then :

$$y(t) = \sum i = 1^{i=N} \gamma_i(t) \in DMA(\gamma_{DMA}(t))$$

If we note t_e the time when the mission end (the DMA area reach the point B), the we can compute the overall objective function :

$$y = \int_{t=t_s}^{t=t_e} y(t)$$

This objective has to be minimize. Example of interaction between civilian traffic and the mission trajectory is illustrated in Fig. 6.

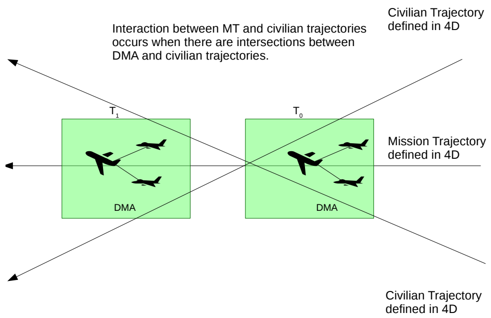

Figure 6 Example of interaction between two civilian trajectories with mission trajectory.

Finally, the proposed mission trajectory optimization problem can be formulated as a mixed-integer optimization problem as follows:

$$\min_{t_s, FL, w} y$$

subject to

$$t_s \in [ts_{min}, ts_{max}],$$
$$FL \in [FL_{min}, FL_{max}],$$
$$w_i \in W_i,$$

where w is the vector of waypoint w_i for $i = 1, \dots, M$, where M is the number of waypoints that that are allowed to be used, and W_i is the set that define feasible locations of the waypoints w_i for $i = 1, \dots, M$.

3 Resolution algorithm: Simulated Annealing

Simulated Annealing (SA) is one of the simplest and best-known meta-heuristic methods for addressing the difficult *black box* global optimization problems (those whose objective function is not explicitly given and can only be evaluated via some costly computer simulation). It is massively used on real-life applications.

In the early 1980s three IBM researchers, Kirkpatrick, Gelatt and Vecchi [9], introduced the concepts of annealing in combinatorial optimization.

These concepts are based on a strong analogy with the physical annealing of materials. This process involves bringing a solid to a low energy state after raising its temperature. It can be summarized by the following two steps:

- Bring the solid to a very high temperature until "melting" of the structure;
- Cooling the solid according to a very particular temperature decreasing scheme in order to reach a solid state of minimum energy.

In the liquid phase, the particles are distributed randomly. It is shown that the minimum-energy state is reached provided that the initial temperature is sufficiently high and the cooling time is sufficiently long. If this is not the case, the solid will be found in a metastable state with non-minimal energy; this is referred to as *hardening*, which consists in the sudden cooling of a solid.

In 1953, three American researchers (Metropolis, Rosenbluth, and Teller [11]) developed an algorithm to simulate the physical annealing. Their aim was to reproduce faithfully the evolution of the physical structure of a material undergoing annealing. This algorithm is based on Monte Carlo techniques which consist in generating a sequence of states of the solid in the following way.

Starting from an initial state i of energy E_i, a new state j of energy E_j is generated by modifying the position of one particle.

If the energy difference, $E_i - E_j$, is positive (the new state features lower energy), the state j becomes the new current state. If the energy difference is less than or equal to zero, then the probability that the state j becomes the current state is given by:

$$Pr\{\text{Current state} = j\} = \exp\left(\frac{E_i - E_j}{k_b.T}\right),$$

where T represents the temperature of the solid and k_B is the Boltzmann constant ($k_B = 1.38 \times 10^{-23}$ joule/Kelvin). The acceptance criterion of the new state is called the *Metropolis criterion*.

In the SA algorithm, the Metropolis algorithm is applied to generate a sequence of solutions in the state space S. To do this, an analogy is made between a multi-particle system and our optimization problem by using the following equivalences:

- The state-space points represent the possible states of the solid;
- The function to be minimized represents the energy of the solid.

A control parameter c, acting as a temperature, is then introduced. This parameter is homogeneous to the criterion that is optimized.

It is also assumed that the user provides for each point of the state space, a neighborhood and a mechanism for generating a solution in this neighborhood. We then define the acceptance principle:

The *acceptance criterion* for accepting solution j from the current solution i is given by the following probability:

$$Pr\{\text{ accept } j\} = \begin{cases} 1 & \text{if } f(j) < f(i) \\ \exp\left(\frac{f(i)-f(j)}{c}\right) & \text{else.} \end{cases}$$

Let i be the current state space, j be the neighbor of i defined by a given neighborhood function. And let k be the temperature step, L_k be the number of iterations to be performed at each temperature step, and c_k be the temperature at the k^{th} step. The simulated annealing can be summarized the following way :

Simulated annealing

1. **Initialization** $i := i_{start}$**,** $k := 0$**,** $c_k = c_0$**,** $L_k := L_0$**);**
2. **Repeat**
3. **For** $l = 0$ **to** L_k **do**
   - **Generate a solution** j **from the neighborhood** S_i **of the current solution** i**;**
   - **If** $f(j) < f(i)$ **then** $i := j$ **(**j **becomes the current solution);**
   - **Else,** j **becomes the current solution with probability** $e^{\left(\frac{f(i)-f(j)}{c_k}\right)}$**;**
4. $k := k + 1$**;**
5. **Compute(**L_k, c_k**);**
6. **Until** $c_k \simeq 0$

One of the main features of simulated annealing is its ability to accept transitions that degrade the objective function.

At the beginning of the process, the value of the temperature c_k is high, which makes it possible to accept transitions with high criterion degradation, and thereby to explore the state space thoroughly. As c_k decreases, only the transitions improving the criterion, or with a low criterion deterioration, are accepted. Finally, when c_k tends to zero, no deterioration of the criterion is accepted.

Simulated annealing has been applied to many highly combinatorial problems coming from industry and operations; to mention a few:

- Airline crew scheduling [6]
- Railway crew scheduling [7]
- Traveling salesman problem [2]
- Vehicle routing problem [10]
- Layout-routing of electronic circuits [**?**]
- Large scale aircraft trajectory planing [3, 8]
- Complex portfolio problem [5]
- Graph coloring problem [4]
- High-dimensionality minimization problems [12]

Having introduced the simulated annealing, the next section presents the implementation of such algorithm to our optimization problem.

4 Algorithm Implementation

The overall structure of the resolution algorithm can be summarized by Fig. 7. It consists of two main components which are:

- Simulation environment module. Given input data and a candidate solution (state space), the simulation compute the corresponding mission trajectory and DMA. Then, place them in the airspace. After that it computes the value of objective function.
- Optimization module control the search for optimal solution. It evaluate the decision variables, and apply acceptation criteria. This process is iterated until the maximum number of interaction or the stopping criteria is fulfilled.

The algorithm proposes some decision variables (time of departure t_s, flight level FL, and route of the mission γ) as a candidate solution. Then, it is used to build the mission trajectory that will be placed in the airspace in order to compute the associated objective function.

The process to simulate the mission trajectory and the DMA in the current airspace can be described as follows. Once the point of the state space ($\vec{X}$) is defined :

- the time of departure of the mission : ts
- the cruise FL :fl
- the two waypoints in the unit segment : $\vec{w}_1$,$\vec{w}_2$

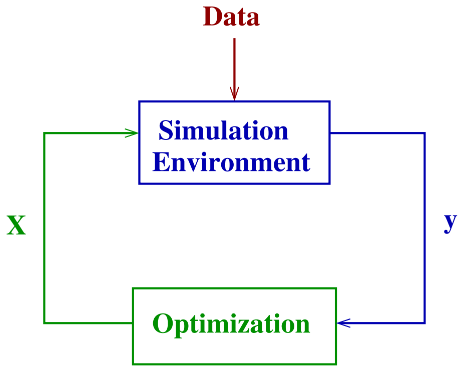

Figure 7 Objective-function evaluation based on a simulation process

Then, $\vec{X} = [ts, FL, \vec{w}_1, \vec{w}_2]$. The algorithm locate the waypoint in the airspace by scaling and rotating the waypoint $\vec{w}_1, \vec{w}_2$ in order to create $\vec{W}_1, \vec{W}_2$. Based on those airspace waypoint an can build the route of the mission. Having the route, the time of departure (ts) the flight level (fl) and the speed of the mission (v_0), on can compute the mission trajectory ($\gamma_{dma}(t)$), which is sampled every 15 seconds. This represents the trajectory on the center of the DMA area. This trajectory is then included in the airspace for evaluation.

In order to run the simulated annealing algorithm, a neighborhood operator is needed. For our application, the algorithm randomly select one of the feature of the state space (time of departure, flight level or route) and apply the following changes :

- Time of departure : $ts^{new} = ts^{old} + \delta$; $delta$ being a random variable; one must ensure that $ts^{new} \in [ts_{min}, ts_{max}]$.
- Flight Level : $fl^{new} = fl^{old} + u$ where is a random variable having values in $-1; +1$; one must ensure that $fl^{new} \in [fl_{min}, fl_{max}]$.
- Route : a waypoint w_i to be modified is first selected. The position of this waypoint is randomly changed in the associated rectangle W_i.

Having developed all the materials for implementing the simulated annealing algorithm to our problem, the next section presents the associated results to some realistic instances of the problem.

5 RESULTS

The simulated annealing algorithm adapted to solve the mission trajectory optimization problem is implemented in Java.

It is tested with two different missions and traffic sets over the French airspace. The overall methodology is implemented on a UNIX platform with 2.40GHz 4 processor and 20 GB DDR4 RAM (personal computer).

The algorithm has been first applied to the French airspace with simulated civilian traffic of 8,836 flights considering en-route traffic. Each trajectory being sampled every fifteen seconds, each sampled point is defined by four coordinates (x, y, z, t). This civilian traffic set is illustrated as blue lines on Fig. 8.

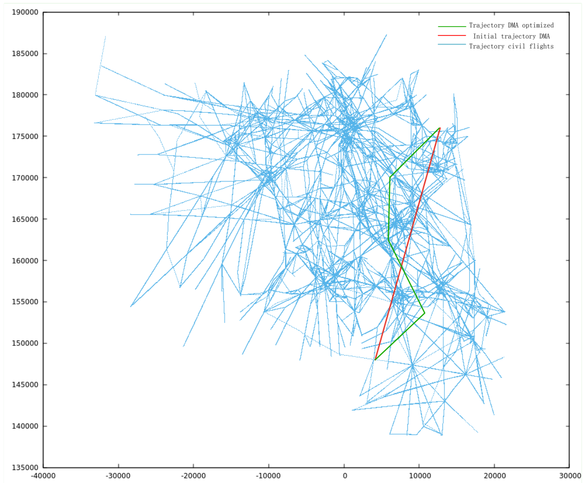

Figure 8 Simulated France traffic consists of 8,836 flights, initial mission trajectory, and optimized mission trajectory.

The traffic is projected to Euclidean plane, and then normalized using separation standard. The initial mission trajectory and the DMA are set as follows:

- The mission is originated at coordinates (40,351) and destination is (13, 462) (in the normalized unit).
- The initial flight level, FL_0, is set to FL350;
- The initial departure time, ts_0 is set to 04:10:00;
- The size of the DMA is as: l_{DMA} = 45 NM, w_{DMA} = 25 NM, and h_{DMA} = 5,000 feet.

To solve this problem, we set the parameters that define the problem as given in Table 1, and empirically set the parameters that define the simulated annealing algorithm as given in Table 2.

The initial mission trajectory involves in interactions with 167 civilian flights (y_{init} = 167). The resolution algorithm was able to find interaction-free solution within 48 seconds. The interaction-free mission trajectory is shown as green line in Fig. 8.

The evolution of the value of objective function for this data set is shown in Fig. 9. One can observe that in the beginning, the algorithm accepts solution that yields higher value of objective function. Then,

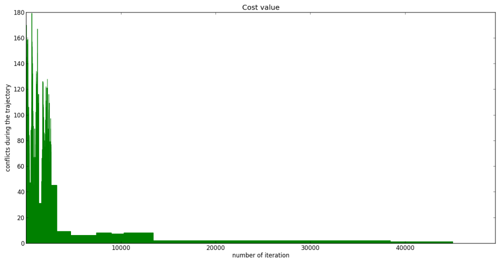

Figure 9 Evolution of the objective function value for data set consists of 8,836 civilian flights.

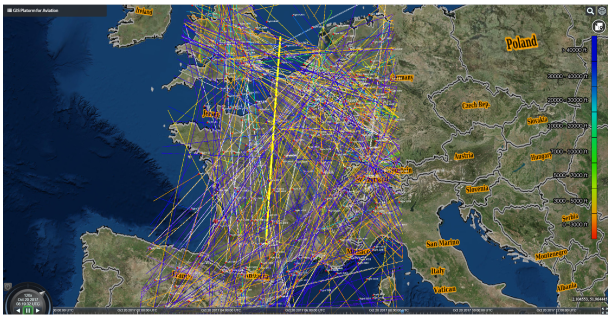

Figure 10 Simulated France traffic consists of 20,734 flights and initial mission trajectory (in thick yellow line).

Parameters	Value
Minimum flight level FL_{min}	$FL_0 - 20$
Maximum flight level FL_{max}	$FL_0 + 20$
Flight level shift δ_{FL}	10
Minimum departure time ts_{min}	$ts_0 - 8$ minutes
Maximum departure time ts_{max}	$ts_0 + 8$ minutes
Departure time shift δ_t	1 minutes
Maximum route length extension	12%

Table 1 Parameters that define the problem.

Parameters	Value
Number of iterations L_k	600
Initial temperature c_0	3,284.2
Temperature decreasing scheme	$c_{k+1} = 0.99 * c_k$
Stopping criteria	$c_k < 0.0001 * c_0$

Table 2 Parameters that define the SA.

this acceptation becomes less and less, until the algorithm is reached towards interaction-free solution.

Then, the resolution algorithm is applied to a larger set of simulated civilian traffic consists of 20,734 flights as illustrated in Fig. 10. The initial mission trajectory and the DMA are set as follows:

- The mission is originated from (44°N,2°E). The destination is set to (55°N,2°E).
- The initial flight level, FL_0, is set to FL370;
- The initial departure time, ts_0 is set to 08:30:00;
- The size of the DMA is as: l_{DMA} = 60 NM, w_{DMA} = 40 NM, and h_{DMA} = 4,000 feet.

The initial mission trajectory interacts with 732 civilian flights (y_{init} = 732). To solve this problem, we use the same parameters that defined the problem and the resolution algorithm as given in Table 1 and 2, except the number of iteration L_k is set to 1,000.

Again, the SA was able to find interaction-free solution for this larger set of civilian traffic within 289 seconds. The interaction-free mission trajectory is shown as thick yellow line in Fig. 11. And the evolution of the objective function value is shown in Fig. 12. Similar to the previous case,the algorithm accepts solution that yields higher value of objective function in the beginning, and then, the rate of acceptation becomes less and less, until the algorithm is reached towards interaction-free solution.

6 CONCLUSION

We have introduced a methodology to address mission trajectory planning in the framework of future airspace management concept that focus on increasing flexibility of the airspace usage by allocating airspace more dynamically, while satisfying the needs of both military and civilian airspace users.

The proposed mission trajectory planning methodology focuses on minimizing the interaction between military mission and civilian air traffic. The interaction is minimized by allocating alternative departure time, flight level, and horizontal flight path to the military mission.

The proposed model is formulated as a mixed-integer optimization problem, for which a resolution algorithm based on simulated annealing has been developed.

Computational experiments on a day instance of traffic over the French airspace, involving up to ≈ 20,000 flights, shows that the proposed methodology is able to find interaction-free mission trajectory.

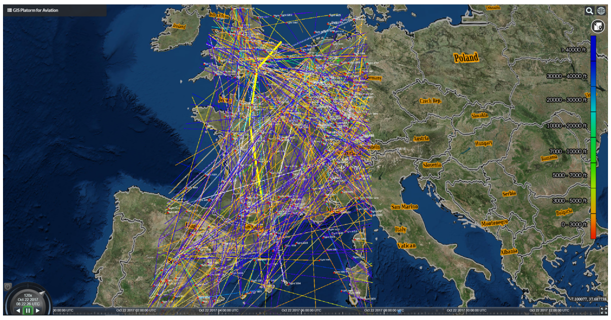

Figure 11 Simulated France traffic consists of 20,734 flights and initial mission trajectory (in thick yellow line).

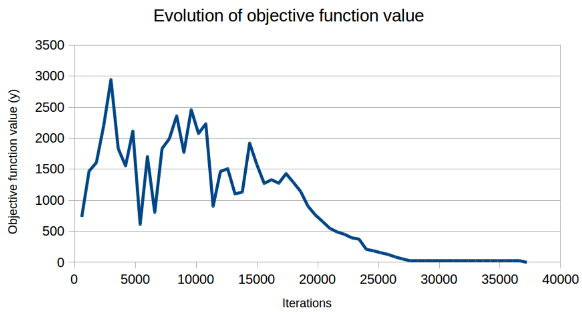

Figure 12 Evolution of the objective function value for data set consists of 20,734 civilian flights.

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