Harmonic and Biharmonic Navigation Functions for Conflict-free Trajectory Planning

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Goal: Compute a trajectory for one aircraft with theoretical proof of **static obstacle avoidance**. The obtained trajectories also have to satisfy Air Traffic Management requirements:

- Bounded speed
- Bounded curvature

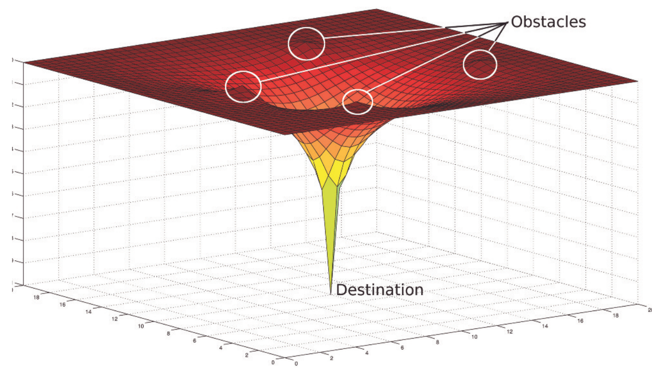
Navigation functions

Navigation functions are maps of values:

- uniformly maximum on the obstacles
- minimum at the destination
- continuous on the space

By following the decreasing values of the navigation function, the aircraft reaches the destination and avoids all the obstacles.

Harmonic functions satisfy the three properties above and are less-suited to be navigation functions. Their drawback is in the lack of bounds on the speed of the aircraft. To tackle this problem, we created the **Biharmonic Navigation Function**.



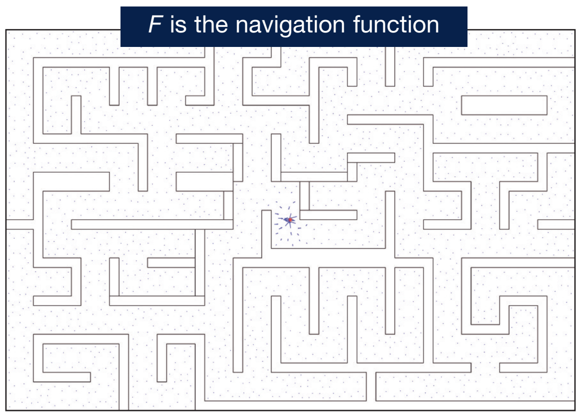
Example of navigation function

Existing methods

Harmonic Dirichlet

$$\begin{cases} \Delta F = 0 & \text{on the space} \\ F = T,\ T > 0 & \text{on the obstacles} \\ F = 0 & \text{at the destination} \end{cases}$$

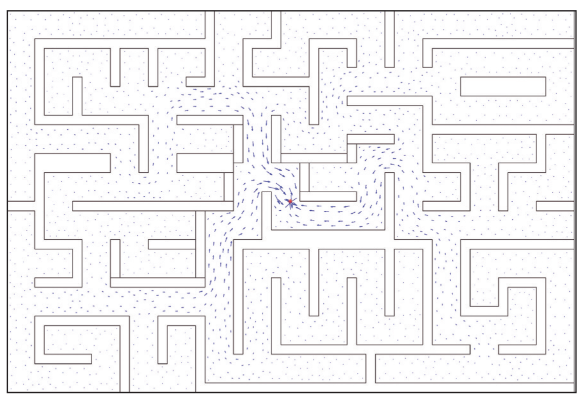
F is the navigation function



Evanescent field: the destination is unreachable from some points

Harmonic Neumann

$$\begin{cases} \Delta F = 0 & \text{on the space} \\ \dfrac{\partial F}{\partial n} = S,\ S < 0 & \text{on the obstacles} \\ F = 0 & \text{at the destination} \end{cases}$$



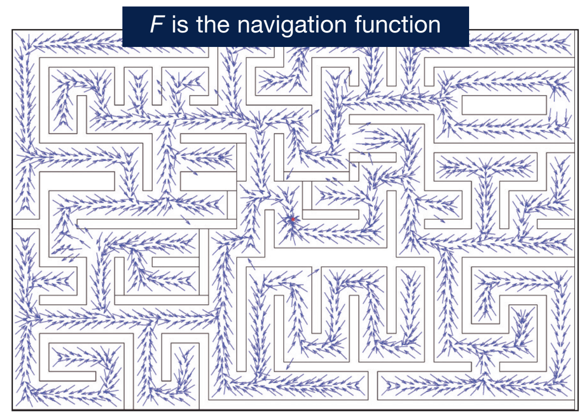
The aircraft speed increases when the aircraft comes closer to the destination

New method

Biharmonic

$$\begin{cases} \Delta^2 F = 0 & \text{on the space} \\ \Delta F = 0,\ F = 0 & \text{on the obstacles} \\ \Delta F = S,\ F = T & \text{at the destination} \end{cases}$$

F is the navigation function

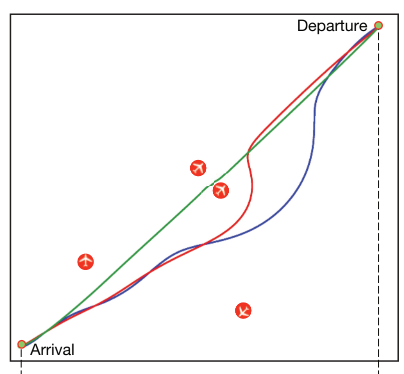


Constant speed

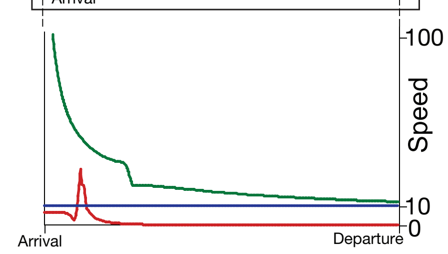
Comparison of trajectories

	ATM requirements	Dirichlet	Neumann	Biharmonic
Provable	Obstacle avoidance	✓	✓	✓
Provable	Bounded Speed			✓
Provable	Bounded Curvature	?	?	?
Observed	Robustness to uncertainties	✓		✓
Observed	Limited Congestion			✓

Trajectories



Speed profile along the trajectories



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