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Fuzzy Risk Assessment for Airport Strategic Planning

Elena M. Capitanul, H. Alfazari, Carlos A. Nunes Cosenza, W.El Moudani, F Mora-Camino

Abstract—Airports are asset-intensive businesses that require a large amount of time to recover the significant financial investments in specific infrastructure such as runways and terminals. Airports investors must perform strategic moves based on calculated risks before taking investment decisions. This communication puts forward a new approach for airport investment risk assessment. The approach takes explicitly into account the degree of uncertainty in activity levels prediction and proposes milestones for the different stages of the project for minimizing risk. Uncertainty is represented through fuzzy dual theory and risk management is performed using dynamic programming.

Keywords—airports, financial risk assessment, uncertainty, fuzzy dual, dynamic programming.

I. INTRODUCTION

Airports are a paramount piece of the economic puzzle with a multiplier economic, social and environmental impact at national, regional and international level. In a highly volatile and uncertain economic environment, airports must be capable to attract sufficient revenues to finance their operations and investments while maintaining a satisfactory quality of service for both their primary clients: airlines and passengers, while maintaining their role as economic drivers supporting in a sustainable manner the local community.

Airports are asset-intensive businesses that require extensive amount of time to recover the significant financial investments in the specific infrastructure, like runways, terminals. This aspect forces airports investors to make strategic moves and to carefully calculate the risks before taking investment decisions. The highly deregulated and liberalized air transportation market determined airports to adopt a more business like operational approach, focusing on non-aeronautical activities as a strategy to achieve self-reliance and financial independence, which will allow them to develop in accordance with the market needs. This process of airport commercialization transformed the passenger as the ultimate beneficiary of airport infrastructure.

In the last decades, airports evolved from being simply infrastructure elements to business oriented service providers, pressured to operate in an optimal manner. They proved to be flexible in turbulent economic times proving they had the

capability to meet the needs of the air transportation industry, sector that has known a sustained high rate of growth of approximately 5% annually in the last decades even through global economic disturbances, with more than 3 billion passengers transported in 2013 [1].

The structure of the article is as follows: section 2 provides a general background for long-term airport planning, section 3 introduces a concise mathematical formulation of the long term airport planning problem with emphasis on the financial aspects and uncertainty degree. In Section 4, a mathematical model is proposed to address airport investment risk assessment. In Section 5 a fuzzy dual dynamic programming approach is discussed to tackle an airport case study. Final conclusions are presented in Section 6.

II. THE AIRPORT PLANNING PROBLEM

As the world economy is going through successive economic downturns, the air transport industry is expected to continue to grow steadily on the long run. Following this trend, airports are expected to expand accordingly.

Airport planning is in general a long term planning issue which has at its core the following objectives: optimized infrastructure development costs and functionality, optimized economic and operational performance and a high degree of flexibility in order to integrate all the shifts in demand and potential disturbances according to the airport future needs and level of growth. The new business culture concepts that airports need to embrace includes strong air service competitor advantages, capability of taking long-term risks, adopting the stakeholder collaborative decision making culture, diversifying the revenues sources and most of all putting the passenger at the core of the business.

The construction of a new airport or the extension of an existing one requires significant investments and many times public-private partnerships are considered in order to make feasible such projects. One characteristic of these projects is uncertainty with respect to financial and environmental impact on the medium to long term. Another one is the multistage nature of these types of projects. While many airport development projects have been a success like Munich Airport or Palma de Mallorca Airport, some others have turned into a nightmare for their promoters.

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Airports were traditionally seen as the responsibility of governments to manage and operate, typically in line with strategic economic and defense policies [2]. In the more recent economic environment, a paradigm shift occurred where private stakeholders emerged as investors evolving from decision makers in airport planning and development to full owners and operators. Privatization of airports emerged as the tool “to go to” for governments looking for strategies to make the local aviation market more dynamic and to achieve their long term planning goals when the costs of funding new infrastructure or maintaining the existing one exceeds their resources. The privatization of airports makes for a governance space where different governance modes intersect and overlap as noted by Donnet and Keast [3].

The long-term airport planning process is a complex endeavor due to the intricacies of the airport system, stakeholders involved and the significant degree of uncertainty. In a highly volatile economic context, the planning process needs to be constantly adjusted to the realities of the market the airport will serve. Quantities such as “demand” and “capacity” need to be re-thought in a dynamic context to compute the operational parameters of the future airport. The fact that long term airport planning is a multibillion-business investment requiring a systemic and flexible approach must be acknowledged.

The demand for air transport services has risen much faster than demand for most other goods and services in the world economy. Since 1970 air travel demand, measured by Revenue Passenger Kilometers flown (RPKs) has increased ten times compared to a three-four expansion of the world economy. Along the same period, international passenger and cargo demand, both reflecting and facilitating the globalization of business supply chains and economies generally, was multiplied forty times [4].

III. GENERAL FORMULATION OF THE AIRPORT PLANNING PROBLEM

A. The Planning Context

The starting point of any airport planning project and its financing are its current state and the potential demand evolution forecast. The forecast generally covers the time horizon of the project and includes potential demands for the annual volumes of international and domestic scheduled and nonscheduled passengers, freight and aircraft movements. Also, daily and monthly traffic distributions are required in order to identify traffic trends and peaking patterns along with the fleet mix. Of paramount importance is the integration of uncertainty in demand forecasting since the decisions taken at a specific step of the development plan can have a long term impact over the general outcome of the project.

Long term airport planning can expand up to 20 years as a time horizon with a proposed six months incremental milestone in order to accurately monitor the progress of the development project. In this way, an important degree of adaptability will allow airport planners to take better-informed decisions over a more controllable period.

B. Adopted Assumptions

Different traffic types leading to costs and revenues can be

considered in airports, they cover passengers and freight flows as well as aircraft traffic that is related with the level of these flows. Let the level of predicted potential demand for traffic type i along the planning horizon K be given by $D_k^i, i \in I, k \in \{1, 2, \dots, K\}$, where I is the set of traffic activities. The necessary aircraft traffic T_k^i to cope with a predicted passenger demand level D_k^i , can be approximated by:

$$T_k^i = D_k^i / (S_k^i \alpha_k^i) \quad (1)$$

where S_k^i is the mean capacity of aircraft type i at time k corrected by the expected mean load factor α_k^i . The rate of return r_k^i , associated with the traffic of type i at time k , depends on the investments made until that period. Let the potential airport passenger processing capacity be C_k^{Pi} and the potential aircraft movements processing capacity be C_k^{Ti} then the estimated level of demand of type i at period k , \bar{D}_k^i , is such as:

$$\bar{D}_k^i = \min\{D_k^i, C_k^{Pi}, S_k^i C_k^{Ti}\} \quad (2)$$

Let L_i be the number of candidate upgrades that can be performed for traffic type i at the considered airport.

Let θ_l^i be the period (an integer) at which upgrade l for traffic type i is planned to be done. When a project is retained, the corresponding value of θ_l^i is within the set $\{1, 2, \dots, K\}$ and when it is not retained $\theta_l^i = K + 1, l \in \{1, 2, \dots, L_i\}$.

Different types of constraints may be found between interrelated projects:

1) Sequential constraints: technical considerations impose in general sequential constraints, so it is supposed that for given a type of traffic i and a pair of projects (l, l') , there may be constraints such as:

$$\exists l, l' \in \{1, \dots, L_i - 1\}, i \in I : \theta_l^i \leq \theta_{l'}^i \dots \dots \dots (3.a)$$

2) Exclusion constraints such as if project l for traffic type i is retained, a set of concurrent or contradictory projects will be dismissed:

$$\theta_l^i \in \{1, 2, \dots, K\} \Rightarrow \theta_{l'}^i = K + 1, l' \in \Lambda_l^i \subset \{1, \dots, L_i\} (3.b)$$

3) Inclusion constraints such as if project l for traffic type i is retained, a set of complementary projects related with other traffic should be performed altogether:

$$\theta_l^i \in \{1, 2, \dots, K\} \Rightarrow \theta_{l'}^i = \theta_l^i, l' \in M_l^i \subset \{1, \dots, L_j\} (3.c)$$

Since the different types of traffic may use common resources in the airport, global capacity constraints must be satisfied.

Let Δ_k be the set of projects which have been retained until period k , then the corresponding capacities with respect to passengers and flights are $C_k^{Pi}(\Delta_k)$ and $C_k^{Ti}(\Delta_k)$.

Let $c_l^{ik}(\Delta_k)$ be the cost of upgrade l with respect to traffic type i when performed at period k . Revenues R_k^i from traffic type i at period k are given by:

$$R_k^i = r_k^i \cdot \bar{D}_k^i(\Delta_k) \dots\dots\dots(4)$$

where r_k^i is the corresponding service rates.

C. Deterministic Problem Formulation

The adopted strategy develops at first a deterministic approach, which leads to the formulation of an optimization problem. Then the parameters and variables subject to significant uncertainty are pointed out and a fuzzy-dual based model of their uncertainty is established. Finally, a fuzzy dual formulation of the airport planning problem is proposed.

The deterministic formulation of the optimal programming problem associated to airport planning can be such as:

$$\max_{\theta_l^i} \pi([\theta_l^i], l \in \{1, \dots, L_i\}, i \in I \dots\dots\dots(5)$$

under constraints (3.a), (3.b) and (3.c).

Here the expected net present value of whole project is given by:

$$\pi([\theta_l^i], l \in \{1, \dots, L_i\}, i \in I = \sum_{i \in I} \left(\sum_{k=1}^K \left(\frac{r_k^i}{(1+\rho)^k} \bar{D}_k^i(\Delta_k) \right) - \sum_{\substack{l=1 \\ \theta_l^i \leq K}}^{L_i} \left(\frac{c_l^{ik}(\Delta_k)}{(1+\rho)^{\theta_l^i}} \right) \right) \dots\dots\dots(6)$$

where ρ is the rate of actualization.

Observe that, according to expression (2) the estimation of demand levels at period k will depend of previous planning decisions.

IV. AIRPORT PLANNING WITH EXPLICIT UNCERTAINTY

Here it is considered that uncertainty regarding the effective levels of demand, the rates of return and the upgrade costs can be represented by fuzzy dual numbers [5].

A. Fuzzy Dual Numbers

A set of fuzzy dual numbers is defined as the set $\tilde{\Delta}$ of numbers of the form $a+\varepsilon.b$, where a is the primal part and b is the dual part of the fuzzy dual number, $\forall a \in \mathfrak{R}, \forall b \in \mathfrak{R}^+$. Here ε represents the unity pure dual number. A fuzzy dual number loses both its dual and fuzzy attributes if b equals zero.

The lower and upper bounds of $a+\varepsilon b$ are given respectively by $B^{low}(a+\varepsilon.b) = a-b \dots\dots\dots(7)$

and $B^{high}(a+\varepsilon.b) = a+b \dots\dots\dots(8)$

The pseudo norm of a fuzzy dual number is given by:

$$\|a+\varepsilon.b\| = |a| + \rho.b \dots\dots\dots(9)$$

Here ρ is a real positively valued shape parameter given by:

$$\rho = (1/b) \int_{a-b}^{a+b} \mu(u) \cdot du \dots\dots\dots(10)$$

where μ is the membership function in the sense of Zadeh [6].

The following properties of the pseudo norm are met no matter the values the shape parameters take:

$$\forall a + \varepsilon \cdot b \in \tilde{\Delta} : \|a + \varepsilon \cdot b\| \geq 0 \dots\dots\dots(11)$$

$$\forall a \in \mathfrak{R}, \forall b \in \mathfrak{R}^+, \|a + \varepsilon \cdot b\| = 0 \Rightarrow a = b = 0 \dots\dots\dots(12)$$

$$\|(a + \varepsilon \cdot b) + (\alpha + \varepsilon \cdot \beta)\| \leq \dots\dots\dots(13)$$

$$\|a + \varepsilon \cdot b\| + \|\alpha + \varepsilon \cdot \beta\| \quad \forall a, \alpha, b, \beta \in \mathfrak{R}^+$$

$$\|\lambda \cdot (a + \varepsilon \cdot b)\| \dots\dots\dots(14)$$

$$= \lambda \cdot \|a + \varepsilon \cdot b\| \quad \forall a \in \mathfrak{R}, \forall b, \lambda \in \mathfrak{R}^+$$

Partial orders between fuzzy dual numbers can be introduced using the above pseudo norm. The strong partial written $\tilde{\geq}$ can be defined over $\tilde{\Delta}$ by:

$$\forall a_1 + \varepsilon \cdot b_1 \text{ and } a_2 + \varepsilon \cdot b_2 \in \tilde{\Delta} : a_1 + \varepsilon \cdot b_1 \tilde{\geq} a_2 + \varepsilon \cdot b_2 \Leftrightarrow a_1 - \rho \cdot b_1 \geq a_2 + \rho \cdot b_2 \dots\dots\dots(15)$$

The weak partial order written $\hat{\geq}$ can be defined over $\tilde{\Delta}$ by:

$$\forall a_1 + \varepsilon \cdot b_1 \text{ and } a_2 + \varepsilon \cdot b_2 \in \tilde{\Delta} : a_1 + \varepsilon \cdot b_1 \hat{\geq} a_2 + \varepsilon \cdot b_2 \Leftrightarrow a_1 - \rho \cdot b_1 \geq a_2 \text{ and } a_2 + \rho \cdot b_2 > a_1 - \rho \cdot b_1 \dots\dots\dots(16)$$

The fuzzy equality, symbolized by, between two fuzzy dual numbers is defined:

$$\forall a_1 + \varepsilon \cdot b_1 \text{ and } a_2 + \varepsilon \cdot b_2 \in \tilde{\Delta} : a_1 + \varepsilon \cdot b_1 \cong a_2 + \varepsilon \cdot b_2 \Leftrightarrow a_2 \in [a_1 - \rho \cdot b_1, a_1 + \rho \cdot b_1] \text{ and } a_1 \in [a_2 - \rho \cdot b_2, a_2 + \rho \cdot b_2] \dots\dots\dots(17)$$

B. Fuzzy Dual Representation of Uncertainty for Airport Planning

Let the fuzzy dual representations of the effective levels of demand, the rates of net return and the upgrade costs be given by:

$$r_k^i = r_k^{iL} + \varepsilon r_k^{iD} \dots\dots\dots(18)$$

$$\bar{D}_k^i(\Delta_k) = \bar{D}_k^{iL}(\Delta_k) + \varepsilon \bar{D}_k^{iD}(\Delta_k) \dots\dots\dots(19)$$

$$c_l^{ik}(\Delta_k) = c_l^{iKL}(\Delta_k) + \varepsilon c_l^{iKD}(\Delta_k) \dots\dots\dots(20)$$

where the likely components are indexed by L and the dual components are indexed by D . In many situations, the likely components can be associated with mean estimated values while the dual components can be associated with their corresponding standard deviations.

The expression of the fuzzy dual net present value is given by:

$$\pi([\theta_l^i], l \in \{1, \dots, L_i\}, i \in I) = \pi^L([\theta_l^i], l \in \{1, \dots, L_i\}, i \in I) + \varepsilon \pi^D([\theta_l^i], l \in \{1, \dots, L_i\}, i \in I) \dots\dots\dots(21)$$

where:

$$\pi^L([\theta_l^i], l \in \{1, \dots, L_i\}, i \in I) = \sum_{i \in I} \left(\sum_{k=1}^K \left(\frac{r_k^{iL}}{(1+\rho)^k} \bar{D}_k^{iL}(\Delta_k) \right) - \sum_{\substack{l=1 \\ \theta_l^i \leq K}}^{L_i} \left(\frac{c_l^{iKL}(\Delta_k)}{(1+\rho)^{\theta_l^i}} \right) \right) \dots\dots\dots(22)$$

$$\text{and} \dots\dots\dots(23)$$

$$\pi^D([\theta_l^i], l \in \{1, \dots, L_i\}, i \in I) =$$

$$\sum_{i \in I} \left(\sum_{k=1}^K \left(\frac{r_k^{iL} \cdot \bar{D}_k^{iD}(\Delta_k) + r_k^{iD} \cdot \bar{D}_k^{iL}(\Delta_k)}{(1+\rho)^k} \right) - \sum_{\substack{l=1 \\ \theta_l^i \leq K}}^{L_i} \left(\frac{c_l^{ikD}(\Delta_k)}{(1+\rho)^{\theta_l^i}} \right) \right)$$

C. Decision Making with Fuzzy Dual Framework

In the case in which only sequencing decision are taken into account for the set of possible projects, the problem reduces to a scheduling problem.

Then, once a development scenario has been chosen by settling the decision variables $[\theta_l^i], l \in \{1, \dots, L_i\}, i \in I$, the likely net present value as well as its attached uncertainty can be computed according to a step by step process (Fig. 1), where current capacity and current and future demand for each type of airport traffic are estimated. Then sensitivity analysis can be performed with respect to the timing of different projects.

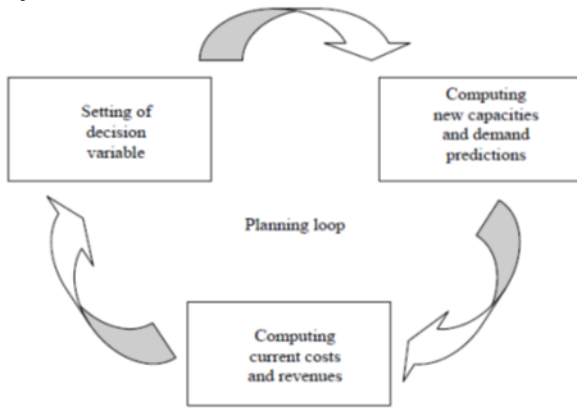


Fig. 1 The airport planning loop

Now, the programming problem associated to airport planning which takes into account the level of uncertainty can be taken as a multi criteria problem by considering on one side the maximization of the likely net present value and on the other side the minimization of uncertainty on this value. However, by introducing a maximum uncertainty level, it can be formulated as:

$$\max_{\theta_l^i} \pi^L([\theta_l^i], l \in \{1, \dots, L_i\}, i \in I) \quad (24)$$

under constraints (3) and a global uncertainty level constraint such as :

$$\pi^D([\theta_l^i], l \in \{1, \dots, L_i\}, i \in I) \leq \Delta\pi_{\max} \quad (25)$$

where $\Delta\pi_{\max}$ represent the maximum allowed level of uncertainty.

Observe here that the solution of problem (24) with (3) and (25) is not straightforward since the effective levels of demand and their associated degree of uncertainty are dependent of the timing and size of investment realizations (see expression (2)).

D. Risk assessment

When solving one of the above problems, the global airport investment plan is considered safe in absolute terms when:

$$\pi^{L^*}([\theta_l^i], l \in \{1, \dots, L_i\}, i \in I) > \pi^{D^*}([\theta_l^i], l \in \{1, \dots, L_i\}, i \in I) \quad (26)$$

A risk degree between 0 and 100% is attached to any solution, either optimal or approximate, for obtaining a present net value equal to π^{L^*} :

$$risk = \begin{cases} 0 & \text{if } \pi^{L^*} - \pi^{D^*} > 0 \\ 100 \cdot \frac{\pi^{L^*} - \pi^{D^*}}{2\pi^{D^*}} & \text{if } \pi^{L^*} - \pi^{D^*} < 0 \leq \pi^{L^*} \\ 100 \cdot \frac{1 - (\pi^{L^*} + \pi^{D^*})}{2\pi^{D^*}} & \text{if } \pi^{L^*} < 0 \leq \pi^{L^*} + \pi^{D^*} \\ 100 & \text{if } \pi^{L^*} + \pi^{D^*} < 0 \end{cases} \quad (27)$$

In addition, it can be interesting to consider the risk level at different stages of the planning process.

V. CASE STUDY AND SOLUTION APPROACH

For the numerical illustration, the case of a national airport expected to gain an international position has been considered. The airport is supposed to be managed under a BOT agreement (Build – Operate – Transfer) over a period of thirty years. In this situation, the BOT project financing involves a private entity that has received a concession from the public sector to finance, design, construct, and operate the complex of airport infrastructure facilities, according to the concession contract. The financial risk of the concessionaire is to be unable to recover its investment, operating and maintenance expenses in the project. In this type of situation, the project proponent is facing a significant amount of risk that needs to be assessed and mitigated.

Mean potential passenger demand is supposed to double every eight years with an initial traffic of 300K passengers/year while mean cargo potential demand is supposed to double every five years, starting with 100K tons. These levels are subject to an increasing uncertainty from 5 to 15% along the considered time period.

The project is composed of three main phases:

- 1) An initial phase where the existing runway and terminal are renewed (cost 150 Billion FCFA $\pm 5\%$ where 1Euro = 656 FCFA).
- 2) A second phase where airport ATC and related equipment are upgraded, the length of the runway is augmented while passenger and cargo terminals capacities are increased (cost 250 Billion FCFA $\pm 10\%$).
- 3) A third phase where a new runway and a new passengers and cargo terminals are built (cost 500 Billion FCFA $\pm 15\%$).

In each phases, new arrangement of airside and landside facilities are necessary. It has been supposed that no new land acquisition is necessary to perform the proposed plan. The financial rate of actualization ρ has been taken equal to 5% along the whole period.

The fuzzy dual formalism allows the consideration of three scenarios with respect to each type of demand (low, medium and high) and costs with an increasing uncertainty.

This has led to a planning problem with about 80 decision variables including timing and size of subprojects resulting in a set of rather small-scale optimization problems.

In this case, to solve problem (22), (3), (23), Dynamic Programming was considered since as quoted in [7]: “Dynamic Programming is a mathematical technique for making a sequence of interrelated decisions, providing a systematic procedure for determining the optimal combination of resources”.

Many different approaches to make use of Dynamic Programming (direct or reverse Dynamic programming) and extensions (stochastic Dynamic Programming, Fuzzy Dynamic programming) have been developed to face different characteristics of sequential decision-making. Fuzzy dual programming has been introduced recently [8] to provide a general framework for dealing with uncertainty approached through the fuzzy dual formalism. The paradigm of Dynamic Programming was extended to this situation by adopting the comparison operators (the weak partial order inequality) between fuzzy dual numbers. Observe also that the search tree generated by the dynamic programming process allows performing a straightforward sensitivity analysis.

Coming back to the case study, it appears that the safest program is to start without delay the initial phase, to launch the extension phase after five years and the construction of the new runway after fifteen years with a total benefit in current value in a range between 1300 and 5700 Billion FCFA. The postponing of phase two for three years brings these values to a total benefit in current value in a range between 800 and 5400 Billion FCFA.

VI. CONCLUSIONS

This communication after performing an analysis of the long-term airport planning problem, proposes a new approach for airport investment risk assessment. This approach takes explicitly into account the degree of uncertainty in the prediction of activity levels while proposing milestones for the different stages of the airport project in view of maximizing profit over the planning horizon while assessing the resulting financial risk.

Uncertainty is represented through fuzzy dual numbers which allows limiting the problem complexity and the computational burden to get a solution. Here risk minimization is performed using a fuzzy dual extension of dynamic programming and the applicability of the proposed approach is discussed through a case study.

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