Reduction of the sensor number in distributed vector-antennas for 3D direction finding

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Full Paper — **For 3D direction finding, one can use vector-sensor antennas. For such antennas, a linear system can been introduced to determine the field components at one point from the measured signals. The condition number of this system evaluates the sensitivity of the antenna to small perturbations. The previous method leads to antennas with a large number of elements. To reduce this number, a feeding network is added that is defined from a singular value decomposition. Numerical experiments are realized to test its performance.**

I. Introduction

There are several ways to estimate the direction of arrival (DoA) of an incoming signal in the context of radio direction finding. A possible solution is to use a sensor with polarization diversity, i.e. an antenna constituted by several elements capable of measuring the 6 components of the electromagnetic fields at one point.

In this context, an ideal configuration would consist in 6 co-located elements [1]: Three short electric dipoles and three short magnetic dipoles. They would measure the components of the electric and magnetic fields, respectively. However, the co-location of the 6 elements cannot be simply realized in practical configurations, notably because this implies strong mutual couplings. A way to overcome this difficulty is to spatially separate the elements of the antenna [2]. But in this case, we have to determine which spatial configuration of the vector-sensor antenna may lead to an efficient DoA estimation.

In [3], we have proposed a general method to determine the 6 components of the electromagnetic fields at the antenna center for a vector sensor comprising N_a ports. This method works regardless of the positions and types of the antenna elements, and takes into account their mutual couplings. This implies a linear system of size $N_a \times N_s$ where N_s is the highest order of the spherical harmonics radiated by the antenna. The system is solved in the least square sense, upon assuming that $N_a \geq N_s$, i.e. the number of antenna elements must be greater than or equal to the number of radiated spherical harmonics. We have also demonstrated that the capability of the configuration to estimate the field components in presence of spatial noise (multipath) is related to the condition number of this linear system. In practical applications, the condition $N_a \geq N_s$ correponds to have an antenna with a large number of elements, typically $N_a \geq 16$.

In this article, we propose a feeding network to reduce this number. The feeding network of dimension $6 \times N_a$ is obtained by minimizing the influence of spherical harmonics of order greater than 6 that are not needed for the DoA estimation. This is realized from a singular value decomposition. Besides, we introduce two numbers, that characterize the efficiency of an antenna-network configuration to estimate the DoA.

This article is organized as follows. In Section II, we present the configuration and the modal representations that are employed. Next in Section III, we show how to characterize the vector-sensor antenna in terms of spherical harmonics via reciprocity. In Section IV, we introduce a feeding network to reduce the number of antenna elements. Finally in Section V, we perform numerical tests before some concluding remarks.

II. Vector-sensor antennas and associated modal representations

A. Configuration

The vector-sensor antenna is constituted by N_a elements of any type as shown in Fig. 1. We seek the direction of arrival of an incoming field from the signals measured at the port of each element. To this end, we aim at estimating the 6 components of the electric and magnetic fields at O, from which we can obtain the DoA from the Poynting vector.

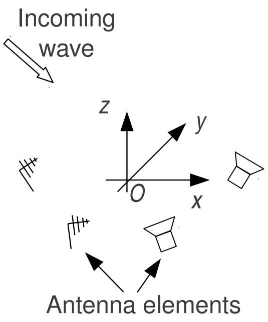


Fig. 1. Antenna configuration .

B. Representations of fields with spherical harmonics

When the antenna is either emitting or receiving, the electromagnetic fields can be represented in terms of ingoing or outgoing spherical harmonics. This modal representation can be cast as

$$\begin{aligned}\mathbf{E} &= \sum_p s_p^+ \mathbf{e}_p^{\mathrm{sph},+} + s_p^- \mathbf{e}_p^{\mathrm{sph},-},\\ \mathbf{H} &= \sum_p s_p^+ \mathbf{h}_p^{\mathrm{sph},+} + s_p^- \mathbf{h}_p^{\mathrm{sph},-},\end{aligned} \tag{1}$$

where $\left(\mathbf{e}_p^{\mathrm{sph},+}, \mathbf{h}_p^{\mathrm{sph},+}\right)$ and $\left(\mathbf{e}_p^{\mathrm{sph},-}, \mathbf{h}_p^{\mathrm{sph},-}\right)$ correspond to the electromagnetic fields of outgoing and ingoing spherical harmonics, respectively. The harmonics amplitudes s_p^- and s_p^+ are given by the following integrals over a sphere S_R of radius R

$$\begin{aligned}s_p^+ &= \frac{1}{2}\int_{S_R}\left[\mathbf{E}.\left(\mathbf{h}_p^{\mathrm{sph},-}\times\hat{\mathbf{r}}\right)+\mathbf{H}.\left(\mathbf{e}_p^{\mathrm{sph},-}\times\hat{\mathbf{r}}\right)\right]dS,\\ s_p^- &= \frac{1}{2}\int_{S_R}\left[\mathbf{E}.\left(\mathbf{h}_p^{\mathrm{sph},+}\times\hat{\mathbf{r}}\right)+\mathbf{H}.\left(\mathbf{e}_p^{\mathrm{sph},+}\times\hat{\mathbf{r}}\right)\right]dS,\end{aligned} \tag{2}$$

with $\hat{\mathbf{r}}$ the outgoing normal to the sphere.

C. Representations of fields with waveguide modes

At the n-th antenna port, we assume that the fields can be represented by one ingoing or outgoing waveguide mode, denoted $\left(\mathbf{e}_n^+, \mathbf{h}_n^+\right)$ and $\left(\mathbf{e}_n^-, \mathbf{h}_n^-\right)$, respectively. The modal amplitudes can be expressed as

$$\begin{aligned}a_n^+ &= \frac{1}{2}\int_{S_n}\left[\mathbf{E}.\left(\mathbf{h}_n^-\times\hat{\mathbf{n}}\right)+\mathbf{H}.\left(\mathbf{e}_n^-\times\hat{\mathbf{n}}\right)\right]dS,\\ a_n^- &= \frac{1}{2}\int_{S_n}\left[\mathbf{E}.\left(\mathbf{h}_n^+\times\hat{\mathbf{n}}\right)+\mathbf{H}.\left(\mathbf{e}_n^+\times\hat{\mathbf{n}}\right)\right]dS,\end{aligned} \tag{3}$$

with S_n the transverse surface of the port, and $\hat{\mathbf{n}}$ the associated outgoing normal.

III. Characterization of vector-sensor antennas via reciprocity

A. Emitting state of excitation

In a first state of excitation, the antenna is excited at its ports by outgoing waves of amplitudes $\mathbf{a}^+ = [a_1^+, \cdots, a_{N_a}^+]^T$ for $n \in \{1, \cdots, N_a\}$. We expand the fields associated with this excitation into outgoing spherical harmonics defined from a coordinate system centered at O. Theoretically, there is an infinite number of spherical harmonics. Nevertheless, since the elements of the antenna are distributed inside a limited area around O, only the lowest-order harmonics are radiated so that we can consider $s_p^+ = 0$ for $p > N_s$. Note that the number of significant harmonics N_s increases with the total size of the antenna. Finally, the radiated fields can be represented by the vector $\mathbf{s}^+ = [s_1^+, \cdots, s_{N_s}^+]^T$. Using the linearity of Maxwell equations, we define a matrix $\mathbf{M}^+$ that links the vector of excitation to the radiated spherical harmonics, i.e.

$$\mathbf{s}^+ = \mathbf{M}^+\mathbf{a}^+. \tag{4}$$

To compute the n-th column of this matrix, we consider that only the n-th antenna element is excited by $a_n^+ = 1$. The other elements are assumed matched, i.e. $a_{n'}^+ = 0$ for $n \neq n'$. We compute the fields radiated by this excitation on a discretized sphere of radius R. This can be performed using any antenna computation software. On the sphere, the numerical integration of (2) yields the column elements of $\mathbf{M}^+$.

B. Receiving state of excitation

In a second state of excitation, the antenna is excited by an external source. This corresponds to the intial state of interest, for which we seek the direction of arrival. The ingoing field can be represented by ingoing spherical harmonics. Similarly to the emitting state, because of its limited size, the antenna is only capable of receiving the first N_s harmonics. We note $\mathbf{s}^- = [s_1^-, \cdots, s_{N_s}^-]^T$ the vector of excitation. The fields induced by $\mathbf{s}^-$ at each antenna port can be represented via fundamental modes of amplitudes a_n^- given by (3). Using linearity, we can define the matrix $\mathbf{M}^-$ such that

$$\mathbf{a}^- = \mathbf{M}^-\mathbf{s}^-. \tag{5}$$

This matrix is of primary importance in our method. Indeed, if we determine $\mathbf{M}^-$ and invert this linear system, we can obtain the ingoing spherical harmonics from the measured signal. The field at O can then be obtained by summing the 6 lowest-order harmonics. Indeed, they are the only ingoing harmonics that do not vanish at O.

To determine $\mathbf{M}^-$ from $\mathbf{M}^+$, we make use of the reaction theorem [3]. This yields

$$\mathbf{M}^- = -\mathbf{M}^{+T}. \tag{6}$$

C. Appication to 3D direction finding

Upon assuming $N_a \geq N_s$, i.e. the number of elements is equal to or greater than the number of significant harmonics, the linear system (5) can be solved in the least square sense. Thus, from the measured signals a_n^-, we can obtain the associated ingoing spherical harmonics s_p^-, from which we can deduce the electric and magnetic fields at O, and the DoA.

The inversion of the linear system (5) is the main step to estimate the components of the electric and magnetic fields at O from the signals measured at the output of the antenna. The condition number of $\mathbf{M}^-$ gives the sensitivy of the result to variations of $\mathbf{a}^-$. If the system is ill-conditioned, the determination of the ingoing harmonics, and consequently, the field at O, will lack of accuracy for any small perturbation of $\mathbf{a}^-$. Thus, the condition number of $\mathbf{M}^-$ is an estimation of the capability of the configuration to estimate the DoA in presence of spatial noise (multipath).

IV. Reduction of the sensor number via an SVD decomposition

A. Definition of the feeding network

The hypothesis $N_a \geq N_s$ means that the number of antenna elements must be equal to or greater than the number of radiated spherical harmonics. In practical applications, most antennas with non co-located elements are such that $N_s \geq 16$. This implies that the antenna should have a large number of elements. The method actually estimates more than what we need. Indeed, the solution of the linear system gives the amplitude of the N_s lowest-order spherical harmonics while we only need the 6 first ones to obtain the field components at O. Thus, we can expect that the number of elements is reductible.

We now consider the case $N_a < N_s$, for which the number of elements is smaller than the number of significant harmonics. We propose to add a reciprocal and linear feeding network, so that when the antenna is fed through this network, only the 6 lowest-order outgoing spherical harmonics can be radiated. Reciprocally, when excited by a source to be localized, the antenna combined with the network will only be sensitive to the 6 lowest-order ingoing spherical harmonics.

(a) Emitting state of excitation

(b) Receiving state of excitation

Fig. 2. Antenna with the feeding network in both states of excitation .

To define this network, we consider the emitting state of excitation. The network is such that

$$\mathbf{a}^+ = \mathbf{P}\mathbf{b}^+, \tag{7}$$

where $\mathbf{b}^+$ corresponds to the input of the network as illustrated in Fig. 2. The matrix $\mathbf{P}$ of dimension $6 \times N_a$ characterizes the passage through the network. This matrix must be chosen so that the radiation of spherical harmonics of order greater than 6 is minimized. To this end, we firstly split the linear system (4) into two blocks

$$\mathbf{s}_1^+ = \mathbf{M}_1^+\mathbf{a}^+, \qquad \mathbf{s}_2^+ = \mathbf{M}_2^+\mathbf{a}^+. \tag{8}$$

The first block is associated with the 6 spherical harmonics of interest, the second one corresponds to higher-order harmonics. They are denoted with the subscripts 1 and 2, respectively. If we introduce the feeding network in these expressions, we obtain

$$\mathbf{s}_1^+ = \mathbf{M}_1^+\mathbf{P}\mathbf{b}^+, \qquad \mathbf{s}_2^+ = \mathbf{M}_2^+\mathbf{P}\mathbf{b}^+. \tag{9}$$

In order to minimize the radiation of harmonics of order greater than 6, we must choose $\mathbf{P}$ such that $\|\mathbf{s}_2^+\|$ is minimal. To achieve this goal, the matrix $\mathbf{M}_2^+$ is subjected to a singular value decomposition. The matrix $\mathbf{P}$ is then defined so that $\mathbf{b}^+$ can only excite the 6 smallest singular values. This will guarantee that $\|\mathbf{s}_2^+\|$ is minimal. Explicitly, the decomposition can be written as

$$\mathbf{M}_2^+ = \begin{bmatrix}\mathbf{u}_1 & \cdots & \mathbf{u}_{N_s}\end{bmatrix} \left[\begin{array}{ccc} \sigma_1 & & \\ & \ddots & \\ & & \sigma_{N_a} \\ \hline & 0 & \end{array}\right] \begin{bmatrix}\mathbf{v}_1^T \\ \vdots \\ \mathbf{v}_{N_a}^T\end{bmatrix} \tag{10}$$

where $\sigma_1 \geq \sigma_2 \geq \cdots \geq \sigma_{N_a}$ are the singular values. Besides, $\mathbf{u}_i$ and $\mathbf{v}_i$ correspond to the i-th left and right singular vectors, respectively. The matrix P can be expressed as

$$\mathbf{P} = \begin{bmatrix} \mathbf{v}_{N_a-5} & \cdots & \mathbf{v}_{N_a} \end{bmatrix}. \tag{11}$$

B. Application to 3D direction finding

When the antenna is emitting, if $\mathbf{P}$ renders $\|\mathbf{s}_2^+\|$ negligible with respect to $\|\mathbf{s}_1^+\|$, the radiation of the antenna in the presence of the network amounts to a linear system of dimension 6×6 given by $\mathbf{s}_1^+ = \mathbf{M}_1^+\mathbf{P}\mathbf{b}^+$. By reciprocity, when the antenna is receiving, the 6 ingoing spherical harmonics of interest $\mathbf{s}_1^-$ can be estimated by solving the linear system

$$\mathbf{b}^- = -(\mathbf{M}_1^+\mathbf{P})^T\mathbf{s}_1^- \tag{12}$$

where $\mathbf{b}^-$ are the signals measured at the network ports. Thus, from the measured signals, the solution gives the 6 ingoing spherical harmonics, from which we can deduce the electric and magnetic fields at the origin, and the DoA.

C. Efficiency of an antenna-network configuration

As for the method without the svd presented in [3], we can introduce figures of merits that determine the ability of a configuration to estimate the DoA. We introduce here to 2 numbers for that purpose.

The first number is associated with the efficiency of the feeding network to remove the influence of harmonics of order greater than 6. This corresponds to have $\|\mathbf{s}_2^+\|$ small compared to $\|\mathbf{s}_1^+\|$ for any excitation. Mathematically, this can be characterized by the number

$$e_{\text{svd}} = \frac{\sigma_{N_a-5}}{\sigma'}, \tag{13}$$

where σ' is the smallest singular value of $\mathbf{M}_1^+\mathbf{P}$. Note that the smaller e_{svd} is, the better the feeding network works.

The second number is associated with the sensitivity of the result to variations in the measured signals $\mathbf{b}^-$. This corresponds to the condition number of $(\mathbf{M}_1^+\mathbf{P})^T$, denoted e_{cond}.

V. Numerical experiments

We consider an antenna constituted by $N_a = 12$ short electric dipoles placed as shown in Fig. 3. The distance between the dipoles and the origin is denoted d.

Using classical analytic expressions for the radiation of short dipoles we determine the field radiated by each dipole on a discretized sphere of radius R. As explained in Section II D, the computation of (2) on the sphere yields the columns of the matrix $\mathbf{M}^+$, from which we compute the matrix $\mathbf{P}$ associated with the feeding network as defined in (11).

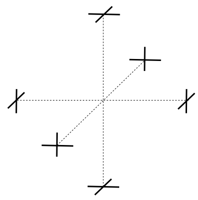
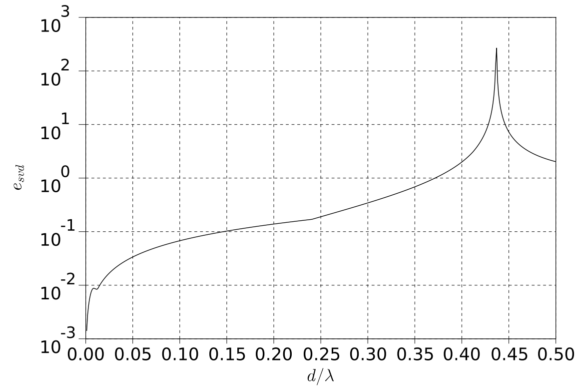
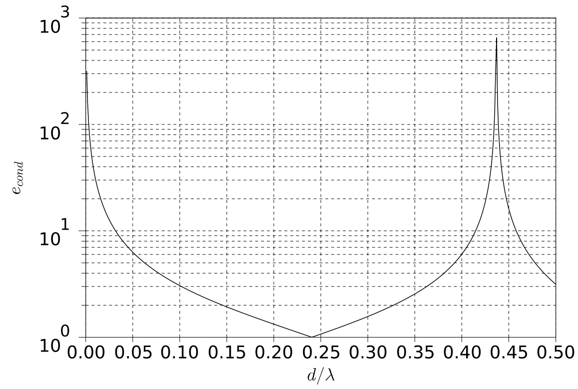


Fig. 3. Antenna constitued by $N_a = 12$ electric short dipoles .



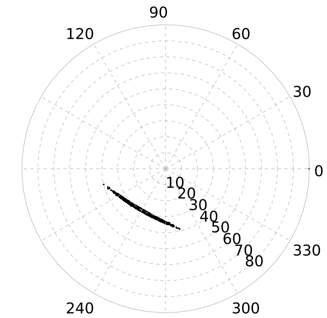
(a) Efficiency of the feeding network



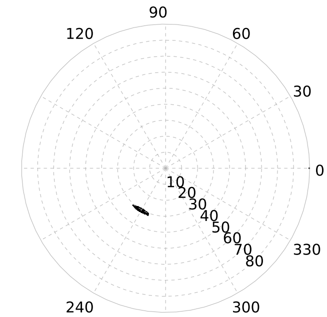
(b) Condition number of the linear system

Fig. 4. Efficiency of the antenna-network configuration with respect to the antenna size d.

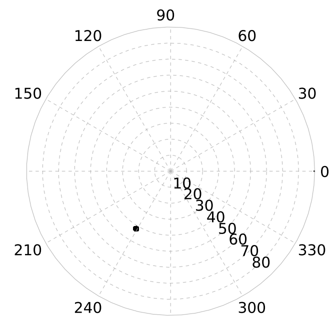
In Fig. 4, we display e_{svd} and e_{cond} with respect to d. We observe that e_{svd} increases with d until a pick localized at $d \approx 0.43\lambda$. Hence in this configuration, when the antenna size increases, the feeding network becomes less efficient to remove harmonics of order greater than 6. Moreover, we see that the condition number of the linear system e_{cond} is poor for $d \to 0$ and $d \approx 0.43\lambda$. This confirms that there may be configurations for which the antenna is sensitive to small variations of the measured signals.



(a) $d = 0.01\lambda$



(b) $d = 0.05\lambda$



(c) $d = 0.25\lambda$

Fig. 5. Monte Carlo analysis of the DoA estimation in presence of an additive noise for 3 sizes of antenna.

We now test whether e_{svd} and e_{cond} characterize the efficiency of the antenna-network system for DoA estimations. We consider the previous antenna illuminated by a source localized in the direction $\theta = 30°, \phi = -120°$ in the presence of an additive gaussian noise of standard deviation 0.01. For three values of d, we apply our method to estimate the DoA with 15000 noise samples.

For $d = 0.01$, the estimated DoA are centered on the true value. In this case, e_{svd} is small, thus the network works correctly. On the other hand, the estimated DoA are very sensitive to the noise because e_{cond} is large. For $d = 0.05$, the estimated DoA are centered on the true DoA with a moderate influence of the noise because both e_{svd} and e_{cond} are relatively small. For the last case $d = 0.25$, even if e_{cond} is almost ideal, the value of e_{svd} becomes large so that the estimated DoA are not centered on the true DoA.

VI. Conclusion

We have presented a general method to determine the 6 components of the electromagnetic fields at one point from the signal measured at the N_a ports of a vector-sensor antenna. This method implies a linear system of size $N_a \times N_s$ where N_s is the highest order of the spherical harmonics radiated by the antenna.

We have proposed a feeding network to reduce the number of the antenna elements. The feeding network of dimension $6 \times N_a$ has been obtained by minimizing the influence of spherical harmonics of order greater than 6 that are not needed for the DoA estimation. This has been realized from a singular value decomposition. Besides, we have introduced two figures of merit. The first one characterizes the efficiency of the feeding network to remove the influence of high-order harmonics. The second one characterizes the influence of small perturbations in the measured signals, i.e. noise, on the DoA estimation.

This method has been successfully tested on an antenna constituted by 12 short dipoles. The figures of merits have been confronted to a Monte Carlo analysis in the presence of noise.

For future works, we intend to apply this method to a realistic vector-sensor antenna, including the couplings of the elements.

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