# 1-bit processing of composite BOC (CBOC) signals

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# 1-BIT PROCESSING OF COMPOSITE BOC (CBOC) SIGNALS

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## ABSTRACT

The latest development in the definition of Galileo L1 OS signal and GPS III L1C signal led to a joint EU/US statement showing the intention of broadcasting signals with common Power Spectral Density (PSD) functions when computed using all signal components including pilot and data components. This common normalized PSD, denominated Multiplexed BOC (MBOC) PSD, is

given by: $\phi_{L1}(f) = \dfrac{10}{11}\phi_{BOC(1,1)} + \dfrac{1}{11}\phi_{BOC(6,1)}$. To

fulfil this constraint, two possible signal generation options were proposed in [Hein et al., 2006]:

- the Time-Multiplexed BOC (TMBOC), that uses an alternating series of full cycle of BOC(1,1) and BOC(6,1) sub-carrier to modulate the spreading code. The choice of the alternating sequence is made on a chip-by-chip basis and has to respect the final MBOC PSD constraint.
- The Composite BOC (CBOC), that uses a temporal weighted sum of the BOC(1,1) and BOC(6,1) sub-carriers to create a new composite sub-carrier modulating the spreading sequence. The Composite BOC signal is an innovation of CNES and UniBwM, and the (6,1) configuration of the CBOC as well. This configuration is mentioned in [Hein et al, 2005].

These two MBOC-compliant signals offer different advantages and drawbacks in terms of payload architecture or adaptation to a BOC(1,1) receiver for instance. The purpose of this paper is to provide a solution to one drawback inherent to CBOC tracking: its multi-level waveform. Indeed, as already mentioned, the CBOC sub-carrier generation uses a weighted sum

of two binary sub-carriers. This means that the result is not binary anymore. It is well known that the optimal correlation process used in the typical receiver acquisition and tracking stages supposes the generation of the exact replica of the incoming signal. This means that a CBOC receiver will have to encode this local replica on more than 1 bit and have correlators that can accept a multi-bit signal. Thus, more complex receiver architectures are needed, and that could be detrimental to the widespread of a signal using this modulation. To make the CBOC modulation more attractive with that respect, the idea is then to find a solution with performances close to optimal CBOC tracking, but with a binary local replica.

This can be achieved by correlating the incoming CBOC signal with two local replicas in parallel: one using a pure BOC(1,1) sub-carrier, and one using a pure BOC(6,1) sub-carrier. The resulting correlator outputs can then be linearly (and thus coherently) combined to re-form the optimal correlation values. In such a case, optimal tracking can be realized. The problem with this method is that it requires twice as many correlators as the traditional CBOC tracking.

To reduce the number of required correlators, the method presented in this paper uses a sub-carrier inspired from a local TMBOC sub-carrier replica (referred to as TM61 local replica) to track a CBOC signal. Using this replica, the local waveform is binary and can then be encoded on 1 bit. It also requires the same number of correlators as traditional CBOC tracking. However, this technique implies that the correlation process is not optimal and thus tracking will not be optimal either (in terms of mitigation of thermal noise). In particular, the choice of the BOC(1,1)/BOC(6,1) sub-carrier alternating sequence for the TM61 local replica has to be thoroughly studied to obtain the best tracking performances in terms of tracking noise, multipath rejection or loop sensitivity. In particular, the relative time length between the BOC(1,1) part and the BOC(6,1) part in the local replica will strongly influence the magnitude of the correlation loss.

The goal of this paper is to assess the performance of this new CBOC tracking technique with respect to traditional tracking. It first introduces the CBOC modulation and its expected theoretical tracking performances. A brief comparison with the TMBOC modulation is then given. The new TM61 tracking technique is then introduced. Its induced correlation loss is expressed, and a theoretical formulation of the resulting code tracking error, assuming a dot-product-like discriminator, is then given and confirmed by extensive experimental tests. This formula is used to understand the influence of the respective weight of the

BOC(1,1) and BOC(6,1) sub-carriers in the local replica in the tracking performance. It is then shown that, using this method, a multipath rejection capacity equivalent to optimal CBOC tracking can be achieved. Consequently, it is seen that this technique provides tracking capabilities close to traditional CBOC tracking with a lower circuitry complexity.

## INTRODUCTION

The Galileo E1 OS and GPSIII L1C signals are still under a definition phase. The desire of interoperability of both signals has led to a common US/EU agreement that defines a common normalized Power Spectral Density (PSD) for both civil signals referred to as Multiplexed Binary Offset Carrier (MBOC). This PSD includes the whole GPSIII L1C or Galileo E1 OS civil signals, which means both their data and pilot components. Its expression is given as follows:

$$G_{OS}(f) = \frac{10}{11} G_{BOC(1,1)}(f) + \frac{1}{11} G_{BOC(6,1)}(f)$$

Note that the MBOC PSD is defined as a weighted linear combination of the BOC(1,1) and BOC (6,1) normalized PSDs.

Since the MBOC is defined only in the frequency domain, different compliant temporal signals can be used. In the literature, two different modulations were proposed to implement the MBOC:
- the Time-Multiplexed BOC (TMBOC) modulation [Hein, 2006; Betz, 2006], that multiplexes in the time domain BOC(1,1) and BOC(6,1) sub-carriers, and
- the Composite BOC (CBOC) [Hein 2006, Avila-Rodriguez, 2006] modulation, that linearly combines the BOC(1,1) and BOC(6,1) sub-carriers (both components being present at all times).

Thus, the philosophy behind these two modulations is very different, and although they would theoretically bring equivalent tracking when used with a TMBOC or CBOC receiver (considering pilot and data channels), they can result in different performances in other configurations.

A major difference between the TMBOC and CBOC modulations is that the CBOC sub-carrier, as the weighted sum of two squared-wave sub-carriers, will have 4 different levels. Consequently, this means that an optimal CBOC receiver has to generate a local replica that also has 4 levels, resulting in a local replica encoded on more than just 1 bit. This, obviously, will significantly complicate the CBOC receiver architecture and might be detrimental to the wide-spread use of this modulation, if retained as the Galileo E1 OS modulation. This paper aims at proposing an innovative

technique that only requires a 1-bit local replica to track CBOC signals. Thus its performances are compared to the other candidate's performance, the TMBOC, in order to compare both approaches with a receiver with comparative complexities.

The first part of this paper will describe the possible CBOC and TMBOC candidates for Galileo E1 OS modulation. The second part looks at the traditional tracking performances of these two modulations in terms of thermal noise and multipath-induced errors. Finally, the new 1-bit tracking technique is introduced and compared against optimal TMBOC tracking in terms of tracking noise and multipath resistance.

## CBOC AND TMBOC IMPLEMENTATION OF MBOC FOR GALILEO

As seen in the expression of the MBOC PSD, the power of the BOC(6,1) component has to represent 1/11 of the total OS signal. As explained in [Hein, 2002], the Galileo E1 OS signal will be composed of a data and a pilot channel. It is understood that the implementation of the MBOC will be dependent, among other parameters, upon the power share between the pilot and data channels, as well as the percentage of BOC(6,1) sub-carrier used on each of these channels.
It seems that a preferred configuration for Galileo is to split equally the OS power between these two channels [Avila-Rodriguez et al., 2006]. The exact choice of the modulation will then depend upon the part of the BOC(6,1) sub-carrier used in the data and pilot signals to fulfill the MBOC constraint.

According to [Avila-Rodriguez, 2006], the two main candidate implementations are:
- 2/11 of BOC(6,1) power and 9/11 of BOC(1,1) power to form the pilot channel sub-carrier and a pure BOC(1,1) sub-carrier on the data channel, or
- 1/11 of BOC(6,1) power and 10/11 of BOC(1,1) power to form the sub-carrier on both the data and pilot channels.

These two cases can be both applied to a TMBOC and a CBOC modulation. To illustrate the two implementations, let us take the example where the BOC(6,1) sub-carrier is only on the pilot channel:
- For the TMBOC implementation, this means that within one code length, 2/11 of the chips should be modulated by a BOC(6,1) sub-carrier while the rest is modulated by a BOC(1,1) sub-carrier. Examples of such cases are given in [Hein, 2006; Betz, 2006]. In such a case, the signal is said to have a TMBOC(6,1,2/11) modulation.
- For the CBOC case, this means that the pilot sub-carrier is the sum of a BOC(6,1) sub-carrier with a weight of $\sqrt{2/11}$ and a BOC(1,1) sub-carrier with a

weight of $\sqrt{9/11}$. In such a case, this signal is said to have a CBOC(6,1,2/11) modulation.

It is well-known that at the payload level, in order to minimize amplifier losses, the transmitted signal should be generated with a constant envelope. This puts constraints on the Galileo L1 OS part of the whole Galileo L1. These constraints, applied to the CBOC modulation, are different according to the use of the BOC(6,1) sub-carrier on only the pilot channel or on both channels. Using an interplex modulation for the whole Galileo E1 signal, if the BOC(6,1) sub-carrier is used on both channel, the baseband OS part will have the following form:

$$s_{OS}(t) = \begin{bmatrix} c_D(t)d(t)(Px(t)+Qy(t))+ \\ + c_P(t)(Px(t)-Qy(t)) \end{bmatrix}$$

where $c_D$ and $c_P$ are the data and pilot channels spreading code sequences,
$d$ is the navigation message,
$x$ and $y$ are the BOC(1,1) and BOC(6,1) sub-carrier waveforms, and
$P$ and $Q$ are the respective weights of the BOC(1,1) and BOC(6,1) waveforms.

Note that the sign of the BOC(6,1) sub-carrier is different between the data and pilot channels. This is necessary to fulfil the MBOC constraint (removal of cross-terms appearing from the cross-correlation between the BOC(1,1) and BOC(6,1) sub-carriers).

In the case when the BOC(6,1) sub-carrier is only used on the pilot channel, the OS signal will have the following form:

$$s_{OS}(t) = \begin{bmatrix} c_D(t)x(t)+ \\ + c_P(t)(Px(t)-Qy(t)) \end{bmatrix} \text{ if even chip}$$

$$s_{OS}(t) = \begin{bmatrix} c_D(t)x(t)+ \\ + c_P(t)(Px(t)+Qy(t)) \end{bmatrix} \text{ if odd chip}$$

As observed, in this case, in order to cancel the cross-terms, the sign of the BOC(6,1) sub-carrier switches from one chip to the next. Obviously, this method has the drawback to be more challenging at the generation level.

Thus, assuming the use of a CBOC modulation, the possible implementations of the MBOC that will be considered in this paper are:
- The use of a CBOC(6,1,1/11) modulation on the data and pilot channels. In this case, each channel has a BOC(6,1) sub-carrier with opposite signs. In

this case, the notation used will be CBOC(6,1,1/11,'+') for the data channel and CBOC(6,1,1/11,'-') for the pilot channel.

- The use of a CBOC(6,1,2/11) modulation on the pilot channel with a BOC(6,1) sub-carrier with alternating sign, while the data channel is a pure BOC(1,1). In this case, the notation used will be CBOC(6,1,2/11,'+/-') for the pilot channel.

For the sake of completeness, another scenario is considered herein where a CBOC(6,1,1/11,'+/-') is used both on the data and pilot channels.

Since the main interest of this article is the tracking performance of the CBOC or TMBOC signals, it is normal to look at each channel separately. In this case, the three following CBOC signal models can be used:

$$CBOC(6,1,p,'-')(t) = c(t)\{P\,x(t) - Q\,y(t)\}$$
$$CBOC(6,1,p,'+')(t) = c(t)\{P\,x(t) + Q\,y(t)\}$$
$$CBOC(6,1,p,'+/-')(t) = \begin{cases} c(t)\{P\,x(t) - Q\,y(t)\} & even \quad chips \\ c(t)\{P\,x(t) + Q\,y(t)\} & odd \quad chips \end{cases}$$

where $p = \dfrac{Q^2}{P^2 + Q^2}$ represents the percentage of

BOC(6,1) in the channel considered (by opposition to the power in the whole OS signal).

For the TMBOC case and following the same logic, the model to use is simpler since there is no BOC(1,1)/BOC(6,1) cross-term appearing on a single channel [Avila-Rodriguez, 2006] and the sign of the BOC(6,1) sub-carrier does not influence the TMBOC performances (although it can if a non-TMBOC receiver is used). In this case, the TMBOC signal model used is:

$$TMBOC(6,1,p)(t) = \begin{cases} c(t)x(t) & if \quad t \in S_1 \\ c(t)y(t) & if \quad t \in S_2 \end{cases}$$

where $S_1$ is the union of the segments of time when a BOC(1,1) sub-carrier is used, while $S_2$, the complement of $S1$ in the time domain, is the union of the segments of time when a BOC(6,1) sub-carrier is used. Note that a relevant choice of the segments $S_1$ and $S_2$ has been shown to potentially reduce by 1 dB the auto and cross-correlation main peak isolation [Avila-Rodriguez et al., 2006].

The next step is to compare the CBOC and TMBOC optimal tracking performances as candidate modulations for the future Galileo E1 OS. Thus, the comparison will be done for the different possible MBOC implementations. Thus:

- the CBOC types allowing the presence of BOC(6,1) on both the data and pilot channel (CBOC(6,1,1/11,'+'), CBOC(6,1,1/11,'-'), and CBOC(6,1,1/11,'+/-')) will be compared with a TMBOC(6,1,1/11), and
- the CBOC(6,1,2/11,'+/-') will be compared with the TMBOC(6,1,2/11).

## CBOC AND TMBOC ACHIEVABLE PERFORMANCES

As it is well-known, the traditional tracking of a signal using spread spectrum techniques uses the correlation of the incoming signal with the same local replica as the useful incoming signal. Thus, most of the tracking performances are dependent upon the autocorrelation of the useful signal. The autocorrelation function of the three CBOC waveforms presented in the previous section is given by:

$$R_{CBOC('-')}(\tau) = \left(P^2 R_x(\tau) + Q^2 R_y(\tau) - 2PQR_{x/y}(\tau)\right)$$
$$R_{CBOC('+')}(\tau) = \left(P^2 R_x(\tau) + Q^2 R_y(\tau) + 2PQR_{x/y}(\tau)\right)$$
$$R_{CBOC('+/-')}(\tau) = \left(P^2 R_x(\tau) + Q^2 R_y(\tau)\right)$$

In the same way, it can be shown that:
$$R_{TMBOC}(\tau) = \left(P^2 R_x(\tau) + Q^2 R_y(\tau)\right)$$

The CBOC('+') and CBOC('-') autocorrelation functions include a cross-correlation term between the BOC(1,1) and BOC(6,1) parts. This cross-correlation function is represented in Figure 1. It can be seen that if the weight of this cross-term is negative, the autocorrelation main peak will become sharper, while if negative, the main autocorrelation peak will become wider.
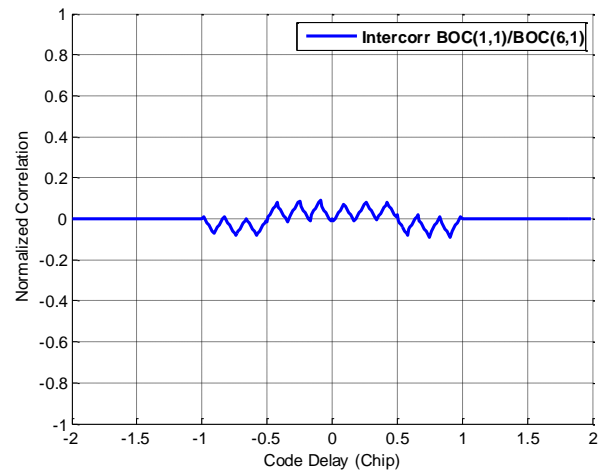


**Figure 1 – Cross-Correlation Between BOC(1,1)- and BOC(6,1)-modulated PRN signals**

Figure 2 shows the autocorrelation functions of each of the CBOC type. It is compared to the corresponding TMBOC autocorrelation functions. It can be seen that:

- The percentage of BOC(6,1) power in the signal channel (data or pilot) total power will shape the correlation function. The higher the value of $p$, the more the autocorrelation function will have the undulations of the pure BOC(6,1) autocorrelation function and its main peak will become narrow.
- The sign of the BOC(6,1) component will also shape the correlation function: it can be seen that a negative sign, the main peak of the autocorrelation function is narrower, as already mentioned.
- The TMBOC and the CBOC('+/-') have very close (not to say the same) autocorrelation functions, at least as far as the part within ±1 chip is concerned.
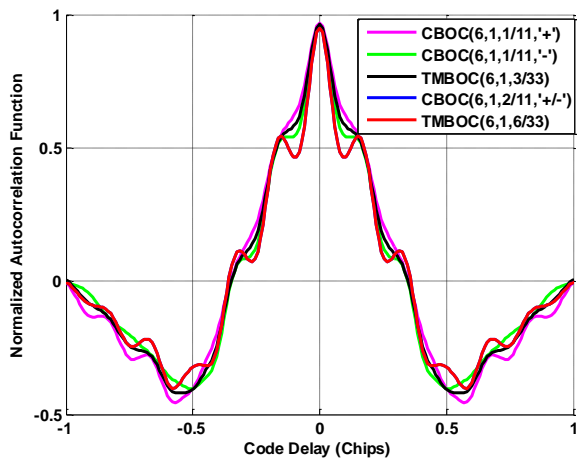


**Figure 2 – CBOC and TMBOC Autocorrelation Functions**

Since, according to the CBOC type, the incoming useful signal autocorrelation is different, it can be expected that the different CBOC will also have different tracking performances. Three main criterions are studied herein: (1) the risk of biased tracking, that will influence the existence of potential false lock points, (2) the code tracking noise induced by thermal noise, and (3) the multipath-induced code tracking error.

**False Tracking Points**

The existence of false tracking points is conditioned by the shape of the code discriminator output used by the Delay Lock Loop (DLL) [Julien 2005]. The dot-product (DP) discriminator is a commonly used discriminator because it has low squaring losses [Kaplan, 1997, Van Dierendonck, 1996] and can be implemented with one common replica 'Early-minus-Late' instead of two separated Early and Late replicas, thus requiring less correlators. It will be investigated herein as the reference discriminator. Its expression is given by:

$$D_{DP} = (I_E - I_L)I_P + (Q_E - Q_L)Q_P$$

where $I_X$ and $Q_X$ represent the in-phase and quadra-phase correlators' output where $X = E$ for the early correlator, $X = P$ for the prompt correlator, and $X = L$ for the late correlator.

Looking at the autocorrelation function in Figure 2, it is easy to understand that the existence of secondary peaks can lead to stable false lock points. With that respect, it can be seen that the CBOC(1/11) is not likely, in any of its types, to lead to stable false lock points close to the expected lock point (located on the main peak). Indeed, the first false lock point would be approximately for a code delay of 0.6 chips, resulting in a bias of around 175 metres, thus easily detectable. For the CBOC(6,1,2/11,'+/-'), on the other hand, the existence of false lock points seems unavoidable. The closest one is around 0.15 chips, equivalent to a measurement bias of 43 meters. Thus, it might be more difficult to detect.

Note that in any case, due to the dominant BOC(1,1) component and its secondary peak located at 0.5 chips, a false lock detector is necessary in order to make sure that the receiver is tracking the signal based on the autocorrelation main peak [Julien, 2005].

**Thermal Noise-Induced Code Tracking Error**

Assuming a DP discriminator, the theoretical thermal noise-induced tracking error variance is given by:

$$\sigma_{DP}^2 = \left( \frac{\dfrac{B_L(1 - 0.5B_L T_I)(\tilde{R}_{CBOC}(0) - \tilde{R}_{CBOC}(d))}{\dfrac{P}{2N_0}\left( \left. \dfrac{d\tilde{\tilde{R}}_{CBOC}(x)}{dx}\right|_{x=\frac{d}{2}} \right)^2} \times}{\left( 1 + \dfrac{\tilde{R}_{CBOC}(0)}{\dfrac{PT_I}{N_0}\tilde{\tilde{R}}_{CBOC}^2(0)} \right)} \right)$$

where  $B_L$ is the DLL loop bandwidth,

$T_I$ is the coherent integration time,

$d$ is the early-late spacing,

$P$ is the incoming useful signal power (in a single channel),

$N_0$ is the thermal noise PSD level,

$\tilde{R}_{CBOC}$ is the filtered correlator output noise correlation function, and

$\tilde{\tilde{R}}_{CBOC}$ is the filtered correlation function of the incoming signal.

Figure 3 shows the theoretical code tracking noise for a DP discriminator for the different considered CBOC cases, assuming a 1 Hz DLL loop bandwidth, a 1/12 of a chip early-late spacing, a 4-ms integration time, and a 12 MHz front-end filter. The use of a pure BOC(1,1) as the incoming signal (with equal signal power) is also shown as a reference because it is still the current Galileo E1 baseline signal.

It can be seen that the best performer is the CBOC(6,1,2/11'+/-'), taking full advantage of its higher power at high frequencies. Within the different CBOC(1/11) cases, it can be observed that the CBOC(1/11,'+') has the lowest performance, while the CBOC(1/11,'-') has the best, as expected according to their autocorrelation function's main peak sharpness. Still, all the CBOC modulations bring a significant improvement compared to the tracking of a pure BOC(1,1) modulation.
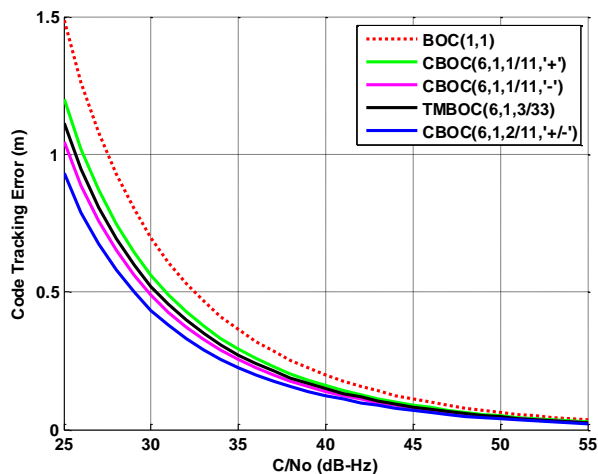


**Figure 3 – BOC(1,1), CBOC and TMBOC DLL Tracking Performance Assuming a DP Discriminator, a 1 Hz Loop Bandwidth, a 1/12 Chip Early-Late Spacing, a 4 ms Integration Time, and a 12 MHz One-Sided Filter**

This improvement, in terms of equivalent $C/N_0$, is shown in Table 1. It can be seen that according to the CBOC or TMBOC modulation chosen, the improvement is between 1.9 and 4.2 dBs, which is significant, regarding the small amount of BOC(6,1) used by the different CBOC/TMBOC signals.

Assuming that code tracking will be performed using the pilot channel only, and according to the possible CBOC or TMBOC implementations for Galileo E1 OS, the tracking noise improvement would be at least 3.1 dBs in terms of equivalent $C/N_0$ over a pure BOC(1,1) tracking.

**Table 1 – CBOC and TMBOC Code Tracking Noise Improvement vs BOC(1,1) in Terms of Equivalent C/N0 (dB)**

| Tracking Error Improvement vs BOC(1,1) in Terms of Equivalent $C/N_0$ (dB) | | | |
|---|---|---|---|
| CBOC(6,1, 1/11,'+') | CBOC(6,1, 1/11,'-') | CBOC(6,1, 1/11,'+/-') or TMBOC(6,1, 1/11) | CBOC (6,1, 2/11,'+/-') or TMBOC(6,1,2/11) |
| 1.9 | 3.1 | 2.5 | 4.2 |

**Multipath-Induced Tracking Error**

Multipath-induced tracking errors are also dependent upon the autocorrelation function shape. It is thus interesting to compare the performance of the different CBOC candidates against multipath. A common figure of merit is the multipath running average error introduced in [Hein et al. 2006]. It is plotted in Figure 4 for an early-late spacing of 1/12 chips and a one-sided front-end filter of 12 MHz. Once again, it can be observed that the CBOC(6,1,2/11,'+/-') tracking has the best performance. This is mostly due to the narrow peaks constituting its autocorrelation function. Comparing the CBOC(1/11) cases, multipath rejection is more effective for the CBOC(6,1,1/11,'-') tracking case, followed by CBOC(6,1,1/11,'+/-') tracking and then CBOC(6,1,1/11,'+') tracking. Note that all these multipath results for the CBOC are much better than for pure BOC(1,1) tracking.
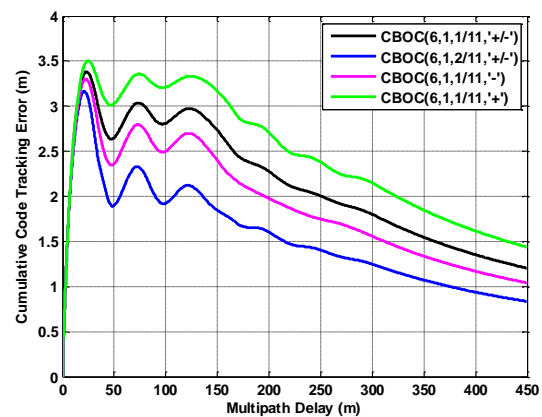


**Figure 4 – CBOC Multipath Running Average Error Assuming a 1/12 Chip Early-Late Spacing, and a 12 MHz One-Sided Filter**

In a more general way, the multipath rejection capability of CBOC or TMBOC signals is dependent upon their autocorrelation function shape, and thus upon the ratio $p$ between the weights of the BOC(1,1) and BOC(6,1) autocorrelation functions. Although fixed by the MBOC PSD, and thus not currently implementable, a more optimal multipath mitigation capacity (according

to the running average envelope figure of merit) could be obtained for other values of $p$.

**Conclusions on Optimal CBOC Tracking**

It has been seen that for the CBOC(1/11) cases, the best tracking performances were achieved for the CBOC(6,1,1/11,'-') modulation with excellent multipath mitigation and low tracking noise. It is appropriated for pilot-only tracking. However, in this case, a CBOC(6,1,1/11,'+') modulation has to be used on the data channel to meet the MBOC specification. This latter one exhibits the worst performances within the CBOC(1/11) family, but is still significantly better than a pure BOC(1,1) modulation.

Another option is the use of a CBOC(6,1,1/11,'+/-') on the data and pilot channels, in which case both channels would have the same tracking performances, and would also offer excellent interoperability. The main problem in this case is the more challenging signal generation architecture for the alternating sign of the BOC(6,1) sub-carrier.

Finally, the use of a CBOC(6,1,2/11,'+/-') on one channel and a pure BOC(1,1) on the other channel would allow having a pilot channel with excellent tracking performances (4.2 dB higher equivalent $C/N_0$ than pure BOC(1,1) for tracking noise and best multipath performance), while the data channel would just use a pure BOC(1,1) modulation, which would be fine if this channel is mainly used for data demodulation. Note, once again, that the alternating sign of the BOC(6,1) sub-carrier complicates the generation of this modulation option.

The choice of the best candidate, among the CBOC modulation has thus, among other criterions, to be done assessing the impact on the different user needs, and on receiver architectures.

However, the traditional processing of a CBOC signal, as shown in this section, implies that a replica of the CBOC signal has to be locally generated by the receiver. As the CBOC is a linear combination of two sub-carriers, it has more than two levels. This means that the local replica has to be encoded on at least 2 bits, which implies the need for a more challenging receiver architecture. This could be detrimental to the use of this signal and it is then interesting to look at techniques that would only use local replicas encoded on 1-bit, while maintaining interesting tracking performances.

An example of such a method is the separate correlations of the incoming CBOC signal with, on one side a pure BOC(1,1) replica, and on the other side, a pure BOC(6,1) replica. A simple linear combination of these two correlation values would result in the exact same correlation value than the CBOC autocorrelation value and thus the exact same tracking performances. However, this processing requires twice as many correlators as the traditional CBOC tracking. The following part introduces a new CBOC tracking technique that intends to remove that problem.

**PROPOSED 1-BIT PROCESSING OF CBOC**

The idea behind the proposed 1-bit processing is that both the BOC(1,1) and BOC(6,1) components should be present in the locally generated signal in order to use their properties:
- Most of the incoming power is in the BOC(1,1) component,
- The narrow BOC(6,1) autocorrelation function implies excellent tracking performances.

Then, one way to have this is to locally generate a signal close to a TMBOC modulation, with an alternating sequence of BOC(1,1) and BOC(6,1) sub-carriers modulating the PRN sequence. However, in order to avoid confusion between the incoming TMBOC and this local replica, it will be referred to as TM61 replica. By extension, the tracking technique will also be referred to as TM61. The local replica can then be expressed as:

$$TM61(\alpha)(t) = \begin{cases} c(t)x(t) & if \quad t \in S_3 \\ \pm c(t)y(t) & if \quad t \in S_4 \end{cases}$$

where $S_3$ is the union of the segments of time when a BOC(1,1) sub-carrier is used, while $S_4$, the complement of $S_3$ in the time domain, is the union of the segments of time when a BOC(6,1) sub-carrier is used. The parameter $\alpha$ represents the percentage of time when the BOC(6,1) sub-carrier is used. The choice upon the sign of the BOC(6,1) sub-carrier in the TM61 local replica depends upon the associated sign of the BOC(6,1) sub-carrier in the incoming CBOC signal. If it is a CBOC('-') signal that is received, the BOC(6,1) sub-carrier in the TM61 replica will have a negative sign.

The resulting correlation function between the TM61 and the different CBOC types is then given by:

$$R_{CBOC('-')/TM61(\alpha)}(\tau) = \begin{pmatrix} \beta P R_x(\tau) + \alpha Q R_y(\tau) \\ -(\beta Q + \alpha P) R_{x/y}(\tau) \end{pmatrix}$$

$$R_{CBOC('+')/TM61(\alpha)}(\tau) = \begin{pmatrix} \beta P R_x(\tau) + \alpha Q R_y(\tau) \\ +(\beta Q + \alpha P) R_{x/y}(\tau) \end{pmatrix}$$

$$R_{CBOC('+/-')/TM61(\alpha)}(\tau) = \frac{1}{2} \begin{pmatrix} (\beta^- + \beta^+) P R_x(\tau) \\ +(\alpha^- + \alpha^+) Q R_y(\tau) \\ +((\alpha^- - \alpha^+)(Q - P) R_{x/y}(\tau)) \end{pmatrix}$$

where $\alpha^- = 1 - \beta^-$ is the percentage of BOC(6,1) with a negative sign used in the TM61 replica (with respect to the negative BOC(6,1) part of the CBOC('+/-')), and $\alpha^+ = 1 - \beta^+$ is the percentage of BOC(6,1) with a positive sign (with respect to the positive part of the CBOC('+/-')).

It can be seen that the different cross-correlation functions are also a linear combination of the BOC(1,1) autocorrelation function, the BOC(6,1) autocorrelation function, and the BOC(1,1)/BOC(6,1) cross-correlation function. However, this time, the ratio between these three components is dependent upon the parameter $\alpha$ and could thus be controlled by the receiver implementation. Since a local replica different from the incoming signal is used, it is important to quantify the associated correlation losses. Assuming an infinite front-end filter bandwidth, and since the BOC(1,1)/BOC(6,1) correlation function equals 0 at the origin, then for each considered CBOC modulation type:

$$R_{CBOC/TM61(\alpha)}(0) = P + \alpha(Q - P)$$

In presence of thermal noise only, the noise power at the correlator output is the same using a CBOC local replica, a TMBOC local replica or a TM61 local replica. Thus, the post-correlation SNR can be quantified, assuming infinite front-end filter, as:

$$\deg_{SNR} = \left(\frac{R_{CBOC/TM61(\alpha)}(0)}{R_{CBOC}(0)}\right)^2 = \left(\frac{(P + \alpha(Q - P))}{(P^2 + Q^2)}\right)^2$$

This SNR degradation due to the use of a TM61 local replica is represented in Figure 5 for the CBOC(6,1,1/11) and CBOC(6,1,2/11) cases. It can be seen that choosing a small value for $\alpha$ allows a minimization of the correlation losses, as expected. A high SNR degradation at the correlator outputs means numerous degradations of the receiver performance at several stages:

- phase tracking performances that uses the in-phase and quadra-phase prompt correlator outputs
- code tracking,
- data demodulation, ....

However, using a TM61 local replica with a small value for $\alpha$ means that the correlation function will be close to a BOC(1,1) correlation function and thus the tracking performance will be significantly degraded compared to optimal CBOC tracking. Since phase tracking and data demodulation are done using the prompt correlator outputs only, it might be interesting to use different TM61 local replicas for the prompt correlator and the

early and late correlators to have a more flexible tracking architecture. Thus, a new DP discriminator is defined using:

$$D_{DP} = \begin{pmatrix} \left(I_E^{TM61(\alpha)} - I_L^{TM61(\alpha)}\right)I_P^{TM61(\alpha')} \\ + \left(Q_E^{TM61(\alpha)} - Q_L^{TM61(\alpha)}\right)Q_P^{TM61(\alpha')} \end{pmatrix}$$
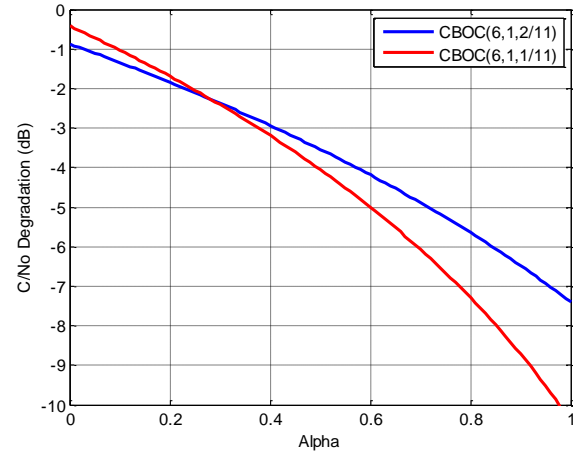


**Figure 5 – TM61-Induced SNR Degradation for CBOC(6,1,1/11) and CBOC(6,1,2/11) Signals**

Using that approach, and assuming that all the correlation functions are symmetric, the theoretical tracking noise standard deviation can be written equal to:

$$\sigma_{DP,TM61(\alpha)}^2 = \left( \frac{B_L\left(1 - 0.5 B_L T_I\right)\left(\tilde{R}_{TM61_{E,L}(\alpha)}(0) - \tilde{R}_{TM61_{E,L}(\alpha)}(d)\right) \times}{\frac{P}{2N_0}\left(\left.\frac{d\tilde{\tilde{R}}_{CBOC/TM61_{E,L}(\alpha)}(x)}{dx}\right|_{x=\frac{d}{2}}\right)^2} \left(1 + \frac{\tilde{R}_{TM61_P(\alpha')}(0)}{\frac{PT_I}{N_0}\tilde{\tilde{R}}_{CBOC/TM61_P(\alpha')}^2(0)}\right) \right)$$

where $R_{TM61_{E,L}(\alpha)}$ is the TM61 autocorrelation function that corresponds to the TM61 local replica used for the early and late correlators, and $R_{TM61_P(\alpha')}$ is the TM61 autocorrelation function that corresponds to the TM61 local replica used for the prompt correlator.

It can be observed that the separation of the early and late TM61 replicas from the prompt TM61 local replicas leads to very interesting conclusions:

- The squaring loss only depends upon the prompt TM61 local replica

- The asymptotical variance (when no squaring losses are present) depends upon the early and late TM61 local replicas only.

Thus, in order to minimize the squaring losses, it is important to have minimal correlation losses for the prompt correlator. Consequently, from now on, the local TM61 replica used for the prompt correlator will use $\alpha'=0$, which means that it is a pure BOC(1,1) local replica. This implies that the correlation losses are minimum (with an infinite front-end 0.4 dBs for CBOC(6,1,1/11) and 0.8 dBs for CBOC(6,1,2/11)) and it is thus very interesting for data demodulation and phase tracking purposes. Simulations showed that for a CBOC(6,1,2/11), the degradation was around 0.5 dBs for a 12 MHz front-end filter double-sided bandwidth.

**Tableau 2 – SNR Post-Correlation SNR Degradation Obtained through Simulations for a CBOC(6,1,2/11,'+/-')**

| $C/N_0$ (dB-Hz) | Post-Correlation SNR Degradation (dB) |
|---|---|
| 30 | -0.49 |
| 35 | -0.54 |
| 40 | -0.55 |
| 45 | -0.56 |
| 50 | -0.56 |

On the other hand, the asymptotical tracking variance depends upon the TM61($\alpha$) autocorrelation values (obtained from the early and late TM61 replicas) in 0 and $d$, and the TM61($\alpha$)/CBOC cross-correlation slope in $d/2$. Thus, a more thorough analysis has to be undertaken. Figure 6 shows the TM61($\alpha$) tracking noise asymptotical behaviour for the different possible incoming CBOC signals and different values of $\alpha$. The results are compared to the optimal TMBOC asymptotical value. According to the CBOC signal, the degradation is different. So, for a CBOC(6,1,1/11), the optimal value for $\alpha$ (for the early and late TM61 local replicas) seems to be 0 for each case. However, the case when $\alpha$ is close to 1 seems interesting as well. It is interesting to understand that these extreme cases mean that only a pure BOC(1,1) or a pure BOC(6,1) could be generated locally for the early and late correlators which would significantly reduce receiver architecture since in this case, even BOC(1,1)/BOC(6,1) sub-carriers multiplexing would not be necessary. For the CBOC(6,1,2/11,'+/-') case, $\alpha$ =1 (for the early and late TM61 local replicas) seems to be the best solution.

In order to better quantify the tracking degradation in thermal noise, Table 3 shows the degradation of the TM61 technique compared to the associated TMBOC method in terms of equivalent $C/N_0$. It is reminded that the prompt local replica is assumed to be a pure BOC(1,1) and thus the values of $\alpha$ only affect the TM61($\alpha$) early and late local replicas. It can be seen that for the case of the CBOC(6,1,1/11,'+'), the TM61 tracking technique seems not to perform so well since at the lowest it exhibits a 4-dB $C/N_0$ loss. On the other hand, the CBOC(6,1,1/11,'+/-') (3-dB loss), CBOC(6,1,1/11,'-') (1.9-dB loss) and CBOC(6,1,2/11,'+/-') (1.6-dB loss) have only slight degradations. This represents limited losses that compensate the use of much simpler receiver architecture. Indeed, using $\alpha$ =1 means that only pure sub-carriers are used and thus no multiplexing is required. In the case when code tracking is done on the pilot only, the degradation are the lowest, since in this case, the pilot channel will be modulated either by a CBOC(6,1,1/11,'-') or a CBOC(6,1,2/11,'+/-').
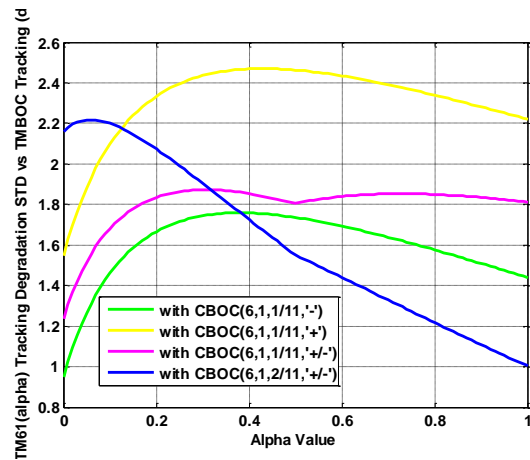


**Figure 6 – Degradation of TM61(α) Tracking Error STD wrt TMBOC Tracking Error STD**

**Table 3 - TM61(α) Tracking Error Degradation wrt TMBOC in Terms of Equivalent C/N₀ for an early-late Spacing of 1/12 Chips, a 12 MHz Double-Sided Front-End Filter, and a 4-ms Integration Time**

| Value of α for Early and Late TM61(α) Local Replicas | TM61(α) Tracking Error Degradation wrt TMBOC in Terms of Equivalent C/N0 (dB) | | | |
|---|---|---|---|---|
| | CBOC(6,1,1/11,'x') vs TMBOC(6,1,1/11) | | | CBOC(6,1,2/11,'x') vs TMBOC(6,1,2/11) |
| | + | - | +/- | +/- |
| 0 | 4 | 2 | 2.9 | 5 |
| 0.2 | 5.1 | 2.9 | 3.6 | 4.2 |
| 0.4 | 5.1 | 2.8 | 3.4 | 3.3 |
| 0.6 | 4.9 | 2.6 | 3.3 | 2.6 |
| 0.8 | 4.6 | 2.3 | 3.2 | 2.1 |
| 1 | 4.3 | 1.9 | 3 | 1.6 |

When comparing these values to Table 1, it can be seen that the use of the optimal values for $\alpha$ allows a tracking noise lower than or equivalent to optimal pure BOC(1,1) tracking (except for the CBOC(6,1,1/11,'+') case).

The last criterion studied for this TM61 tracking technique is its inherent resistance to multipath. As in the optimal tracking case, average multipath envelope error will be investigated. Figure 7 shows the multipath resistance of the TM61 technique for different values of $\alpha$ used for the early and late local replicas (the prompt replica uses a pure BOC(1,1) sub-carrier) assuming incoming CBOC(6,1,1/11,'-') and CBOC(6,1,2/11,'+/-') signals.
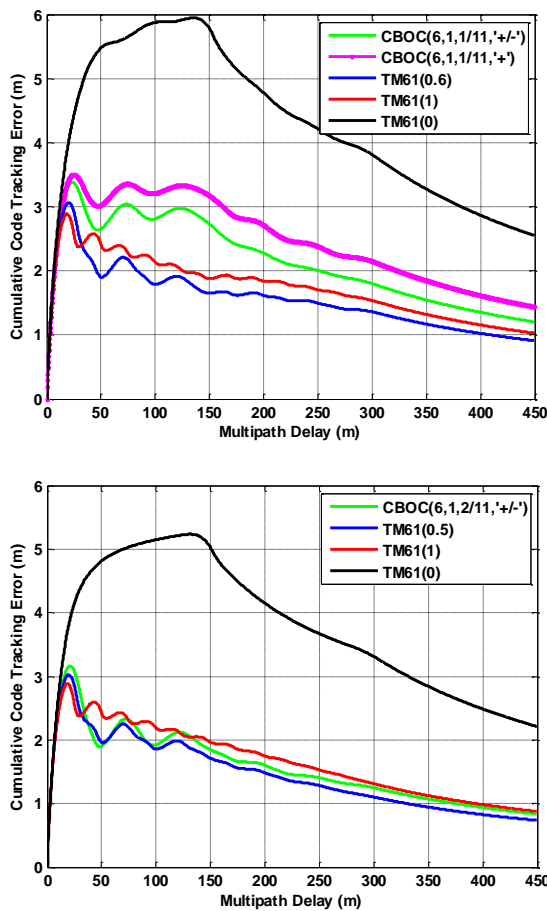




**Figure 7 – Running Average Multipath Error using TM61 Tracking Technique for a CBOC(6,1,1/11,'-') (Top) and CBOC(6,1,2/11,'+/-') (Bottom) for an Early-Late Spacing of 1/12 Chips and a 12 MHz Double-Sided Front-End Filter**

It can be seen that the case when $\alpha$ equals 0 has, by far, the lowest performances which was expected since in this case the TM61/CBOC correlation function is close to a pure BOC(1,1) one. The case around $\alpha$ =0.5 seems

to be optimal although, when $\alpha$ =1 the performance is comparable. In the case of a CBOC(6,1,1/11), the multipath resistance is even significantly increased. This result is very interesting and corroborates what was foreseen with traditional CBOC tracking which is that an optimum (in terms of running average multipath error) can be reached for a certain ratio between the weights affected to the BOC(1,1) and BOC(6,1) autocorrelation functions. Since, unlike in the traditional CBOC tracking case, the TM61 allows setting this ratio to any value (through the parameter $\alpha$ ), this optimum can be reached using the new proposed method. The TM61 multipath resistance improvement is particularly important when a CBOC(6,1,1/11) signal is used.

**Conclusion on CBOC Tracking using the TM61 Method**

It thus seems that a preferred implementation for the TM61 tracking method is to locally generate a pure BOC(1,1) replica for the prompt correlator, and a pure BOC(6,1) replica for the early and late correlators. This results in a very simple tracking architecture with no multiplexing involved. Obviously, a degradation in terms of code tracking noise is observed, but multipath rejection is increased. Whatever the CBOC modulation chosen, this implementation of the TM61 tracking seems particularly recommended for pilot tracking (only the CBOC(6,1,1/11,'+') that could be only on a data channel does not perform as well).

**CONCLUSIONS**

It has been shown that the use of a CBOC modulation to fulfil the MBOC constraint led to different candidate implementations. It particularly means that if a CBOC modulation is used by Galileo, different types of CBOC should be available on the data and pilot channel. In particular, the sign of the BOC(6,1) sub-carrier will play an important role for signal tracking. It has been seen that in any configuration and using a traditional tracking scheme, the pilot component will bring better resistance to thermal noise and multipath compared to the data channel. In any case, a significant tracking improvement in terms of noise and multipath resistance with respect to the choice of a pure BOC(1,1) has been demonstrated.

A new tracking technique, referred to as TM61, has also been proposed in order to be able to track the CBOC modulation with a 1-bit only locally generated replica. This method uses a time-multiplexing of BOC(1,1) and BOC(6,1) sub-carrier on the same model as the TMBOC modulation. It has been seen that a preferred implementation of TM61 is the use of a pure BOC(1,1) sub-carrier for the prompt correlators and a pure BOC(6,1) sub-carrier for the early and late correlators

(a DP discriminator being assumed). This brings a great simplicity of the receiver architecture since it requires only pure sub-carriers with no-multiplexing (differently from TMBOC receivers), 1-bit local replicas (unlike a CBOC local replica) and a minimum of correlators. Note that it is also possible to use other implementation of the TM61 tracking methods with time-multiplexing.

In its preferred implementation, TM61 brings only a slight post-correlation SNR degradation (around 0.5 dBs) enabling good phase tracking. Code tracking performance, compared to optimal TMBOC tracking, has been shown to be dependent upon the CBOC modulation type. Although TM61 tracking does not work very well for CBOC(6,1,1/11,'+'), it would only be degraded by 1.9 and 1.6 dBs approximately compared to TMBOC tracking for a CBOC(6,1,1/11,'-') or a CBOC(6,1,2/11,'+/-') signal, which are the main candidates for the Galileo OS pilot channel. This has to be put into perspective considering the high gain in receiver complexity. Finally, the TM61 tracking technique has been demonstrated to provide, in its preferred implementation, a better or equivalent multipath resistance compared to traditional CBOC tracking. Consequently, it seems to be a very good tracking technique to be implemented in CBOC receivers.

Although some has still to be done, TM61 tracking technique seems to offer a high interoperability with TMBOC tracking as well, and thus would be an interesting common tracking technique for GPS/Galileo receivers.

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