Air Traffic Complexity Versus Control Workload

Daniel Delahaye, Stéphane Puechmorel

Air Traffic Complexity Versus Control Workload

D.Delahaye and S.Puechmorel

Abstract T
his paper presents a new air traffic complexity metric based on dynamical systems. Based on a set of radar observations (position and speed) a vector field interpolating these data is constructed. Once the field has been obtained, the Lyapunov spectrum of the associated dynamical system is computed on points evenly spaced on a spatial grid. The results of the computations are summarized on complexity maps, with high values indicating areas to avoid or to carefully monitor. A first approach based on linear dynamical system enable to compute an aggregate complexity metric. In order to produce complexity maps, an extension of the previous approach have been developed. Based on such complexity metric, a control workload predictor has been developed validated on a real operational airspace.

1 Introduction

In a control sector, the higher the number of aircraft, the more the control workload increases (in a non-linear manner). A limit exists after which the controllers in charge of a control sector are unable to accept additional aircraft, obliging these new aircraft to travel around the sector, moving through less charged neighboring sectors. In this case, the sector is said to be saturated. This critical state should be avoided, as it provokes a cumulative overloading phenomenon in preceding sectors which can back up as far as the departure airport. The saturation threshold is very difficult to estimate, as it depends on the geometry of routes traversing a sector,

D.Delahaye
MAIAA , French Civil Aviation University, 7 Ave Edouard Belin 31055 Toulouse FRANCE, e-mail: delahaye@recherche.enac.fr

S.Puechmorel
MAIAA , French Civil Aviation University, 7 Ave Edouard Belin 31055 Toulouse FRANCE, e-mail: stephane.puechmorel@.enac.fr

the geometry of the sector itself, the distribution of aircraft along routes, the performances of the control team, etc. One widely accepted threshold is fixed at 3 conflicts and 15 aircraft for a given sector. This maximum load should not last for more than ten minutes as it places the controllers under considerable stress, with the risk that they will no longer be able to manage traffic in optimal safety conditions.

The control workload measurement is critical in many domains of ATM as it is at the heart of optimization processes. Examples include the following applications:

- Airspace comparison (US/Europe).
- Validation of future concepts (SESAR,NEXTGEN,etc.).
- Analysis of traffic control action modes (situation before and after control).
- Optimization of sectorization.
- Optimization of sector grouping and de-grouping (pre-tactical alert: anticipation of an increase in congestion in a group of sectors in order to carry out degrouping in an optimal manner).
- Optimization of traffic assignment.
- Determination of congestion pricing zones.
- Organic control assistance tools.
- Generation of 4D trajectories.
- Prediction of congested zones.
- etc.

The operational capacity of a control sector is currently measured by the maximum number of aircraft able to traverse the sector in a given time period. This measurement does not take account of the orientation of traffic and considers geometrically structured and disordered traffic in the same manner. Thus, in certain situations, a controller may continue to accept traffic even if operational capacity has been exceeded (structured traffic); in other situations, controllers may be obliged to refuse additional traffic even though operational capacity has not yet been reached (disordered traffic). Thus, a measurement in terms of the number of aircraft per unit of time constitutes an insufficient metric for the representation of the difficulty level associated with a particular traffic situation.

In the context of operational control, the ideal would be to find a metric which precisely measures the level of mental effort needed to manage a set of aircraft. Without going quite so far, it is possible to find complexity metrics which go beyond a simple measurement of the number of aircraft. We shall begin by clarifying two essential notions for use in the rest of this chapter:

- **Control workload**: measurement of the difficulty for the traffic control system of treating a situation. This system may be a human operator or an automatic process. In the context of operational control, this workload is linked to the cognitive process of traffic situation management (conflict prediction and resolution, trajectory monitoring, etc.).

- **Traffic complexity**: intrinsic measurement of the complexity associated with a traffic situation. This measurement is independent of the system in charge of the traffic and is solely dependent on the geometry of trajectories. It is linked to sensitivity to initial conditions and to the inter-dependency of conflicts. Incertitude with respect to positions and speeds increases the difficulty of predicting future trajectories. In certain situations, this incertitude regarding future positions can increase exponentially, making the system extremely complex in that it is virtually impossible to reliably extrapolate a future situation. When a future conflict is detected, a resolution process is launched which, in certain situations, may generate new conflicts. This inter-dependency between conflicts is linked to the level of mixing between trajectories. As an example, figure 1 presents three traffic situations classed according to increasing level of difficulty as a function of the level of predictability and of inter-dependency between trajectories.

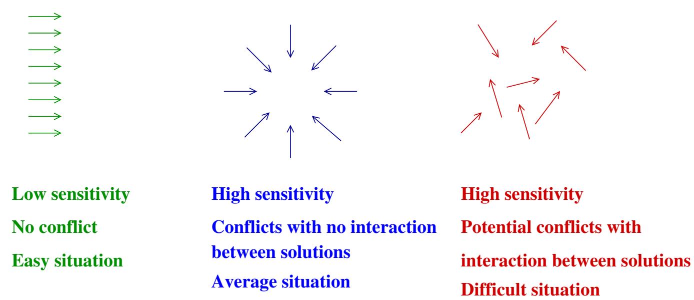

Fig. 1 Three traffic situations classed by increasing order of complexity

One way of interpreting these notions is to imagine oneself to be in charge of each situation in a context where the radar imaging equipment has ceased to work. Naturally, our attention is immediately focused on the situation on the right, as it is difficult to predict (in terms of the appearance of conflicts) and presents a high level of inter-dependency between trajectories. The middle solution, which presents a significant risk of conflict, is easy to manage as the same direction order must simply be given to all of the aircraft (+90 or -90 degrees) in order to place them into safe roundabout trajectories. Finally, in the situation on the left, the trajectories do not present any difficulties and the relative distance between the aircraft will be maintained, at least for the immediate future.

Research into air traffic complexity metrics has attracted considerable attention in recent years, particularly in the United States and in Europe. The first projects were launched in Germany in the 1970s, and since then the subject has continued to develop. Currently, NASA, MIT and Georgia Tech are involved in work on the subject within the framework of the NextGen project. In Europe, the DSNA, the DLR and the NLR are involved in similar activities linked to SESAR.

The objective of most of this work is to model the control workload associated with given traffic situations. The main approaches are as follows:

- Workload model based on traffic level [9]. This approach defines the workload as the proportion of control time over an hour, taking account of the average duration of routine control tasks for an aircraft, the average time taken to resolve conflicts per aircraft, the average rate of arrivals in a sector per hour and the average rate of conflicts in a sector per hour.
- Queue-based model [8]. In this case, a control sector is modeled as a system receiving an input (airplanes) and providing a service, allowing the aircraft to traverse the sector in safety. The sector may then be modeled as a service center including one or more servers and an airplane queue. By applying queuing theory, this approach allows us to determine a maximum acceptable arrival rate for a sector.
- Model based on airspace structure [5]. In this case, the capacity of a sector is based solely on its structure (flight levels, routes, route intersections, etc.).
- Dynamic density [6, 10, 2]. This model, developed by NASA, consists of measuring a set of traffic characteristics (number of changes in direction, changes in speed, changes in altitude, etc.) and the workload experienced by a controller, then carrying out linear regression in order to adjust the model to the experienced workload as precisely as possible.

  This model of control workload presents the two following drawbacks:

  1. incapacity for generalization to new sectors
  2. modeling is highly dependent on the controllers used to infer the model.

Other approaches [4, 7] model the complexity of a traffic situation using automatic conflict resolution algorithms, for which we measure the number of trajectory modifications required in processing a given situation. In the same way as before, these methods are highly dependent on the type of algorithm used to resolve conflicts. These considerations have led us to develop intrinsic traffic complexity metrics which are only linked to trajectory structure, and not to the system used to process them.

The next section presents the new trajectory complexity metric we have developed based on a non linear dynamical system modeling of the air traffic. An application the the real operation Reims airspace is then presented in the following section.

2 Trajectory Complexity Metric Based on Non Linear Dynamical Systems

2.1 Non linear Dynamical System modeling of Air traffic

A non linear dynamical system is summarized by the following equation :

$$\dot{\mathbf{X}}(t) = \mathbf{f}(\mathbf{X}) \tag{1}$$

where $\mathbf{X}$ is the state vector of the system ($\mathbf{X} = [x,y,z]^T$) and $\mathbf{f}$: C^2 vector field, describe systems which integral curves may fit the observed trajectories. This equation associates a vector speed $\dot{\mathbf{X}}$ to a position in the space coordinate $\mathbf{X}$ and then synthesis a particular vector field. Based on the observations of the aircraft (positions, speed vectors), the dynamical system has to be adjusted with the minimum error.This fitting is done with a Least Square Minimization (LMS) method for which the following criterion is used :

$$E_1 = \sum_{i=1}^{i=N} \|\mathbf{V_i} - \mathbf{f}(\mathbf{X}_i)\|^2 \tag{2}$$

where N is number of observations.

If we consider criterion E_1 only, it can be shown that there is an infinite number of vector fields $\mathbf{f}$ which can be adjusted to the observations. In order to keep the smoothest one, another criterion is added which has to be minimized, the so-called "div-curl" criterion :

$$E_2 = \int_{R^3} \alpha \|\nabla div\mathbf{f}(\mathbf{X})\|^2 + \beta \|\nabla curl\mathbf{f}(\mathbf{X})\|^2 d\mathbf{X} \tag{3}$$

with α, β positive weights controlling the smoothness of the approximation by focusing on constant divergence or constant curl. In the following, we will consider $\alpha = \beta = 0.5$; in such case :

$$E_2 = \|\Delta \mathbf{f}(\mathbf{X})\|^2$$

where $\Delta \mathbf{f}(\mathbf{X})$ is the Laplacian of the vector field $\mathbf{f}$.

The joint minimization of E_1 and E_2 induces a unique optimum vector field [1] :

$$\mathbf{f}(\mathbf{X}) = \sum_{i=1}^{N} \phi(\|\mathbf{X} - \mathbf{X_i}\|).\mathbf{a}_i + \mathbf{A}.\mathbf{X} + \mathbf{B}$$

with $\mathbf{a}_i$ parameter vectors (one for each observation),

$$\phi(\|\mathbf{X} - \mathbf{X_i}\|) = \mathbf{Q}(\|\mathbf{X} - \mathbf{X_i}\|^3)$$

and

$$Q = \begin{bmatrix} \gamma\ 0\ 0 \\ 0\ \gamma\ 0 \\ 0\ 0\ \gamma \end{bmatrix}$$

with $\gamma = \partial^2_{xx} + \partial^2_{yy} + \partial^2_{zz}$
The resulting adjustment is done without error ($\Rightarrow minE_1 = 0$).
When $\alpha = \beta = 0.5$, the vector spline function ϕ has the following structure :

$$\phi(\|\mathbf{X} - \mathbf{X_i}\|) = 12.\|\mathbf{X} - \mathbf{X_i}\|$$

($r_i = \|\mathbf{X}_i - \mathbf{X}\|$). It must be noticed, that farthest observations has more weight in such calculation. Then, in order to compute the smoothest vector field which fit exactly the measures, all observations have to be taken into account in the computation.

2.2 Lyapunov Exponents

The metric chosen for complexity computation relies on a measure of sensitivity to initial conditions of the underlying dynamical system called Lyapunov exponents. In order to figure out what Lyapunov exponents are, let consider a point and look at its evolution when transported by the dynamical system. Let $\mathbf{x_0}$ be fixed (initial point) and let γ be a point trajectory of the dynamical system associated to the vector field $\mathbf{f}$ given by :

$$\gamma(t, \mathbf{x_0}) = \mathbf{x_0} + \int_0^t \mathbf{f}(u, \gamma(u, \mathbf{x}))du \tag{4}$$

Assume now that trajectory is disturbed by a small perturbation ε, we have :

$$\gamma(t, \mathbf{x}_0 + \varepsilon) = \gamma(t, \mathbf{x}_0) + \nabla_{\mathbf{x}}\mathbf{f}(\gamma(t, \mathbf{x})).\varepsilon + o(\|\varepsilon\|)$$

where $\nabla_{\mathbf{x}}\mathbf{f}(t, \gamma(t, \mathbf{x}))$ is the differential of the vector field $\mathbf{f}$ at $\mathbf{x}$. Divergence to nominal trajectory with respect to time is thus $\|\gamma(t, \mathbf{x}_0) - \gamma(t, \mathbf{x})\| = D(t, s)$ (see figure 2). $\gamma(t, \mathbf{x})$ being defined as a flow :

$$\frac{\partial \gamma(t, \mathbf{x})}{\partial t} = \mathbf{f}(t, \gamma(t, \mathbf{x})) \quad \gamma(0, \mathbf{x}) = \mathbf{x}$$

with $\mathbf{f}$ a smooth vector field, it is possible to show that $D(t)$ satisfies a differential equation also. Given a nominal trajectory $\gamma(t, \mathbf{x}_0)$, then divergence of nearby trajectories can be found up to order one in $\|\mathbf{x} - \mathbf{x}_0\|$ by solving :

$$\frac{\partial D(t, \mathbf{x})}{\partial t} = \nabla_{\mathbf{x}}\mathbf{f}(t, \gamma(t, \mathbf{x})).D(t, \mathbf{x}) \quad D(0, \mathbf{x}) = \|\mathbf{x} - \mathbf{x}_0\|$$

If the three space dimensions are considered (x, y, z), and since the previous equation is linear, it can be extended to the matrix form :

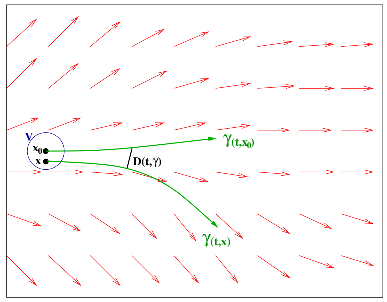

Fig. 2 Time evolution of a reference trajectory and a perturbed trajectory

$$\frac{dM(t)}{dt} = \nabla_{\mathbf{x}}\mathbf{f}(t,\gamma(t,\mathbf{x})).M(t) \quad M(0) = Id$$

Where each column of the M matrix corresponds to the divergence associated to the principal coordinate axis. This equation is called the variational equation of the flow. The variational equation describes in some sense a linear dynamical system "tangent" to the original one. Let $U^t(t)\Sigma(t)V(t) = M(t)$ be the singular value decomposition of $M(t)$. The Lyapunov exponents are mean values of the logarithms of the diagonal elements of $\Sigma(t)$:

$$\kappa(s) = -\frac{1}{T}\int_0^T \log(\Sigma_{ii}(t))dt \;\; \forall \Sigma_{ii}(t) \leq 1$$

When Lyapunov exponents are high, the trajectory of a point under the action of the dynamical system is very sensitive to initial conditions (or to the parameters on which the vector field may depend), so that, situation in the future is unpredictable. On the other hand, small values of the Lyapunov exponents mean that the future is highly predictable (expected to be comfortable for a controller).

So, the Lyapunov exponent map determines the area where the underlying dynamical system is organized. It identifies the places where the relative distances between aircraft do not change with time (low real value) and the ones where such distance change a lot (high real value).

Let us now describe the practical algorithm for computing complexity maps.

1. **Dynamical regression based on radar observations.**
2. **The vector field and its associated gradient is then computed on each point of a cube of airspace.**
3. **Lyapunov exponents are computed at each point of the cube by the mean of a Runge-Kutta integration.**

The figure 3 shows an example of Lyapunov exponents map for which full organized miles in trail trajectories (from south west to north east) cross two random traffic situations. This figure shows clearly a complexity valley on the miles in trail

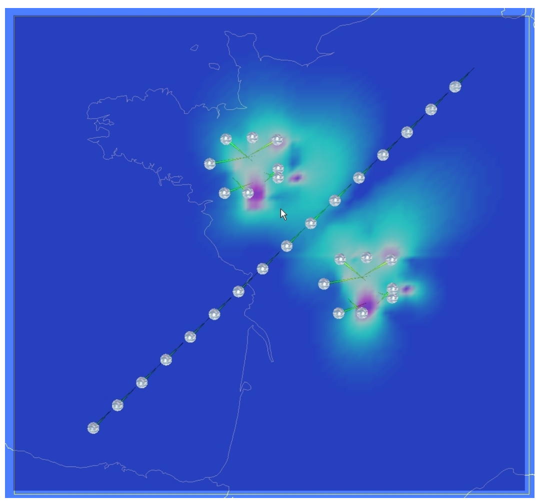

Fig. 3 Miles in trail traffic between disordered areas

direction. This organization may have been detected even if the miles in trail trajectories would have been structured on a curve trajectory. That is the strong point of this metric: **Lyapunov exponents are able to identify any kind of trajectory organization** (when aircraft follow the same trajectory at the same speed).

This approach is able to produce a picture of the complexity at a given time. By summing the Lyapunov exponents in a given airspace on can extract a scalar metric which has been used in the Reims experiments presented in the next section.

3 Workload Prediction based on complexity metric

Based on the previous complexity algorithm, we propose to compare our metric to the workload measured on a real control sector. To do such experiments, we have used some real traffic sample of control sector belonging to the Reims airspace area (see figure 4).

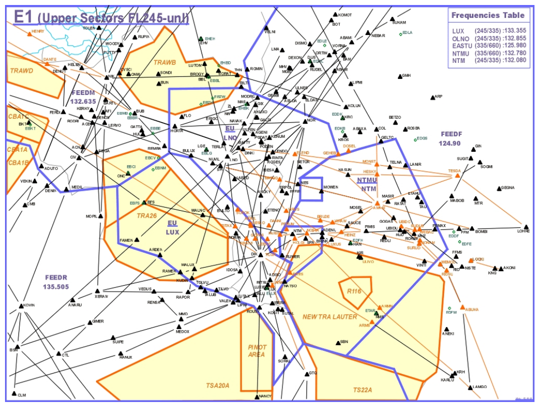

Fig. 4 Airspace used for our experiments

Then, we have considered the traffic crossing this airspace for the 14 February 2013 from 10:00 till 12:00AM. The controller in charge of such traffic have been invited to quantify the workload they feel by using a dedicated HMI. Such workload measurements have been done every five minutes. These measurements have been recorded and aggregated in vector a $\{X_i\}_{i=1..N}$. For the same traffic samples we have measured the associated traffic complexity at the same step and a vector of complexity measures has also been built : $\{Y_i\}_{i=1..N}$.

The objective of the study is to establish the level of correlation between those two time series. The first guess was to used the standard correlation measure :

$$C_{XY} = \sum_{i=1}^{N} (X_i - \bar{X})(Y_i - \bar{Y}) \tag{5}$$

It is maximum when $(X - \bar{X}) = \lambda(Y - \bar{Y})$.

However, since workload values given by the controllers are only relative, such a correlation may gives false results, we have decided to used a rank correlation approach which indicates if both series evolve in the same direction.

First, we consider any pair of samples in both series and we introduce the following variable :$\varepsilon_{ij} = (X_i - X_j)(Y_i - Y_j)$ and four situations can be identified :

ε_{ij}	$(X_i - X_j)$	$(Y_i - Y_j)$
+	+	+
+	+	+
-	+	-
-	-	+

In the first two cases the method classify the times series samples in the same way.

Then, we introduce the variable $\sigma = \frac{1}{N(N-1)} \sum_{i>j} \varepsilon_{ij}$ which indicate how often both series vary in the same direction. If σ is close to one or minus ones both series are classified in the same way but if σ is close to zero no conclusion can be given.

The confidence level was checked using a Spearman test [3].

$H_0 : \sigma = 0$

$Pr\{$ reject H_0/H_0 true$\}$

To quantify such hypothesis the P_{value} is computed. If such P_{value} is close to zero, then the correlation is not linked to hazard.

Based on the traffic samples the overall rank correlation value reaches 0.78 which is very good number based on the data we get from operation.

The associated $P_{Value} = 10^{-6}$ meaning that such correlation meaningful.

The evolution of both series is given on figure 5.

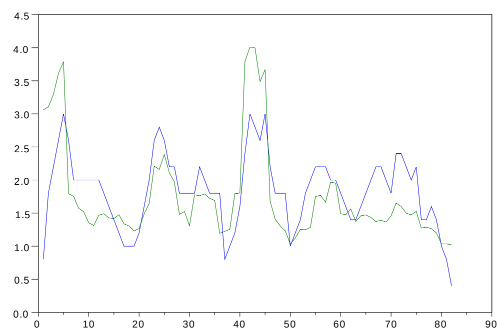

Fig. 5 Times series evolution. Those curves represent the evolution of the control workload and the associated complexity with time. To built those curves we consider the Reims airspace and the associated real traffic. Every two minutes we request the controller to rank the workload he feels. Those measures are then aggregated on the blue curve (with a time window filtering in order to get a smooth curve).The green curve represents the complexity measures by our algorithm for the same time samples.

4 Conclusion

We have presented in this paper a new air traffic complexity metric based on vector field model of air traffic. Most of the work has been devoted to improvements in interpolating splines used to fit a vector field to the observation. Unlike linear models which produce mean complexity indicators, the non-linear one may give local information, thus providing a way of displaying maps of complexity. Based on this complexity metric, a new control workload predictor has been developed and validated on a real operational airspace.

References

1. L Amodei and M.N Benbourhim, *A vector spline approximation*, Journal of Approximation Theory **67** (1991), 51–79.
2. G.B Chatterji and B. Sridhar, *Measure for air traffic controller workload prediction*, Proceedings of the First AIAA Aircraft Technology, Integration, and Operation Forum, AIAA, 2001.
3. E. C. Fieller, H. O. Hartley, and E. S Pearson, *ests for rank correlation coefficients*, Biometrika **44** (1957), 470–481.
4. G. Granger and N. Durand, *A traffic complexity approach through cluster analysis*, Proceedings of the US Europe ATM Seminar, Eurocontrol-FAA, 2003.
5. M. Janic and V. Tosic, *En route sector capacity model.*, Transportation Science **25** (1991), no. 4.
6. I.V. Laudeman, S.G. Shelden, R. Branstrom, and C.L. Brasil, *Dynamic density : an air traffic management metric.*, Tech. Report TM-1998-112226, NASA, 1998.
7. K. Lee and A. Feron, E.and Prichett, *Air traffic complexity : An input-output approach*, Proceedings of the US Europe ATM Seminar, Eurocontrol-FAA, 2007.
8. L. Maugis and J.B. Gotteland, *Techniques de détermination de la capacité des secteurs de contrôle de l'espace aérien: statistiques et simulations.*, Tech. report, Centre d'études de la navigation aériene, 1997.
9. D. Schmidt, *A queueing analysis on the air traffic controller's workload.*, IEEE transactions on Systems, Man, and Cybernetics **8** (1978).
10. B. Sridhar, K.S. Seth, and S. Grabbe, *Airspace complexity and its application in air traffic management*, Proceedings of the Second USA/EUROPE ATM R&D Seminar, Eurocontrol/FAA, 2001.