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# DEVELOPMENT AND ILLUSTRATIVE EXAMPLES OF AN AIRCRAFT LONGITUDINAL REFERENCE PROFILE TO COMBINE CONTINUOUS DESCENT OPERATIONS AND TAILORED ARRIVALS 

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## KEYWORDS

Tailored arrival, Continuous Descent Operation, reference speed profile, reference vertical profile


#### Abstract

This paper addresses a specific aspect of air traffic control services, namely the achievement of an orderly and expeditious flow of air traffic under time constrained continuous descent approach. The task of establishing properly spaced landing sequences is quite demanding for air traffic controllers, especially under heavy traffic conditions. Indeed, terminal control area (TMA) air traffic controllers have to merge two or more streams into a single stream by means of radar vectoring and speed instructions. This high level task of sequencing aircraft is not currently communicated to the pilot. In this paper, the task of merging an aircraft over a specified meter fix is addressed through a novel clearance in which air traffic control clears an aircraft to track an ad-hoc computed reference trajectory. This enables the aircraft to merge a specified meter fix at a given time. This kind of application may be envisioned as an enhancement of the use of the ground based Arrival MANager (AMAN), which is a tactical controller assistance system enabling the computation of rendez-vous time at meter fix to meet the runway capacity and absorb the traffic. The time computed by the AMAN is envisioned to be used by the air traffic controller to request the aircrew to overfly the meter fix at the desired time. In comparison with current operations, the change is that the controller would communicate decisions on traffic flow organization at a higher level to the pilot rather than requiring the controller to calculate and communicate lower-level guidance instructions.


While keeping the controller responsible for making the traffic flow decisions, the envisioned application involves new avionics capabilities for merging operations, including the tracking of height and speed. Thus, air traffic control improvement is achieved through a greater involvement of pilots in cooperation with air traffic controllers. This type of application is clearly in the scope of the 4D trajectory
concept promoted by the European programme SESAR and by the US programme NextGen.

In this paper a futuristic 4D trajectory application where the air traffic controller will ask an aircraft to overfly a meter fix at a specific time is addressed. The overflying time is assumed to be specified by the air traffic controller to settle properly the arrival sequence and could be given for example through an Arrival Manager (AMAN). In addition, this clearance is assumed to be given after the Top Of Descent (TOD) of the aircraft. This paper focuses on the issue of computing reference airspeed and height profiles for time constrained descent. The proposed approach is illustrated through some examples.

## INTRODUCTION

Nowadays, environmental impact and efficiency have become the two very important aspects in aviation industry after safety. New operations such as Continuous Descent Operation (CDO) can significantly reduce the noise impact of landing aircraft by keeping them longer at higher altitudes and by avoiding steps during descent. Such new operations need a collaborative work between airlines and Air Navigation Service Providers (ANSPs) to be defined and operated. CDO is defined by ICAO as "an aircraft operating technique aided by appropriate airspace and procedure design and appropriate ATC clearances enabling the execution of a flight profile optimized to the operating capability of the aircraft, with low engine thrust settings and, where possible, a low drag configuration, thereby reducing fuel burn and emissions during descent. The optimum vertical profile takes the form of a continuously descending path, with a minimum of level flight segments only as needed to decelerate and configure the aircraft or to establish on a landing guidance system (e.g. ILS)" [1].

Since 2008 European stakeholders have initiated an European CDO implementation program. Up to now, Basic CDO is already in operation at UK Heathrow Airport [2]. In Sweden the European project NUP2 has enabled SAS and LFV to operate 4D flight paths, or green approaches, at Stockholm-Arlanda
airport with beneficial impacts on environment. Thanks to the collaboration between Air France and French ANSPO, CDOs are operated routinely at Marseille airport in France [3].

In United States a program named Partnership for AIR Transportation Noise and Emission Reduction (PARTNER) conducted tests at Louisville International Airport in 2002 and at Los Angeles International Airport in 2007 [4].

NASA and FAA have been involved in extensive efforts to develop advanced concepts, technologies and procedures for the Next Generation Air Transportation System (NextGen) later on. One aim of NextGen is to develop ground-side automation systems to assist controllers in strategic planning operations. The En Route Descent Advisor (EDA) is one of the Center TRACON Automation System (CTAS) decision support tools under development at the NASA Ames Research Center. EDA generates maneuver advisories for arrival aircraft to meet scheduled arrival times at the arrival meter fix, sometimes $20-25$ minutes ahead of the aircraft's scheduled meter fix arrival time [5]. A research [6] has also been done to develop ground-side automation to enable 4D-Trajectory-Based Operations (4DTBO) in terminal airspace. This research illustrates a computational framework for the design of 4D-Trajectories (4DTs) based on fundamental flight mechanics and nonlinear trajectory optimization techniques with sample scenarios and open-source models.

AIRE Project (Atlantic Interoperability Initiative to Reduce Emissions) is a joint initiative by the European Commission and the FAA to improve energy efficiency and aircraft noise. AURORA project was a project implemented by Airbus, Scandinavian Airlines International (SAS), Swedish ANSP LFV and Stockholm Arlanda Airport. They conducted Continuous Descent Approaches for the first time on SAS transatlantic flights using an Airbus A330.

Another new ATC technique called Point Merge System (PMS) [7] aims to facilitate the merging of traffic from a number of Area Navigation (RNAV) arrival routes. The technique is based upon aircraft flying a quasi-arc, up to 30 NM long, with a radius of more than 20NM from the designated merging point. Each arc has a published height that the aircraft must have reached before establishing on the arc and a predefined speed to fly it. In general the arc nearest to the merging point has the highest height while the other has the lowest height so that the external
sequencing leg is free from traffic from the internal sequencing leg during descent of the traffic. On April 2011 Oslo became the world's first airport to implement a PMS in their airspace. Other countries and airports will follow.

In this paper, the task of merging an aircraft over a specified meter fix is addressed through a novel clearance in which air traffic control clears an aircraft to merge a specified meter fix at a given time. This kind of application may be envisioned as an enhancement of the use of the ground based Arrival MANager (AMAN), which is a tactical controller assistance system enabling the computation of rendez-vous time at meter fix to meet the runway capacity and absorb the traffic [8]. The time computed by the AMAN is envisioned to be used by the air traffic controller to request the aircrew to overfly the meter fix at the desired time. In comparison with current operations, the change is that the controller would communicate decisions on traffic flow organization at a higher level to the pilot rather than requiring the controller to calculate and communicate lower-level guidance instructions.

While keeping the controller responsible for making the traffic flow decisions, the envisioned application involves new avionics capabilities for merging operations including the tracking of height and speed. Thus air traffic control improvement is achieved through a greater involvement of pilots in cooperation with air traffic controllers. This type of application is clearly in the scope of the 4D trajectory concept promoted by the European programme SESAR [9] and by the US programme NextGen [10].

The main benefit expected from this application [11] is to improve flight efficiency by more precise maneuvering resulting from onboard capabilities, and also noise abatement and fuel saving. More precise maneuvers are also expected to increase sector capacity. Indeed moving from radar vectoring to monitoring pre-computed trajectories would contribute to decrease controller's workload, and therefore to increase sector capacity.

This paper presents a new methodology to compute a reference trajectory for time based continuous descent operations. The aircraft is considered as a point mass model and focuses on aircraft longitudinal motion including known wind. As far as time constrained operations are assumed, final time as well as final altitude and along track distance to be flown are imposed. We propose a new methodology to compute a reference indicated
airspeed (IAS) and a reference vertical profile to achieve imposed final position and altitude at a prescribed time. The computed trajectory is a time parametrized trajectory which will be used as a reference trajectory by some envisioned tracking controller installed on board the aircraft. Nevertheless the design of the tracking controller is out of the scope of this paper.

The structure of the paper is the following: first the equations of motion which will be used are presented as well as the constraints to be satisfied. Then the Two-point Boundary Value Problem (TPBVP) to be solved is defined. The core of the paper consists in describing the proposed methodology to compute the reference indicated airspeed (IAS) and reference vertical profile to solve the problem. Then a scenario is proposed and simulation results are provided to illustrate the effectiveness of the proposed approach.

## EQUATIONS OF MOTION

In the following, the aircraft is modeled as a point mass flying on a non-rotating and flat earth. The aircraft is assumed to fly in an time varying atmospheric wind field comprising of both horizontal and vertical components. In addition the side-slip angle is assumed to be zero so that the flight is coordinated as depicted on Figure 1.


Figure. 1. Reference frame and notations
The notations which will be used are the following:

- $\quad V$ is the true airspeed (TAS) of the aircraft
- $\quad x$ and $y$ denote the position of the aircraft in the horizontal plane.
- $\quad W_{h}$ is the horizontal component of the wind
- $\psi_{w}$ is the direction from where the wind is blowing
- $\quad \psi$ is the heading of the aircraft

Kinematic equations in the horizontal plane reads as follows:

$$
\left\{\begin{array}{c}
\dot{x}=V_{p} \cos (\psi)+W_{h} \cos \left(\psi_{w}+\pi\right)  \tag{1}\\
\dot{y}=V_{p} \sin (\psi)+W_{h} \sin \left(\psi_{w}+\pi\right)
\end{array}\right.
$$

Denoting by $V$ the flight path angle the relationship between $V_{p}$ and $V$ is the following:

$$
\begin{equation*}
V_{p}=V \cos (\gamma) \tag{2}
\end{equation*}
$$

- $s$ is the horizontal path length flown by the aircraft. Infinitesimal path length increment $d s$ is linked to infinitesimal positions increments $d x$ and $d y$ by the following relationship

$$
\begin{equation*}
d s^{2}=d x^{2}+d y^{2} \tag{3}
\end{equation*}
$$

- $\quad V_{g}$ is the ground speed of the aircraft

$$
\begin{equation*}
\dot{s}=V_{g}=\sqrt{\dot{x}^{2}+\dot{y}^{2}} \tag{4}
\end{equation*}
$$

- $\quad X$ is the track angle of the aircraft

$$
\left\{\begin{array}{c}
\dot{x}=V_{p} \cos (\psi)+W_{h} \cos \left(\psi_{w}+\pi\right)=V_{g} \cos (X)  \tag{5}\\
\dot{y}=V_{p} \sin (\psi)+W_{h} \sin \left(\psi_{w}+\pi\right)=V_{g} \sin (X)
\end{array}\right.
$$

Rearranging (5) leads to the following relationships:

$$
\left\{\begin{align*}
\dot{s}=V_{g} & =\sqrt{V_{p}^{2}+W_{h}^{2}(t)-2 V_{p} W_{h} \cos \left(\psi-\Psi_{w}\right)} \\
\psi & =X-\arcsin \left(\frac{W_{h}}{V_{p}} \sin \left(X-\Psi_{w}\right)\right) \tag{6}
\end{align*}\right.
$$

- $\quad z$ is the vertical position (altitude) of the aircraft
- $W_{z}$ is the vertical component of the wind

The kinematic equation in the vertical plane reads as follows:

$$
\begin{equation*}
\dot{z}=V \sin (\gamma)+W_{z}(t) \tag{7}
\end{equation*}
$$

Pilots usually use the indicated airspeed on board aircraft. Conversion between true airspeed $V$ and indicated airspeed $V_{i}$ can be done using a nonlinear relationship provided for example within BADA documentation [12]:

$$
\begin{equation*}
V=g\left(V_{i}, z\right) \tag{8}
\end{equation*}
$$

When the indicated airspeed $V_{i}$ is approximated by the equivalent airspeed $V_{e}$ we get:

$$
\begin{equation*}
V=V_{e} \sqrt{\frac{\rho(0)}{\rho(z)}} \approx V_{i} \sqrt{\frac{\rho(0)}{\rho(z)}} \tag{9}
\end{equation*}
$$

Where $\rho(z)$ is the air density which depends on altitude $z$ and $\rho(0)$ is the air density at sea level.

## PROBLEM FORMULATION

In aviation, a standard terminal arrival route or standard terminal arrival (STAR) is a published procedure followed by aircraft on an IFR flight plan just before reaching a destination airport [13].


Figure. 2. Example of standard terminal arrival route
On usual approach chart the flight path to be followed by the aircraft is known by the aircrew. As a consequence the track $X$ of the aircraft can be easily computed as a function of the horizontal path length $s$ meaning that $X(s)$ is known during the all the path to be followed by the aircraft. Thus the path length $s$ will be envisioned as the natural independent variable of the aircraft path rather than time $t$.

Taking the path length $s$ as the new independent variable, we will denote by $t^{\prime}$ the derivative of time $t$ with respect path length $s$ and by $z$ ' the derivative of altitude $z$ with respect path length $s$. Equations (6) and (7) are then rewritten as follows:

$$
\left\{\begin{array}{l}
t^{\prime}=\frac{d t}{d s}=\frac{1}{V_{g}}  \tag{10}\\
z^{\prime}=\frac{d z}{d t} \frac{d t}{d s}
\end{array}\right.
$$

That is:

$$
\left\{\begin{array}{l}
t^{\prime}=\frac{1}{\sqrt{V_{p}^{2}+W_{h}^{2}(s)-2 V_{p} W_{h} \cos \left(\psi-\Psi_{w}\right)}}  \tag{11}\\
z^{\prime}=\frac{V \sin (\gamma)+W_{z}}{\sqrt{V_{p}^{2}+W_{h}^{2}(s)-2 V_{p} W_{h} \cos \left(\Psi-\Psi_{w}\right)}}
\end{array}\right.
$$

Heading $\psi(s)$ can be computed as a function of track $X(s)$, direction $\psi_{w}(s)$ from where the wind is blowing, horizontal component $W_{h}(s)$ of the wind and speed $V_{p}$ :

$$
\begin{equation*}
\psi=X-\arcsin \left(\frac{W_{h}}{V_{p}} \sin \left(X-\psi_{w}\right)\right) \tag{12}
\end{equation*}
$$

Where speeds $V_{p}$ is linked to the indicated airspeed $V_{i}$ and flight path angle $\gamma$ by (2) and (8).

$$
\begin{equation*}
V_{p}=V \cos (\gamma)=g\left(V_{i}, z\right) \cos (\gamma) \tag{13}
\end{equation*}
$$

Airliners are equipped with autopilot systems. Some autopilot modes control the indicated airspeed $V_{i}$ as well as the flight path angle $\gamma$. So the two variables which are chosen to form the control vector $\underline{u}$ of the aircraft are the indicated airspeed $V_{i}$ and the flight path angle $\gamma$ :

$$
\underline{u}=\left[\begin{array}{ll}
V_{i} & \gamma \tag{14}
\end{array}\right]^{T}
$$

Equations (11) can be rewritten as follows:

$$
\begin{equation*}
\underline{x}^{\prime}=f(\underline{x}, \underline{u}, \underline{p}) \tag{15}
\end{equation*}
$$

Where:

- $\quad \underline{u}$ is the control vector defined in (14)
- $\underline{x}$ is the state vector

$$
\underline{x}=\left[\begin{array}{ll}
t & z \tag{16}
\end{array}\right]^{\top}
$$

- $\underline{p}$ is the vector of exogenous parameters depending on path length $s$

$$
\begin{equation*}
p=\left[W_{h}(s) \quad W_{z}(s) \quad \psi_{w}(s) \quad X(s)\right]^{T} \tag{17}
\end{equation*}
$$

In order to satisfy the time constrained maneuver the following constraints are imposed on initial and final values of time $t$ with respect to path length $s$ :

$$
\left\{\begin{array}{l}
t(0)=0  \tag{18}\\
t\left(s_{f}\right)=t_{f}
\end{array}\right.
$$

The preceding constraints indicate that the aircraft starts the maneuver at initial time 0 and
finishes it at the prescribed time of arrival $t_{f}$ after having flown a distance equal to $s_{f}$.

Similarly the constraints on initial and final values of the vertical components of state vector $\underline{x}$ are the following:

$$
\left\{\begin{array}{l}
z(0)=z_{0}  \tag{19}\\
z\left(s_{f}\right)=z_{f}
\end{array}\right.
$$

The preceding constraints indicate that the aircraft starts the maneuver at altitude $z_{0}$ and finishes it at altitude $z_{f}$ at the prescribed arrival path length $s_{f}$.

In terms of vertical speed we impose the following initial and final values:

$$
\left\{\begin{array}{l}
\dot{z}(0)=V_{z 0}  \tag{20}\\
\dot{z}\left(t_{f}\right)=V_{z f}
\end{array}\right.
$$

The preceding constraints indicate that the vertical speed at the beginning and at the end of the maneuver are imposed to be $V_{z 0}$ and $V_{z f}$ respectively.

Moreover the indicated airspeed $V_{i}$ is set to be $V_{i 0}$ at the beginning of the maneuver and $V_{i f}$ at the prescribed time of arrival $t_{f}$ :

$$
\left\{\begin{array}{l}
V_{i}(0)=V_{i o}  \tag{21}\\
V_{i}\left(t_{f}\right)=V_{i f}
\end{array}\right.
$$

Then the problem to be solved consists in finding and expression of controls $V_{i}(s)$ and $\gamma(s)$ such that differential equations (11) hold under constraints (18), (19), (20) and (21). Constraints (18), (19), (20) and (21) associated with the set of coupled ordinary differential equations (11) constitute a Two-Point Boundary Value Problem (TPBVP).

The key step of our approach to solve this TwoPoint Boundary Value Problem is the parametrization of the indicated airspeed $V_{i}$ and flight path angle $\gamma$, that is the control vector $\underline{u}$, along the path.

## CONTROL PARAMETRIZATION

We will denote by $\tau$ the ratio between the actual path length $s$ and the total distance to be flown during the maneuver $s_{f}$ :

$$
\begin{equation*}
\tau=\frac{s}{s_{f}} \tag{22}
\end{equation*}
$$

As a consequence when path length $s$ varies from 0 to $s_{f}$ parameter $T$ varies from 0 to 1 . When identifying $r^{\prime}(s)$ with $r^{\prime}\left(s_{f} \tau\right)$ we get:

$$
\begin{equation*}
\frac{d}{d s} r(s)=r^{\prime}(s) \equiv r^{\prime}\left(s_{f} \tau\right)=\frac{1}{s_{f}} \frac{d}{d \tau} r\left(s_{f} \tau\right) \tag{23}
\end{equation*}
$$

To control the set of coupled ordinary differential equations (11) we shall provide the expression of the commanded indicated airspeed $V_{i}$ and commanded flight path angle $\gamma$. Quadratic Bézier curves will be used to build both control. The expression of the commanded indicated airspeed $V_{i}$ is chosen as follows:

$$
\begin{equation*}
V_{i}(\tau)=(1-\tau)^{2} V_{i 0}+2 \tau(1-\tau) u_{V}+\tau^{2} V_{i f} \tag{24}
\end{equation*}
$$

It is clear that whatever constant parameter $u_{v}$ is the proposed expression for the indicated airspeed $V_{i}$ satisfied the constraints (21) on initial and final values:

$$
\left\{\begin{array}{l}
V_{i}(0)=V_{i 0}  \tag{25}\\
V_{i}(1)=V_{i f}
\end{array}\right.
$$

Similarly the vertical speed $z^{\prime}$ is obtained thanks to a Quadratic Bézier curves.

$$
\begin{equation*}
z^{\prime}(\tau)=(1-\tau)^{2} z^{\prime}{ }_{0}+2 \tau(1-\tau) u_{z}+\tau^{2} z_{f}^{\prime} \tag{26}
\end{equation*}
$$

The values of constants $z_{0}^{\prime}$ and $z_{f}^{\prime}$ are chosen in order to satisfy the constraints on initial and final values in equation (20). From (7) and (11) we get:

$$
\left\{\begin{array}{l}
z^{\prime}(0)=z^{\prime}{ }_{0}=\frac{V_{z 0}}{V_{g}(0)}  \tag{27}\\
z^{\prime}(1)=z^{\prime}{ }_{f}=\frac{V_{z f}}{V_{g}\left(s_{f}\right)}
\end{array}\right.
$$

Where $V_{g}(s)$ is the ground speed::

$$
\begin{equation*}
V_{g}(s)=\sqrt{V_{p}^{2}+W_{h}^{2}(s)-2 V_{p} W_{h} \cos \left(\psi-\psi_{w}\right)} \tag{28}
\end{equation*}
$$

In the preceding equation heading $\psi$ is computed thanks to (12) whereas flight path angle $\gamma$ is obtained thanks to (11):

$$
\begin{equation*}
\gamma(\tau)=\arcsin \left(\frac{z^{\prime}(\tau) V_{g}(\tau)-W_{z}(\tau)}{V_{p}(\tau)}\right) \tag{29}
\end{equation*}
$$

Integrating (28) leads to the expression of altitude $z(T)$ :

$$
\begin{equation*}
z(\tau)=\frac{-(1-\tau)^{3}}{3} z^{\prime}{ }_{0}+\tau^{2}\left(1-\frac{2}{3} \tau\right) u_{z}+\frac{\tau^{3}}{3} z^{\prime}{ }_{f} \tag{30}
\end{equation*}
$$

We choose vertical speed $z^{\prime}$ as the control variable rather than flight path angle $\gamma$ because the value of $z$ is needed to compute the true airspeed $V$.

Indeed as expressed in (8) true airspeed $V$ is a function of indicated airspeed $V_{i}$ and altitude $z$.

Thus the proposed approach consists in computing constant parameters $u_{v}$ and $u_{z}$ such that differential equation (11) holds under constraints (18) and (19):

$$
\text { find }\left[\begin{array}{ll}
u_{z} & u_{V}
\end{array}\right]^{\top} \text { st }\left\{\begin{array}{c}
\int_{0}^{1} t^{\prime}(\tau) d \tau=\frac{t_{f}}{s_{f}}  \tag{31}\\
\int_{0}^{1} z^{\prime}(\tau) d \tau=\frac{z_{f}-z_{0}}{s_{f}}
\end{array}\right.
$$

This is a classical nonlinear equation solving problem which can easily be handled with numerical computational tools such as Scilab. Scilab is an open source platform for numerical computation [14].

Once the reference indicated airspeed $V_{i}$ and the reference vertical profile $z$ have been computed the feasibility of those reference profiles shall be checked in order to know if those profiles are flyable. This feasibility check is done thanks to the dynamics equations of a point mass fixed wing aircraft:

$$
\left\{\begin{array}{l}
\dot{V}=\frac{T-D}{m}-g \sin (\gamma) \\
\dot{\gamma}=\frac{L-m g \cos (\gamma)}{m V} \tag{32}
\end{array}\right.
$$

Where

- $L$ is the lift force

$$
\begin{equation*}
L=\frac{1}{2} \rho(z) S V^{2} C_{L} \tag{33}
\end{equation*}
$$

- $\quad \rho(z)$ is the air density which depends on altitude $z$
- $S$ is the reference wing surface area
- $C_{L}$ is the lift coefficient.
- $D$ is the drag force

$$
\begin{equation*}
D=\frac{1}{2} \rho(z) S V^{2} C_{D} \tag{34}
\end{equation*}
$$

- $C_{D}$ is the drag coefficient. We will assume a parabolic drag coefficient which is specified as a function of the lift coefficient $C_{L}$

$$
\begin{equation*}
C_{D}=C_{D 0}+C_{D 2} C_{L}^{2} \tag{35}
\end{equation*}
$$

- $F$ is the thrust, which is assumed to be in the opposite direction of the drag $D$
- $g$ is the gravitational acceleration
- $m$ is the aircraft mass

Thanks to the knowledge of the reference indicated airspeed $V_{i}$ and reference vertical profile $z$ true airspeed $V$ is known and the values of the required thrust $T$ and required lift coefficient $C_{L}$ can be computed from (32) and (33):

$$
\left\{\begin{array}{l}
T=m \dot{V}+m g \sin (\gamma)+D  \tag{36}\\
C_{L}=\frac{m V \dot{\gamma}+m g \cos (\gamma)}{0.5 \rho(h) S V^{2}}
\end{array}\right.
$$

The last step consists in checking that the required thrust $T$ and the required lift coefficient $C_{L}$ are compatible with aircraft performance.

## SIMULATION RESULTS

The scenario which has been simulated is the following: the aircraft starts the descent at FL 140 with an indicated airspeed (IAS) of 220 kts . The final approach fix (FAF) is assumed to be 35 NM ahead and shall be overflown at 2500 feet with an indicated airspeed of 170 kts . The maneuver duration is set to $T=540 \mathrm{sec}(9 \mathrm{~min})$. The aircraft is assumed to descent in clean configuration. The hypothetical approach path is depicted on Figure 3: the first leg is east oriented and its length is 15 NM; the second leg is north-east oriented and its length is 5 NM and the last leg is north oriented and its length is 10 NM.


Figure. 3. Hypothetical path to be followed
As far as wind is concerned we will assume no vertical wind. The horizontal wind is set to 5 kts blowing from the north in the first scenario and to 30 kts still blowing from the north in the second scenario.

The flight level with respect to path length (in $N M$ ) for the first scenario is depicted hereafter:


Figure. 4. Reference altitude (FL) for the 5 kts North wind scenario

As expected, the aircraft descents smoothly from FL 140 to 2500 feet following a continuous descent.

The following figure presents the reference indicated airspeed (IAS in kts) with respect to path length (in NM) for the first scenario:


Figure. 5. Reference indicated airspeed (kts) for the 5 kts North wind scenario

As expected, the indicated airspeed changes from 220 kts to 170 kts. We can see on the figure that there is a slight increase of the indicated airspeed just before the achieving 170 kts which may be disturbing for the pilot during the approach phase.

The next figure depicts the ground speed $V_{g}$ (in $k t s$ ) with respect to path length (in NM) for the first scenario:


Figure. 6. Groundspeed (kts) for the 5 kts North wind scenario

We can see on the preceding figure two discontinuities; those are obviously the effect of the change in the wind direction.

For the second scenario the flight level with respect to path length (in NM) is depicted hereafter:


Figure. 7. Reference altitude (FL) for the 30 kts North wind scenario

As expected, the aircraft descents smoothly from FL 140 to 2500 feet following a continuous descent. This vertical profile is very similar to the vertical profile computed for the first scenario.

The following figure presents the reference indicated airspeed (IAS in kts) with respect to path length (in NM) for the second scenario:


Figure. 8. Reference indicated airspeed (kts) for the 30 kts North wind scenario
As expected, the indicated airspeed changes from 220 kts to 170 kts. We can see that the indicated airspeed is continuously decreasing for that scenario.

The next figure depicts the ground speed ( $V_{g}$ in $k t s$ ) with respect to path length (in NM) for the second scenario:


Figure. 9. Groundspeed (kts) for the 30 kts North wind scenario

We can see on the preceding figure two discontinuities; those are obviously the effect of the change in the wind direction.

We have also checked that the distance flown is $30 N M$ for both scenario, as expected; this is a question of integrating the ground speed $V_{g}$.

## CONCLUSION

This paper addresses a specific aspect of air traffic control services, namely the achievement of an orderly and expeditious flow of air traffic under time constrained continuous descent approach.

This paper presents a new methodology to compute a reference indicated airspeed (IAS) and a reference vertical profile for time constrained descent which satisfy length and endpoint constraints.

The proposed approach is based on the shaping of Quadratic Bézier curves which are applied to the controlled indicated airspeed (IAS) of the aircraft and its vertical profile. An appropriate choice of the available degree of freedom when computing the reference indicated airspeed (IAS) and the vertical speed enables to solve the Two-point Boundary Value Problem (TPBVP).

Simulation results which include wind illustrate the efficiency of the proposed approach.

Future developments include the test of the robustness of the proposed design with respect to unexpected wind. This may be addressed through the periodic update of the computation of reference indicated airspeed and vertical speed as well as the design of some tracking controller installed on board the aircraft.

The proposed approach can be extended to the case where constraints over multiple fixes are imposed. Indeed, the way to compute the reference speed takes explicitly into account the initial and final values of aircraft's position and speed and can thus be used to accommodate the reference speed to each segment of flight.

## REFERENCES

[1] ICAO, Continuous Descent Operations (CDO) Manual, Doc 9931, 2010
[2] Heathrow Airport Flight Evaluation Data Report Q1 2011-
http://www.heathrowairport.com/static/Heathrow_Noi se/Downloads/PDF/FlightEvaluationDataReport-Q12011_LHR.pdf
[3] Guibaud J., Thépenier P., Montier F., Evaluation des descentes continues à Marseille-Provence, Séminaires de la DSNA, November 2007, Paris
[4] Yi Cao, Tatsuya Kotegawa, Joseph Post, Evaluation of Continuous Descent Approach as a Standard Terminal Airspace Operation, Ninth

USA/Europe Air Traffic Management Research and Development Seminar, June 14-17 2011, Berlin, Germany
[5] Mueller K. T., Bortins R., Schleicher D. R. and Sweet D., Effect of Uncertainty on En Route Descent Advisor (EDA) Predictions, AIAA 4th Aviation Technology, Integration and Operations (ATIO) Forum, 20-22 September 2004, Chicago, Illinois
[6] Sai Vaddi, Gregory D.Sweriduk, Monish Tandale, 4D Green Trajectory Design for Terminal Area Operations Using Nonlinear Optimization Techniques, AIAA, Guidance, Navigation, and Control Conference, 13-16 August 2012, Minneapolis, Minnesota
[7] Boursier L., Favennec B., Hoffman E., Trzmiel A., Vergne F., Zeghal K., Merging Arrival Flows Without Heading Instructions, 7th USA/Europe Air Traffic Management R\&D Seminar, Spain, July 2007
[8] Bourdon B., Maestro: Sequenced Traffic Flow Tool, Direction de la Technique et de I'Innovation, Technical review nº55, November 1998
[9] SESAR Consortium, The ATM Target Concept D3, DLM-0612-001-02-00a, 2007
[10]Merkle J., 3D Path Arrival Management (PAM) Project, High Density Airport Time-based RNAV/RNP NextGen / JPDO Demonstrations, REACT Workshop, June 2008
[11] Miquel T., Manzoni H., Legrand F., Martin D., Millischer M., 4D Green Approach Trajectory: Illustrative Assessment On Toulouse Approach, AIAA, Guidance, Navigation, and Control Conference, 2010
[12] EEC Technical/Scientific Report No. 13/04/16-01, User Manual For The Base Of Aircraft Data (BADA) revision 3.11, Eurocontrol Experimental Centre
[13] https://en.wikipedia.org/wiki/Standard_termin al_arrival_route
[14]Scilab, the open source platform for numerical computation (http://www.scilab.org/)

