Merging Optimization Method with Multiple Entry Points for Extended Terminal Maneuvering Area

Daichi Toratani, Daniel Delahaye, S Ueno, T. Higuchi

[EN-A-010] Merging Optimization Method with Multiple Entry Points for Extended Terminal Maneuvering Area

+D. Toratani*, D. Delahaye**, S. Ueno***, T. Higuchi***

*Graduate School of Environment and Information Sciences
Yokohama National University (YNU)
Yokohama, Japan
toratani-daichi-jk@ynu.jp

**Applied Math Laboratory (MAIAA)
Ecole National Aviation Civil (ENAC)
Toulouse, France
delahaye@recherche.enac.fr

***Faculty of Environment and Information Sciences
Yokohama National University (YNU)
Yokohama, Japan
s-ueno@ynu.ac.jp, higuchi@ynu.ac.jp

Abstract: To implement the Continuous Descend Operation (CDO) to the high-density flow, Flight-deck Interval Management (FIM) is proposed by some researchers. To enhance the FIM, the merging optimization method with the multiple entry points for the Extended Terminal Maneuvering Area (E-TMA) is developed in this study. The simultaneous optimization method for trajectory and sequence is employed as the merging optimization method. The simultaneous optimization method is able to optimize the trajectory and the sequence simultaneously by introducing the criterion function which expresses the relation between the terminal time and the value of the criterion of the trajectory. However, the simultaneous optimization method for trajectory and sequence is not enough for the merging optimization method for the E-TMA, because aircraft enter the TMA from some entry points to merge the arrival flow, and the previous method is able to treat only one merging point. To enable the simultaneous optimization method to treat the multiple entry points, the criterion function is modified in this paper. The ability of the proposed method is demonstrated by the simulations. The simulation results show the merging optimization method is able to optimize the merging with the multiple entry points.

Keywords: Merging Optimization, E-TMA, Optimal Control, Mixed-Integer Linear Programming

1. INTRODUCTION

Continuous Descent Operation (CDO) is the one of the hot topics in the field of the Air Traffic Management (ATM) [1]. The CDO is expected to be able to reduce fuel consumption and noise pollution. However, the CDO is currently applied to only low-density air traffic flow to avoid conflicts between aircraft. To implement the CDO in high-density flow, the autonomous arrival flow control system as Flight-deck Interval Management (FIM) is developed. The FIM is the one of the airborne time separation system utilizing Aircraft Surveillance Applications System (ASAS). Itoh et al. developed the FIM for the arrival flow with the CDO and demonstrated the ability of the FIM in practical simulations [2].

To enhance the FIM, the merging optimization method is useful. Before the aircraft take the arrival flow, the aircraft fly from various directions to the merging point to make the sequence. Then, the aircraft must merge keeping the minimum time interval to avoid wake turbulence. If the sequence keep the minimum time interval at the merging point by the merging optimization method, it is easy for the FIM to keep the time interval in the arrival flow. Therefore, the merging optimization method is developed in this study. The merging optimization method optimizes the trajectories outside the Terminal Maneuvering Area (TMA). This area is called Extended TMA (E-TMA), and the image of the E-TMA is shown in Fig. 1. In this study, the merging optimization for the E-TMA is developed to enhance the FIM.

There are some previous studies with respect to the merging optimization method. However, these studies have problems to implement to the practical air traffic control. These problems are roughly classified as shown below.

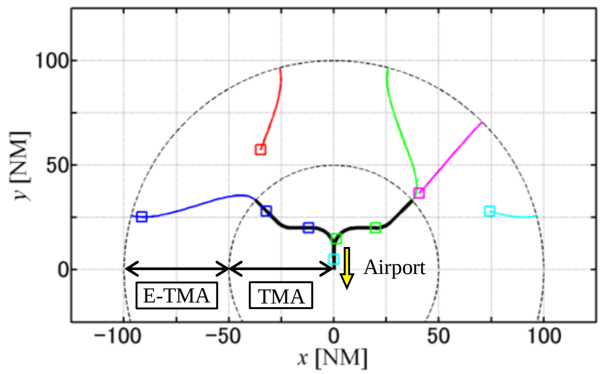

Figure 1 Image of the E-TMA.

- Some studies are not able to optimize trajectory and sequence simultaneously [3].

Mathematically, trajectory is continuous system, and sequence is discrete system. Therefore, it is difficult to optimize them simultaneously.

- Others are not able to treat the nonlinear model [4].

One of the solution to optimize the trajectory and the sequence simultaneously is to employ Mixed-Integer Linear Programming (MILP). The MILP is able to treat both real numbers and integers. Accordingly, the MILP is able to optimize both the continuous and discrete system simultaneously. Generally, the commercial aircraft model is formulated as the nonlinear function, for example, Base of Aircraft Data (BADA) developed by EUROCONTROL [5]. However, the MILP is able to treat only linear function. In the previous studies, the aircraft is modeled as the simple vehicle model.

To resolve these problems, the authors developed the simultaneous optimization method for trajectory and sequence [6]. This method combines the optimal control and the MILP to optimize the trajectory and the sequence simultaneously. Employing the optimal control to optimize the trajectory, the method is able to treat the nonlinear functions. By the simulation, it is confirmed that the simultaneous optimization method is able to optimize the trajectory and the sequence simultaneously.

However, the simultaneous optimization method for trajectory and sequence is not enough for the merging optimization method for the E-TMA. That is why the E-TMA has several entry points (These points is called merging points in the simultaneous optimization method.) to the TMA as shown in Fig. 1, but in Ref. [6], the merging point was only one. In the E-TMA, it is better for the red aircraft to enter the TMA form the left entry point, but it may be optimal for the red aircraft to enter from the right entry point depending on the conditions as rough weather and congestion. Accordingly, the simultaneous optimization method for trajectory and sequence is improved to the merging optimization method with the multiple entry points in this paper.

This paper is composed of 5 sections. Section 2 presents the trajectory optimization method for single aircraft. Section 3 presents the merging optimization method for the E-TMA. Section 4 shows the simulations. Section 5 presents the conclusion the future plans.

2. TRAJECTORY OPTIMIZATION MTHOD FOR SINGLE AIRCRAFT

2.1. Aircraft Model

In this study, the BADA is employed as the aircraft model. Generally, the model is represented as the differential equation whose independent variable is time. However, to treat the temporal constraint along with the spatial constraint, the space-time coordinate system (STCS) is employed in this study [7]. In the STCS, the time is treated as the state variable, and the independent variable of the differential equation is the length of the trajectory. The aircraft model based on the BADA in the STCS is formulated as shown below.

$$\frac{d}{ds}\begin{pmatrix}\psi_s\\ \psi_t\\ x\\ y\\ H_p\\ t\end{pmatrix}=\begin{pmatrix}\dfrac{g_0}{\cos\gamma}\sin\psi_t\tan\psi_t\tan\phi\\ -\dfrac{\sin^3\psi_t}{m}(Thr-D-mg_0\sin\gamma)\\ \cos\psi_t\cos\psi_s\cos\gamma\\ \cos\psi_t\sin\psi_s\cos\gamma\\ \cos\psi_t\sin\gamma\\ \sin\psi_t\end{pmatrix}, \tag{1}$$

where s denotes the length of the trajectory in the STCS, ψ_s denotes the spatial angle which is same as the azimuth angle, ψ_t denotes the temporal angle which represents the velocity in the STCS, x and y are the position of the aircraft, H_p denotes the pressure altitude, g_0 denotes the gravitational acceleration, γ denotes the flight path angle, ϕ denotes the roll angle, , m denotes the mass of the aircraft, Thr denotes the thrust, and D denotes the drag. D is defined as shown below.

$$D=C_{D0}\frac{\rho V_{TAS}^2 S}{2}+C_{D2}\frac{2m^2g_0^2}{\rho V_{TAS}^2 S\cos^2\phi}, \tag{2}$$

where C_{D0} and C_{D2} denote the drag coefficient provided by the BADA, V_{TAS} is the true air speed, and the S denotes the wing reference area. ρ denotes the air density and defined as shown below.

$$\rho=\frac{p_0}{RT_0}\left(1+\frac{\beta}{T_0}H_p\right)^{-1-\frac{g_0}{\beta R}}, \tag{3}$$

where p_0 denotes the standard pressure, R denotes the real gas coefficient, T_0 denotes the standard temperature, and β denotes the temperature gradient with the altitude. Thr is defined in the BADA as shown below.

$$Thr_{cl}=C_{Tc1}\left(1-\frac{H_p}{C_{Tc2}}+C_{Tc3}H_p^2\right), \tag{4}$$

$$Thr_{des}=C_{Tdes}Thr_{cl}, \tag{5}$$

$$Thr_{cr}=D, \tag{6}$$

where the subscripts $_{cl}$, $_{des}$, and $_{cr}$ denote climb, descent, and cruise phase respectively, C_{Tc1}, C_{Tc2}, and C_{Tc3} denote the maximum climb thrust coefficient, and C_{Tdes} denotes the descent thrust coefficient. The fuel flow is derived by the following equation.

$$FF = C_{f1}\left(1 + \frac{V_{TAS}}{C_{f2}}\right)Thr, \tag{7}$$

where C_{f1}, C_{f2} denote the thrust specific fuel consumption coefficient. Here, the cruise fuel flow is corrected as shown below.

$$FF_{cr} = C_{fcr}FF, \tag{8}$$

where C_{fcr} denotes the cruise fuel flow correction coefficient.

2.2. Optimal Control for Trajectory Optimization

In this study, the model as shown in 2.1. is employed to optimize the trajectory. The objective function is set as the fuel flow to calculate the minimum fuel consumption trajectory. To optimize the trajectory, the optimal control is employed. The optimal control problem is transformed to the Two Point Boundary Value Problem (TPBVP), and the optimal trajectory is derived by solving the TPBVP [8].

3. MERGING OPTIMIZATION METHOD FOR EXTENDED TERMINAL AREA

3.1. Simultaneous Optimization Method for Trajectory and Sequence

In this section, the summary of the simultaneous optimization method for trajectory and sequence is explained. To optimize the trajectory and the sequence, criterion function is introduced in this method. Fig. 2 shows the image of the criterion function. In the left figure of Fig. 2, the horizontal axis denotes time, and the vertical axis denotes the distance from the initial position to the merging point. When one trajectory is optimized without constraints with respect to the terminal time, the trajectory is optimal with respect to the terminal time t_f, where the optimal terminal time is denoted as t_{fopt}. Then, the value of the criterion of the trajectory J is minimized with respect to t_f as the blue circle in the right figure of Fig. 2, where the optimal value of the criterion is denoted as J_{opt}. When the trajectory is optimized with specified terminal time constraints as earlier or later than t_{fopt}. Then, the value of the criterion will increase. Here, the function $f_J(t_f)$ which expresses the relation between t_f and J is called criterion function in this method.

When three aircraft are in the airspace, the trajectories

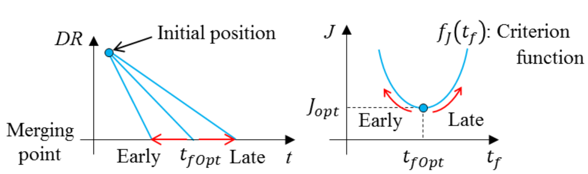

Figure 2 Trajectories and criterion function.

and the criterion functions are expressed as Fig. 3. Then, all aircraft must keep the minimum time interval Δt_{min}. Therefore, the merging optimization problem is transformed to the problem to minimize the sum of the value of the criterion keeping Δt_{min} in Fig. 3. This optimization problem is formulated as shown below.

$$\begin{aligned} &\min_{t_f} \sum_{i=1}^{N_{AC}} J_i \\ &\text{subject to: } J_i = f_{Ji}(t_f i) \quad (i = 1, \cdots, N_{AC}) \\ &\qquad t_{fi} - t_{fi+1} \geq \Delta t_{min} \\ &\qquad \text{or } t_{fi+1} - t_{fi} \geq \Delta t_{min} \\ &\qquad\qquad (i = 1, \cdots, N_{AC} - 1), \end{aligned} \tag{9}$$

where N_{AC} denotes the number of the aircraft. Eq. (9) includes “or”, and it is difficult to solve this kind of the problem. To solve the problem formulated as Eq. (9), the constraints with respect to the minimum time interval is transformed to Eq. (10).

$$\begin{aligned} &-t_{fp} + t_{fq} - Mc_1 \leq -\Delta t_{min} \\ \text{and } &t_{fp} - t_{fq} - Mc_2 \leq -\Delta t_{min} \\ \text{and } &\sum_{k=1}^{2} c_k = 1 \\ &(p, q = 1, \cdots, N_{AC} \mid p < q), \end{aligned} \tag{10}$$

where p and q denote the index of the aircraft, M denotes the positive number that is much larger than any value in the problem, and c denotes the binary variable. Eq. (10) is formulated as the linear inequalities. Therefore, the MILP is able to treat Eq. (10). Here, the criterion function is nonlinear function, and the MILP is able to treat the only linear function. To enable the MILP to treat the criterion function, the criterion function is piecewise-linearized as Eq. (11) and Fig. 4.

$$\begin{aligned} &-a_{ij} + J_i - Me_{ij} \leq b_{ij} \\ \text{and } &a_{ij} - J_i - Me_{ij} \leq -b_{ij} \\ \text{and } &-t_{fi} - Me_{ij} \leq -t_{fLij} \\ \text{and } &t_{fi} - Me_{ij} \leq -t_{fUij} \\ \text{and } &\sum_{j=1}^{N_{sec}} e_{ij} = 1 \\ &(i = 1, \cdots, N_{AC}), (j = 1, \cdots, N_{sec} + 1), \end{aligned} \tag{11}$$

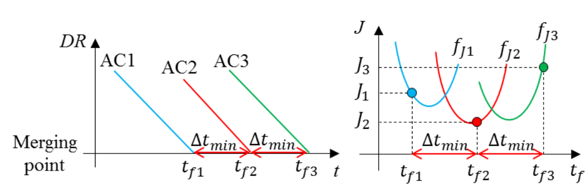

Figure 3 Criterion functions of 3 aircraft.

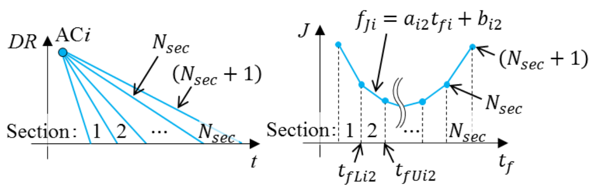

Figure 4 Piecewise-linearized criterion function.

where a and b are the slope and the intercept of the piecewise-linear function, e denotes the binary variable, and N_{sec} denotes the number of the piecewise-linearizing section of the criterion function. Then, the optimization problem as Eq. (9) is transformed to the linear equations and inequalities. This formulation is able to be optimized by the MILP. The form of the MILP is shown below.

$$\begin{aligned}
&\min_x f^T x \\
&\text{subject to: } Ax \leq b \\
&\qquad A_{eq}x = b_{eq} \\
&\qquad l_b \leq x \leq u_b \\
&\qquad x_{int} \in \mathbb{Z} \\
&\qquad x_{ibin} \in \{0,1\}
\end{aligned} \tag{12}$$

In the simultaneous optimization method for trajectory and sequence, the criterion function is derived by the trajectory optimization, and the optimal terminal times for all aircraft keeping Δt_{min} is derived by the MILP. Afterward, the all trajectories are reoptimized with the optimal terminal times to derive the optimal merging.

3.2. Multiple Entry Points

In this paper, the simultaneous optimization method for trajectory and sequence is improved to enable to treat the multiple entry points. In the previous study, the merging point was only one, and the merging point is set as the entry point in the merging optimization method for the E-TMA. Fig. 5 shows the image of the merging optimization problem with the multiple entry points. In Fig. 5, it is better for the red aircraft to enter the TMA from the left entry point, but it may be better to enter from the right entry point depending on the condition, for example rough weather and congestion. It is better that the selection of the entry points is also optimized with the trajectory and the sequence simultaneously, because it depends on the conditions that the entry points from which to enter is better. Therefore, the entry point optimization problem is also introduced to the merging optimization problem. To introduce the entry point optimization, the criterion function is modified as shown Fig. 6. In Fig. 6, the red aircraft has the two criterion functions for the left and right entry points. Selecting either entry point, the aircraft merge to one flow finally, and the aircraft must keep the minimum time constraints. The two criterion functions are not connected. Therefore, the aircraft selects the one criterion function by the MILP. Then, the selection of the entry points is also optimized.

By this modification, the simultaneous optimization method for trajectory and sequence is improved to the merging optimization method with the multiple entry points. Then, the merging optimization method is able to be applied to the E-TMA.

4. SIMULATIONS

The ability of the merging optimization method with the multiple entry points is demonstrated by the simple simulations. In this simulations, some aircraft enter the E-TMA and enter the TMA form entry points. The obstacle as storm is set to demonstrate the entry point optimization. Without the entry point optimization, namely all the entry points are specified, the red aircraft takes roundabout trajectory to avoid the obstacle and enter the TMA from the left entry point as Fig. 7. On the other hand, with the entry point optimization, the red aircraft enter the TMA from the right entry point as Fig. 8. Selecting the right entry point by the entry point optimization, the trajectory of the red aircraft is smoother than the trajectory optimized without the entry optimization.

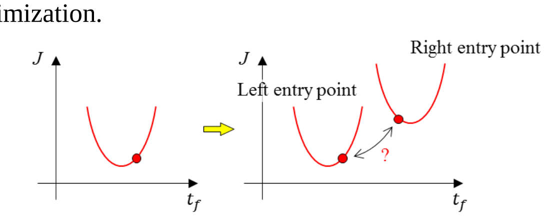

Figure 6 Criterion function for multiple entry points.

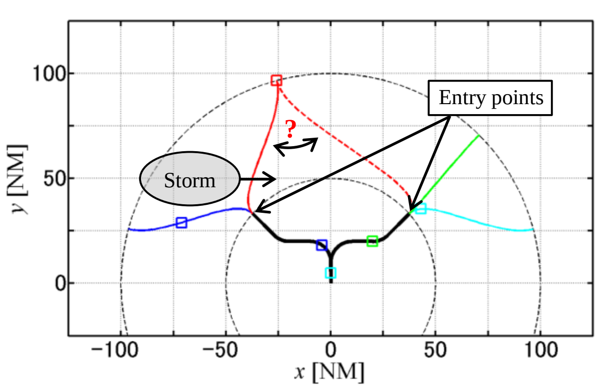

Figure 5 Multiple entry points.

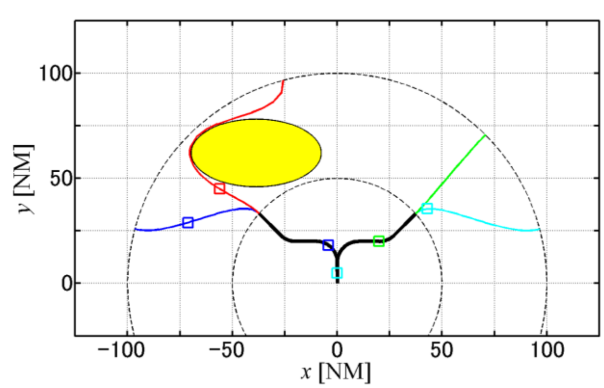

Figure 7 Simulation without entry point optimization.

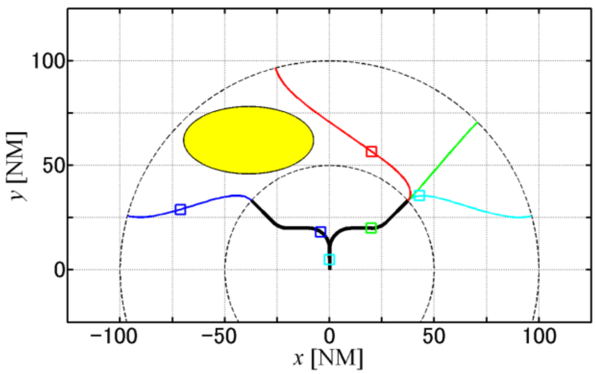

Figure 8 Simulation with entry point optimization.

By the simulations, it is confirmed that the merging optimization method is able to treat the merging problem with the multiple entry points. Then, the trajectory, the sequence, and the entry point are optimized simultaneously.

5. CONCLUSION AND FUTURE PLANS

To enhance the ability of the FIM, the merging optimization method for the E-TMA is developed in this paper. The simultaneous optimization method for trajectory and sequence is applied to the merging optimization method. Employing the simultaneous optimization method, the merging optimization method is able to optimize the trajectory and the sequence simultaneously and treat the nonlinear function as the model of the aircraft. The aircraft is modeled based on the BADA, and the fuel consumption is minimized by the optimal control. To enable the merging optimization method to treat the entry point optimization, the criterion function is modified.

By the simple simulation, the ability of the merging optimization method for the E-TMA is demonstrated. In this paper, two simulations are held. One is the case without the entry point optimization, and another is the case with the entry point optimization. In the simulation, the obstacle is set as storm. Without the entry point optimization, the aircraft takes the roundabout trajectory to avoid the obstacle and enter the specified entry point. On the other hand, with the entry point optimization, the aircraft is able to enter the TMA from better entry point. The merging optimization method for the E-TMA is able to treat the multiple entry point in the simulations. By the merging optimization method, not only the trajectory and the sequence but also the entry point is optimized simultaneously.

In this paper, only the simple simulations are held. Therefore, the future plans on this study are more practical simulations and the quantitative evaluation of the optimization method. Especially, the effect of the entry point optimization with respect to the fuel consumption will be evaluated.

ACKNOWLEDGMENTS

A big part of this work was accomplished at the MAIAA, ENAC. We are grateful to the members of the MAIAA for helpful discussions.

REFERENCES

[1] J.E. Robinson, III, M. Kamgarpour, “Benefits of Continuous Descent Operations in High-Density Terminal Airspace Under Scheduling Constraints” 10th AIAA Aviation Technology, Integration, and Operations (ATIO) Conference, 2010.

[2] E. Itoh, K. Uejima, “Applying Flight-deck Interval Management based Continuous Descent Operation for Arrival Air Traffic to Tokyo International Airport”, 10th USA/Europe Air Traffic Management Research and Development Seminar (ATM seminar), 2013.

[3] J. Zhou, S. Caeri, D. Delahaye, M. Sbihi, “Optimization of Arrival and Departure Routes in Terminal Maneuvering Area”, 6th International Conference on Research in Air Transportation (ICRAT), 599, 2014.

[4] H. Chen, Y. J. Zhao, “Sequential Dynamic Strategies for Real-Time Scheduling of Terminal Traffic”, Journal of Aircraft, Vol. 49, No. 1, pp. 237-249, 2012.

[5] EUROCONTROL, “User Manual for the Base of Aircraft Data (BADA) revision 3.10”, EEC Technical/Scientific Report, No. 12/04/10-45, 2012.

[6] D. Toratani, S. Ueno, T. Higuchi, “Simultaneous Optimization Method for Trajectory and Sequence for Receding Horizon Guidance in Terminal Area”, SICE Journal of Control, Measurement, and System Integration, Vol. 8, No. 2, pp. 144-153, 2015.

[7] D. Toratani, S. Ueno, T. Higuchi, “A Study on Optimal Conflict-Free Trajectory for Climbing Aircraft in Terminal Area”, 航空宇宙技術, Vol. 14, pp. 105-112, 2015.

[8] A.E. Bryson, Jr., Y.C. Ho, “Applied Optimal Control: Optimization, Estimation, and Control”, pp. 42-89, Taylor & Francis Group, 1975.