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***Abstract*—This paper presents a discussion about the RF link budget for a communication system based on unusual waves carrying a non-zero orbital angular momentum (OAM). A typical configuration using circular antenna arrays of isotropic sources to transmit and receive OAM waves is considered. We point out issues when usual assumptions are made to express the link budget. Then, we propose to express the power budget associated with OAM in an elementary MIMO matrix approach. Using a single mode communication configuration, some basic numerical calculations and case studies based on the developed formulations can then be led and discussed, highlighting some intrinsic limitations. Indeed, the asymptotic behavior of the link budget leads to question the concepts of gain and propagation losses.**

***Index Terms*—antenna, link budget, OAM, gain, MIMO, SVD.**

I. Introduction

From fundamental physics, it has been proven that electromagnetic waves are carrying both linear and angular momentum [1]. Since the thirties, the mechanical properties of the angular momentum have been demonstrated theoretically and experimentally [2]. The total angular momentum can be divided into spin angular momentum (SAM), which refers to polarization, and orbital angular momentum (OAM), which is related to the spatial distributions of the field magnitude and phase [3].

On the one hand, the SAM, or polarization, with two orthogonal modes is now well known and widely exploited in operating communication systems. On the other hand, the OAM has not yet been used, even though it represents a fundamental new degree of freedom that might be a powerful asset for many applications like improving radio capacities as controversially discussed in several recent papers [4], [5], [6].

During the last two decades, the photon OAM has been widely studied in optics with Laguerre-Gaussian beams [3], [7]. For some years, it has been proven that OAM can be used in the radiofrequency domain, theoretically [8], [9] and experimentally [4]. A circular antenna array [9], a plane or spiral phase plate [10], [11] or a helicoidal parabolic antenna [4], with dedicated modifications, can generate an electromagnetic wave carrying non-zero OAM. To receive such a wave, a kind of an interferometer appears to be sufficient [4]. However, the way experiments are made has to be discussed with the background of traditional communication using multiple-input-multiple-output (MIMO) antenna system [5], [6].

In this article, we propose to analyze the link between two antennas designed to transmit and receive OAM, and to discuss its efficiency for RF communications. Similar to [4], [5] and [6], a basic configuration using circular antenna arrays both to transmit and receive one single OAM mode will be studied. The link budget is then developed in details and used to question the antenna gain and its asymptotic behavior for different mode orders.

The paper is organized as follows. In Section II, we introduce the classical theory for OAM and link budget. In Section III, the basic geometrical parameters are introduced. The power transfer in an OAM link is then expressed in its simplest form. In Section IV, we perform case studies for different positions of the receiving antenna to question the asymptotic behavior which lead to criticize the concepts of antenna gain and propagation losses.

II. Classical theory

A. Orbital angular momentum

An OAM mode is defined by its integer order l. Remembering the results of [9] regarding modal radiation patterns, we can see in Fig. 1 that, assuming $l = 0$, the main lobe is oriented as usual in the aimed direction of the receiving antenna.

For other values of l, the waves are of vortex shapes with no axial main lobe. The mode mainly radiates energy on a cone which the opening angle of this cone is slightly different when modifying l: the greater the OAM mode, the larger the cone angle.

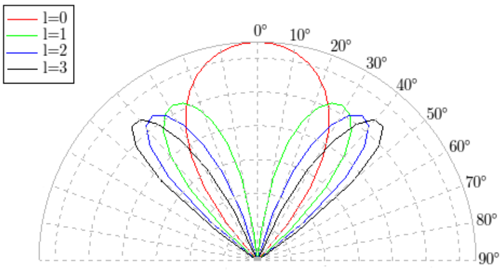

Fig. 1. Gain patterns of modes $l = 0, 1, 2, 3$.

B. Link budget

A link budget addresses the quality of a communication system. It sums all the gains and losses from the transmitter to the receiver, associated to both the antennas and the propagation medium (Fig. 2).

In its most concise form, its equation is given by

$$P_r = \frac{P_e G_e G_r}{L_{FS}} \tag{1}$$

where P_e is the emitted power, G_e the transmitting antenna gain, P_r the received power, G_r the receiving antenna gain and L_{FS} the free space losses.

Firstly, we want to calculate this link budget for two monomode OAM antennas facing each other [4], [6]. The Tx antenna radiates an OAM mode of order l and the Rx antenna receives the same OAM mode.

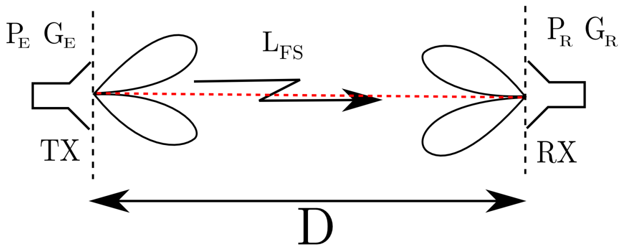

Fig. 2. Standard configuration for the transmission of waves with non-zero OAM.

This configuration presents issues. When l differs from 0, no field is radiated in the direction of the receiver as shown in Fig. 2. So, no power is transmitted to the receiver, because both $G_e = 0$ and $G_r = 0$ in (1).

Even for vortex waves, if we consider that we are in the very far field, for a classical system we would consider local plane waves coming normally to the receiver. In that case, we could not be able to detect any radio-OAM power at the ports of the receiving antenna because the OAM of a plane wave is equal to zero [12].

In this section, we have pointed out some problems in the calculation of the link budget. Indeed with classical considerations, it appears that power cannot be neither transmitted nor received with OAM waves. This asymptotic far-field behavior questions the capability of OAM waves to support communications.

III. Extended Theory

A. Configuration/Setup

Antennas usually do not generate electromagnetic waves with non-zero orbital angular momentum.

To radiate and receive fields carrying a non-zero OAM, we can use a configuration with two circular phased arrays. For the transmitting antenna, the N_{Tx} array elements are distributed equidistantly along the perimeter of a circle. They are phased with a beam forming network (BFN) in such a way that the phase at element $n \in \{0, \ldots, N_{\text{Tx}} - 1\}$ is given by

$$\phi_n = 2\pi \frac{ln}{N_{\text{Tx}}} \tag{2}$$

where l is the desired OAM order [8]. For the sake of simplicity, we further consider that each array element is an isotropic antenna and mutual couplings are negligible. Non-isotropic radiation patterns could be considered for more realistic system analysis. This would not impact the main conclusions of this paper. The receiving antenna is of the same type but may have a different radius R_{Rx} or number of elements N_{Rx}. No hypothesis is made here on the relative position of the receiver with respect to the transmitter in order to achieve a generic formulation. The global configuration is depicted in Fig. 3. In [4], [5], [6], the same configuration is used but the arrays are facing each other, *i.e.* both are centered and orthogonal to a common axis.

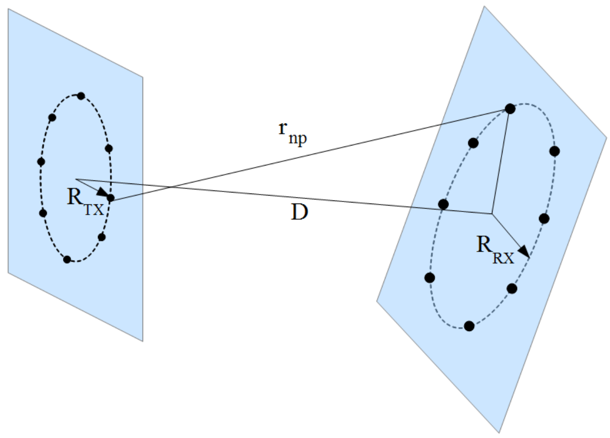

Fig. 3. Global configuration of the system with two circular arrays.

B. OAM link budget

In Fig. 4 we describe the configuration with which OAM modes of orders l are sent and OAM modes of orders l' are detected.

The transmission can be divided into three blocks: the beam forming network (BFN) of the transmitter, the propagation channel, and the BFN of the receiver. We will study one by one the contribution of each block.

The first block is the BFN of the transmitter. When we want to transmit one OAM mode of order l and amplitude a_l^{OAM}, the

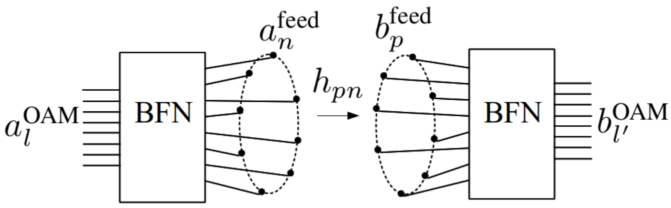

Fig. 4. Link schematic for OAM.

BFN must be built in such way that the n-th antenna element is fed with

$$a_n^{\text{feed}} = \frac{a_l^{\text{OAM}}}{\sqrt{N_{\text{Tx}}}} \exp\left(j2\pi \frac{ln}{N_{\text{Tx}}}\right), n \in 0, \ldots, N_{\text{Tx}} - 1 \quad (3)$$

We can generalize this result by considering that the BFN has N_{Tx} input ports associated with the transmission of OAM modes of order $l \in \{0, \ldots, N_{\text{Tx}} - 1\}$, considering usual sampling results. N_{Tx} is the number of output ports that feed the array elements. It can also be noticed that although the number of modes is limited with N_{Tx}, the orders can be conveniently shifted to negative values as well.

From (3), the matrix that relates the output a_n^{feed} to the output a_l^{OAM} is the matrix of the discrete Fourier transform of size N_{Tx}, denoted $\mathbf{U}$.

The second block is the propagation channel, which can be characterized by the channel matrix $\mathbf{H}$. Its terms h_{pn} correspond to the propagation from the element n of the transmitter to the element p of the receiver. Because, we have assumed ideal isotropic array elements, we simply have

$$h_{pn} = \exp\left(-jkr_{np}\right) \frac{\lambda}{4\pi r_{np}} \quad (4)$$

where r_{np} is the distance from element n of the transmitter to element p of the receiver.

The third block is the BFN of the receiver. In order to receive at its output the OAM mode of order l', it must be so that

$$b_{l'}^{\text{OAM}} = \frac{1}{\sqrt{N_{\text{Rx}}}} \sum_{p=0}^{N_{\text{Rx}}-1} b_p^{\text{feed}} \exp\left(-j2\pi \frac{pl'}{N_{\text{Rx}}}\right), \quad (5)$$
$$p \in 0, \ldots, N_{\text{Rx}} - 1$$

Generalizing to the reception of OAM modes of order $l' \in \{0, \ldots, N_{\text{Rx}} - 1\}$, the BFN can be characterized by the matrix $\mathbf{V}^H$ where $\mathbf{V}$ is the matrix of the discrete Fourier transform of size N_{Rx}.

Finally taking into account the 3 blocks, the OAM link can be characterized by the matrix

$$\mathbf{H_t} = \mathbf{V}^H \mathbf{H} \mathbf{U} \quad (6)$$

This matrix takes into account all the parts of the link. Thus, it can be used to derive the power transfer in the context of various OAM transmissions, regarding the versatility of the geometry and the multimode formulation.

We consider the transmission and reception of one single mode of order l. Developing the matrix-vector product, the ratio between the output and input powers is explicitly given by

$$\frac{P_r}{P_e} = \left|\frac{b_l^{\text{OAM}}}{a_l^{\text{OAM}}}\right|^2 = \left| \sum_{p=0}^{N_{\text{Rx}}-1} \sum_{n=0}^{N_{\text{Tx}}-1} \frac{1}{\sqrt{N_{\text{Rx}} N_{\text{Tx}}}} \right. \quad (7)$$
$$\left. \exp\left(j2\pi l \left(\frac{n}{N_{\text{Tx}}} - \frac{p}{N_{\text{Rx}}}\right)\right) \exp\left(-jkr_{np}\right) \frac{\lambda}{4\pi r_{np}} \right|^2$$

This expression of the received power is unusual compared to (1), even asymptotically when $D \to \infty$. The link budget may depend on the mode order. The capability to define antenna gains G_e, G_r, and free space losses L_{FS} will be numerically addressed in the next section.

IV. Simulations for Identical Arrays Facing Each Other

We have proposed a basic but general tool to focus on the mode-per-mode OAM link budget. In this section, simulations are performed for two identical arrays facing each other. As in [9], we choose $N_{\text{Tx}} = N_{\text{Rx}} = 12$ and $R_{\text{Tx}} = R_{\text{Rx}} = \lambda$. Besides, the frequency is 2.45 GHz so that $\lambda = 0.12$ m and the Fraunhofer distance is $d_{\text{Fraunhofer}} = 0.48$ m.

In the Fig. 5, the achieved radiation pattern is depicted for $l = 1$. We observe a conical mainlobe, and also sidelobes that are due to the 2λ diameter of the antenna.

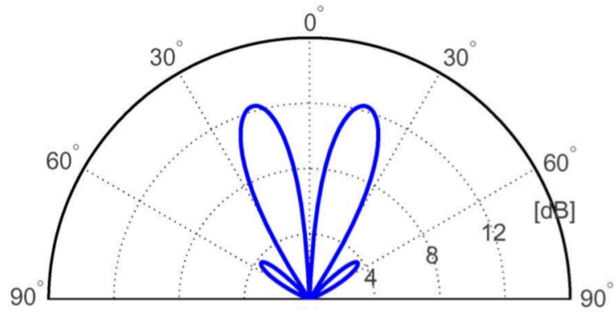

Fig. 5. Radiation pattern for a $N_{\text{Tx}} = 12$ element array antenna and radius $R_{\text{Tx}} = \lambda$. (From Mohammadi *et al* [9])

As a first link budget result, the received to emitted powers ratio (7) is plotted with respect to the distance in Fig. 6.

The asymptotic behavior appears to be decreasing as expected in the far field zone. To define a global antenna gain K we simply compensate the free space losses in (7). We obtain

$$K = \frac{P_r}{P_e} \times L_{FS} \quad (8)$$

The value of K is displayed for various mode orders in Fig. 7.

For an OAM mode $l = 0$, the gain K is constant in the far field, so that the link budget attenuation is only due to free space losses. In this case, K is asymptotically equal to $G_e G_r$.

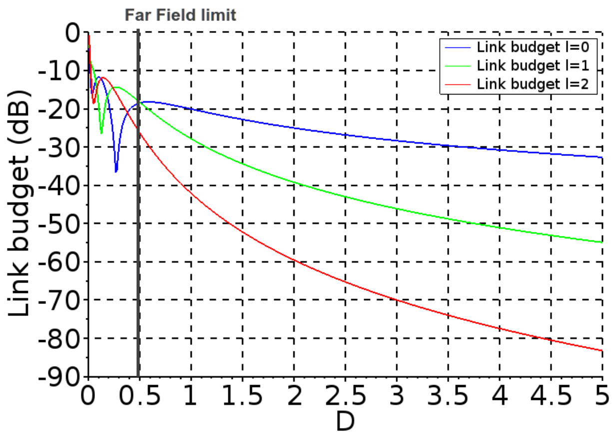

Fig. 6. Link budget as a function of the distance and the mode order l.

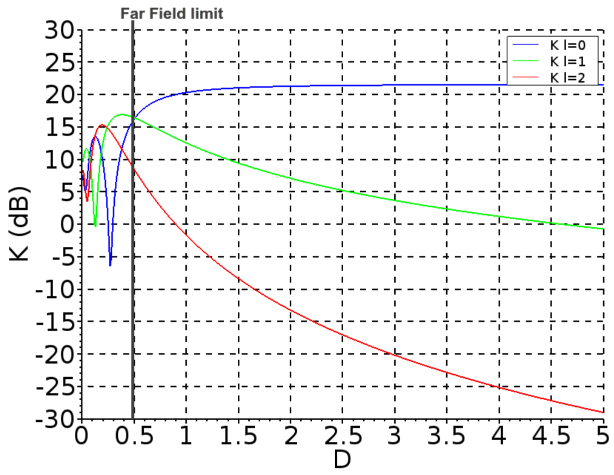

Fig. 7. Mode to mode link gain K in dB.

This is a classical result of link budget without OAM. This leads here to $K(dB) = 10\log(N_{\text{Tx}} \times N_{\text{Rx}}) = 21.6$ dB.

For a non-zero mode order ($l \neq 0$), the gain K is no longer constant and falls rapidly. The gains are dependent to D and l, which is obviously not a standard behavior.

Therefore, communications with a non-zero OAM order ($l \neq 0$) using the same type and size of transmitting and receiving antennas have a poor link budget because of its asymptotic behavior in the far-field.

Such a result has been predicted in [6] for the special case $l = 1$ and for a 2×2 MIMO system. Indeed, in this case, the authors have demonstrated that both electric and magnetic fields have a $1/D^4$ attenuation in power, different from the classical $1/D^2$ attenuation.

Instead of deriving a general demonstration from (7), we here propose an *a priori* attenuation in $1/D^{2l+2}$. In order to test this hypothesis, Fig. 8 shows the evaluation of $P_r/P_e \times D^{2l+2}$.

We observe that the expression $\dfrac{P_r}{P_e} \times D^{2l+2}$ is asymptotically constant when $D \to \infty$ for $l = 0, 1, 2$. Other values of l have been tested and present the same asymptotic behavior

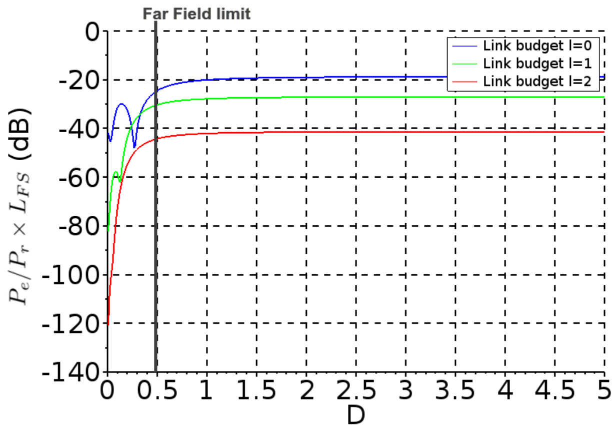

Fig. 8. Test of the $1/D^{2l+2}$ decreasing assumption.

which validates this model for the power attenuation with the distance D. Besides, such a result is consistent with Tamagnone's conclusion for $l = 1$.

V. Conclusion

Applying conventional link budget theory for OAM monomode radio communication in a standard face-to-face configuration directly leads to zero received power due to zero antenna gains. A careful development of the non asymptotic link budget allows testing this conclusion as it has been done numerically in the present paper. A $1/D^{2l+2}$ decrease is then observed. Unusually, this result depends on the modal order l and decreases faster than the standard $1/D^2$. A special attention has been paid on a progressive presentation of the results and interpretations considering the controversial context of this topic.

It could be of great interest to extend this study to other configuration, especially when antennas are not facing each other. Several cases like the ones shown in Fig. 9 could be investigated with the presented tool.

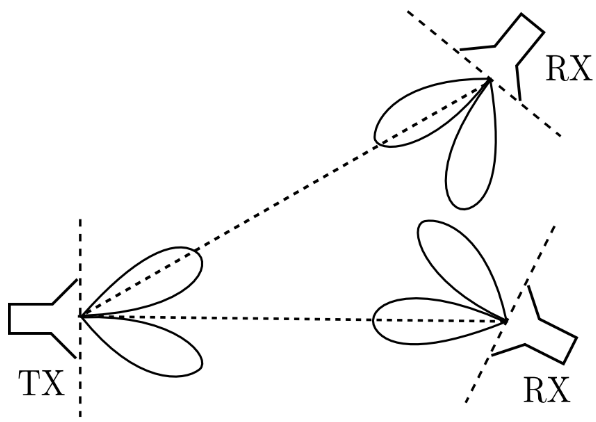

Fig. 9. Possible configuration for the transmission of OAM waves with non-zero power for at least one of the antennas Tx/Rx.

Besides, once the propagation losses have been correctly removed from the link budget, the remaining coefficient could be studied for various antenna parameters, *e.g.* the array radii, the numbers of sources, the modal orders.

Furthermore, as for MIMO, an analysis based on a singular value decomposition (SVD) of the matrix (6) can be performed to decompose the link in terms of several orthogonal modes. They are given by the singular vectors. The mode-per-mode link budget is simply achieved by the squared singular values. This would also enable to express the MIMO capacity gain.

Note that, in the case where the two antennas are aligned and facing each other, Edfors [5] has demonstrated that singular vectors exactly correspond to OAM modes because **H** is a circulant matrix.

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REFERENCES

[1] J. Poynting, "The wave motion of a revolving shaft, and a suggestion as to the angular momentum in a beam of circularly polarised light," *Proc. R. Soc. Lond. A*, vol. 82, no. 557, pp. 560–567, 1909.

[2] R. A. Beth, "Mechanical detection and measurement of the angular momentum of light," *Phys. Rev.*, vol. 50, pp. 115–125, Jul 1936.

[3] L. Allen, M. W. Beijersbergen, R. J. C. Spreeuw, and J. P. Woerdman, "Orbital angular momentum of light and the transformation of laguerre-gaussian laser modes," *Phys. Rev. A*, vol. 45, pp. 8185–8189, Jun 1992.

[4] F. Tamburini, E. Mari, A. Sponselli, B. Thid, A. Bianchini, and F. Romanato, "Encoding many channels on the same frequency through radio vorticity: first experimental test," *New Journal of Physics*, vol. 14, no. 3, p. 033001, 2012.

[5] O. Edfors and A. Johansson, "Is orbital angular momentum (oam) based radio communication an unexploited area?" *Antennas and Propagation, IEEE Transactions on*, vol. 60, no. 2, pp. 1126–1131, 2012.

[6] M. Tamagnone, C. Craeye, and J. Perruisseau-Carrier, "Comment on encoding many channels on the same frequency through radio vorticity: first experimental test," *New Journal of Physics*, vol. 14, no. 11, p. 118001, 2012.

[7] M. Padgett and L. Allen, "The poynting vector in laguerre-gaussian laser modes," *Optics Communications*, vol. 121, no. 13, pp. 36–40, 1995.

[8] B. Thide, H. Then, J. Sjoholm, K. Palmer, J. Bergman *et al.*, "On the utilization of photon orbital angular momentum in the low-frequency radio domain," *Phys.Rev.Lett.*, vol. 99, p. 087701, 2007.

[9] S. Mohammadi, L. K. S. Daldorff, J. Bergman, R. Karlsson, B. Thide, K. Forozesh, T. Carozzi, and B. Isham, "Orbital angular momentum in radio - a system study," *Antennas and Propagation, IEEE Transactions on*, vol. 58, no. 2, pp. 565–572, 2010.

[10] A. Bennis, R. Niemiec, C. Brousseau, K. Mahdjoubi, and O. Emile, "Flat plate for oam generation in the millimeter band," *Antennas and Propagation (EuCAP)*, pp. 3203–3207, 2013.

[11] M. Beijersbergen, R. Coerwinkel, M. Kristensen, and J. Woerdman, "Helical-wavefront laser beams produced with a spiral phaseplate," *Optics Communications*, vol. 112, no. 56, pp. 321–327, 1994.

[12] J. Humblet, "Sur le moment d'impulsion d'une onde lectromagntique," *Physica*, vol. 10, no. 7, pp. 585–603, 1943.