GNSS RAIM assumptions for vertically guided approaches

Anaïs Martineau, Christophe Macabiau, Mikaël Mabilleau

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A. Martineau, C. Macabiau, *ENAC*
M. Mabilleau, *Egis Avia*

BIOGRAPHIES

Anaïs Martineau graduated in 2005 as an electronics engineer from the Ecole Nationale de l'Aviation Civile (ENAC) in Toulouse, France. Since 2005, she has been working at the signal processing lab of the ENAC where she carries out research on integrity monitoring techniques. She received her Ph.D. in 2008 from the University of Toulouse.

Christophe Macabiau graduated as an electronics engineer in 1992 from the ENAC in Toulouse, France. Since 1994, he has been working on the application of satellite navigation techniques to civil aviation. He received his Ph.D. in 1997 and has been in charge of the signal processing lab of the ENAC since 2000. His research now also applies to vehicular, pedestrian and space applications, and includes advanced GNSS signal processing techniques for acquisition, tracking, interference and multipath mitigation, GNSS integrity, as well as integrated GNSS inertial systems and indoor GNSS techniques.

Mikaël Mabilleau graduated as an electronics engineer in 2006 from the ENAC in Toulouse, France and integrated Egis Avia as CNS project engineer the same year. He has been involved since 2006 in Galileo standardisation activities for Civil Aviation in the frame of his duty in DTI (Direction de la Technique et de l'Innovation). He follows actively the standardisation activities undertaken by the EUROCAE WG 62 and ICAO NSP (Navigation System Panel) and leads a subgroup of the EUROCAE WG 62 in charge of Integrity for future Galileo receiver MOPS (RAIM, Galileo Integrity Concept).

ABSTRACT

Receiver Autonomous Integrity Monitoring (RAIM) is currently a simple and efficient solution for civil aviation applications to check the integrity of GNSS down to Non Precision Approaches. The future introduction of new satellite constellations such as the European satellite navigation system Galileo or modernized Global Positioning System (GPS) will imply great improvements in the number as well as the quality of available measurements. More demanding phases of flight such as approaches with vertical guidance could be targeted using RAIM to provide integrity monitoring.

The targeted probability of missed detection constitutes a major input of RAIM algorithm. This parameter derives from the integrity risk but also depends on the probability of satellite failure. Thus it refers to the threat model and particularly needs to be detailed. Up to now, most of RAIM algorithms assumed that only one satellite failure could occur at the same time and that the rate of occurrence of such a failure was the one of major service failure. But those assumptions have to be reconsidered for multi constellation RAIM designed for approaches with vertical guidance. Indeed, a larger number of available measurements also implies a larger number of potential faulty measurements for the receiver. Moreover, the targeted phases of flight are characterized by smaller horizontal and vertical tolerable position errors compared to NPA. Therefore, the threatening range errors that need to be detected by the fault detection algorithm have to be reconsidered, since they could have smaller amplitude, and a probability of occurrence that is not clearly defined currently.

The aim of the proposed paper is to present some assumptions to be adopted for the design and the evaluation of RAIM algorithms for vertically guided approaches. The way RAIM algorithms can be implemented in order to take into account both civil aviation requirement and threat model is addressed and the way the required probability of missed detection can be set is particularly investigated.

In the first part of the paper, the User Equivalent Range Error variance computation is detailed.

The concept of critical bias which is the smallest bias on a single pseudorange measurement that leads to a positioning failure is then developed. It is based on the fact that integrity monitoring requires that the navigation system detects the presence of an unacceptably large position error for a given mode of flight.

Then we demonstrate that, for the single failure case using GPS + Galileo constellations, the amplitude of pseudo range additional biases that lead to a positioning failure systematically belongs to the major service failure category for both APV I and LPV 200 (VAL=35 m) operations. Therefore even if the targeted phases of flight are characterized by smaller horizontal and vertical tolerable position errors compared to NPA, this effect is

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mitigated by the great number of available measurements that reduces the impact a of single satellite bias on the global positioning error. Thus only Major Service Failures need be taken into account for single failure case consideration using GPS + Galileo constellations.

This assumption is more questionable if an effective monitor threshold is set to 10 meters or 15 meters for LPV200 operations. Some biases, smaller than major service failures, could lead in some worst case situations to dangerous positioning failure. Unfortunately, the rate of occurrence of such biases is not currently known due to a lack of monitoring. This constitutes a major issue for the use of RAIM for approaches with vertical guidance.

Multiple failure case is also addressed in this paper. A method that benefits from the fact that multiple failures are very rare is used. It consists in not trying to detect these multiple failures and setting the probability of detecting an integrity failure caused by multiple faults to zero. This operation leads to more stringent required probability of missed detection for single failure but allows the use of various detection algorithms that have been designed assuming only one pseudorange failure at the same time.

Finally, a review of major RAIM assumption for approaches with vertical guidance operations is proposed.

INTRODUCTION

Autonomous integrity monitoring refers to a technique where a receiver uses the redundancy of satellite measurements to determine whether a fault condition exists that would cause it to have an unacceptable probability to experience a position error outside a specified bound. Today, Receiver Autonomous Integrity Monitoring (RAIM) is worldwide used to provide integrity monitoring down to non precision approaches using GPS constellation with L1 C/A measurements. Indeed, it is a simple and efficient solution to check the integrity of GNSS in civil aviation applications. But its performance is limited at best to NPA up to now. On another hand, approaches with vertical guidance present some safety, operational and environmental benefits that have been widely recognized. In a next future, in a multi constellation context due to the introduction of new satellite constellations, such as Galileo and modernized GPS, great improvements could be expected from RAIM performance.

So the perspective of using RAIM to provide integrity monitoring for approaches with vertical guidance is very attractive. However, this potential needs to be precisely quantified.

Two main facts will significantly improve RAIM capability to monitor integrity during more stringent operations.

First there will be an improvement in the quality of the available measurements that will significantly reduce the nominal error on pseudo range measurements. Indeed both GPS and Galileo will broadcast signals for aeronautical use on two distinct frequencies and that will allow the use of iono free measurements. Moreover, these future signals will offer improved tracking accuracy. Future systems will also provide better satellite clock and ephemeris information. These factors will significantly reduce the nominal error on pseudo range measurements.

Another point is the augmentation in the number of available measurements that will reduce the impact of a single satellite failure on the position estimation error and provide better satellite geometries.

But the targeted phases of flight are characterized by smaller horizontal tolerable position errors compared to NPA, by the introduction of vertical requirements and also by lower acceptable probabilities for these alert limits to be exceeded. Therefore, threatening range errors that need to be detected could have smaller amplitude and a probability of occurrence that is not clearly defined currently.

This is why there is a need to state the assumptions used for RAIM design and for RAIM performance evaluation. Although not unique nor conclusive, we make here our proposition.

For vertically guided approaches, we want to highlight the difficulties to set the values of some RAIM assumption parameters due to the lack of knowledge and validation on the occurrence and monitoring of small range errors.

The main design and evaluation assumptions that have been identified are:

- Pseudorange error model (UERE, nominal biases, correlation time)
- Threat model (single or multiple failure, failure full probability distribution)
- Internal probabilities (from application specifications and threat model)

Other assumptions for RAIM evaluation such as constellation, user grid, time period and time step are not particularly addressed here.

These assumptions will be detailed all along this paper. Assumptions for RAIM design are more particularly addressed here. The way the threat model can take into account the civil aviation requirements is particularly investigated through the computation of the required probability of missed detection.

PSEUDORANGE ERROR MODEL

The fundamental measurement in satellite navigation is the pseudorange which is modeled as:

$$Y(k) = h\big(X(k)\big) + E(k)$$

that is to say as a function of the true user position and clock plus some measurement errors.

The main differences between the true pseudorange and the measured pseudorange are due to several sources of error:

- space vehicle clock error
- equivalent satellite position estimation error
- signal propagation delays caused by the ionosphere and the troposphere
- multipath
- receiver errors which main source is code tracking loop noise

To evaluate GNSS positioning performance, measurement errors have to be modeled as precisely as possible. Systematic errors are gathered in the fault free case. Unusual errors that may cause a dangerous positioning failure and that may have to be detected are addressed in the faulty case.

This section addresses fault free case pseudorange error model. Most of the models assume that the pseudorange error components have a normal distribution with a known variance and a zero mean and that the pseudorange error components are combined by convolving their error distributions.

The User Equivalent Range Error is the value reflecting the error budget and it is based on the computation of the following contributions: orbit determination and synchronization equivalent error, troposphere residual error, ionosphere residual error, multipath residual error and receiver noise residual error.

$$\sigma_{UERE}^2 = \sigma_{URA}^2 + \sigma_{\text{iono}}^2 + \sigma_{\text{noise}}^2 + \sigma_{\text{multipath}}^2 + \sigma_{\text{tropo}}^2$$

Each model adopted for the study is described in this section. The measurement considered here are GPS L1/L5 and Galileo E1/E5b.

Satellite clock and ephemeris error

Satellite clock and ephemeris error components will depend on the considered system. For GPS, User Range Accuracy (URA) is the standard deviation of the range component of clock ephemeris error. The distribution of every satellite's range error is over bounded by a zero mean Gaussian distribution with standard deviation equal to URA. For Galileo, the signal in space error (not necessarily Gaussian) of each satellite will be over-bounded by a nonbiased Gaussian distribution with the minimum standard deviation called Signal In Space Accuracy (SISA). The integrity performance requirement specifies a SISA value for both nominal and degraded mode [ESA, 2005].

In this study we assume that this parameter is equivalent to the GPS URA. For GPS it has to be computed depending on the modernization step and a range of value is available [Lee and McLaughlin, 2007]. It has been decided in this study to convert the SISA value in an URA value and to choose the same value for GPS and Galileo, that is to say:

$$\sigma_{URA} = 0.85\,m$$

Tropospheric residual error

The model for the residual error for the tropospheric delay estimate is [RTCA, 2006]:

$$\sigma_{\text{tropo}} = \frac{1.001}{\sqrt{0.002001 + \sin^2 El}} \times 0.12\,m$$

where El is the elevation angle

This model was adopted for GPS L1 C/A and is assumed for GPS L5 and Galileo E1 and E5b.

Ionospheric residual error

Future civil aviation GNSS receivers will use dual frequency measurements and will combine them into this single composite measurement called the ionospheric-free measurement, corrected for ionospheric error. By this way the ionospheric residual error is not considered as significant anymore:

$$\sigma_{\text{iono}} = 0$$

Receiver noise residual error

The receiver noise residual error will be dominated by code tracking loop error will depend on the choice of the discriminator.

The error variance of the code-tracking loop for the Early – Minus Late Power (EMLP) is given by [Betz and Kolodziejski, 2000]:

$$\sigma_{EMLP}^2 = \frac{B_L(1-0.5B_LT)\int_{-B/2}^{B/2} G(f)\sin^2(\pi f C_S)\,df}{\frac{C}{N_0}\left(2\pi\int_{-B/2}^{B/2} fG(f)\sin(\pi f C_S)\,df\right)^2} \times \left(1 + \frac{\int_{-B/2}^{B/2} G(f)\cos^2(\pi f C_S)\,df}{\frac{C}{N_0}T\int_{-B/2}^{B/2} G(f)\cos(\pi f C_S)\,df}\right)$$

where

$B_L\,(H_z)$ the one sided bandwidth of the equivalent loop filter

$T\,(s)$ the data period

G the power spectrum density of the signal
C/N_0 (dBHz) the signal to noise ratio
C_S (s) the chip spacing
B (H_z) the two sided bandwidth of the front end filter

The error variance of the code tracking loop, error due to noise, can be thus computed for different kind of signals, using for example the following values:

	GPS L1 C/A	GPS L5	Galileo E1	Galileo E5b
C_S	0.25	0.25	0.25	0.25
B_L	1	1	1	1
B	16×10^6Hz	20×10^6Hz	20×10^6Hz	14×10^6Hz
C/N_0	35 dBHz	29 dBHz	36.5 dBHz	29.7 dBHz
T	$0.02\ s$	$0.02\ s$	$0.1\ s$	$0.1\ s$
T_C	1/1.023 MH	1/1.023 MH	1/1.023 MH	1/10.23 MH

Table 1 - Values for code delay tracking error variance computation

Note that worst case C/N_0 are given here and not typical values [Eurocae, 2006]. This drop of the equivalent C/N_0 down to tracking threshold allows to take into account some level of interference in our fault free case model.

The obtained error variance of the code-tracking loop values are gathered in the following table:

	GPS L1 C/A	GPS L5	Galileo E1	Galileo E5b
σ_{EMLP} (m)	2.00	0.53	0.86	0.59

Table 2 - Code-tracking loop error variance

Thus, from GPS L1 – L5, and from GALILEO E1 – E5b, two distinct iono-free measurements are built.

$$\sigma_{\text{code},L1-L5} = \sqrt{2.261^2\sigma^2_{\text{code},L1} + 1.261^2\sigma^2_{\text{code},L5}}$$

$$\sigma_{\text{code},E1-E5b} = \sqrt{2.422^2\sigma^2_{\text{code},E1} + 1.422^2\sigma^2_{\text{code},E5b}}$$

Once elaborated, these two GPS and GALILEO iono-free measurements are then smoothed to reduce the influence of noise and multipath [Hegarty, 1996]:

$$\sigma_{\tilde{P}}{}^2 \approx \frac{\sigma_P^2}{2T_{\text{smooth}}}$$

where
T_{smooth} is the time smoothing constant in seconds
σ_P^2 is the raw code pseudorange measurement error variance
$\sigma_{\tilde{P}}{}^2$ is the smoothed code pseudorange measurement error variance
Finally, the receiver noise residual error variance $\sigma_{noise}{}^2$ of smoothed iono free measurements is obtained.

	GPS L1 C/A /L5	Galileo E1/ E5b
$\sigma_{noise,EMLP}$ (m)	0.32	0.16

Table 3 - Receiver noise residual error variance

Multipath error

The smoothed multipath error for the airborne equipment is described by [RTCA, 2006]:

$$\sigma_{\text{multipath}} = 0.13 + 0.53\, exp\left({}^{-\theta}/_{10\text{deg}}\right)$$

where θ is the elevation angle in degrees of the considered satellite.

Preliminary studies have shown that smaller error can be anticipated for GPS L5, Galileo E1 and E5b since a flat sigma curve referring to a constant deviation of 7 cm for any elevation is proposed [Macabiau et al., 2006]. Nevertheless, to be conservative and before further validation, the L1 C/A SARPs [ICAO, 2006] error curve will be used in the following calculation for the other GNSS signals.

As for the error variance of the code-tracking loop, the smoothed multipath errors of each available signal are affected by the iono free combination:

$$\sigma_{\text{multipath}\ L1-L5} = \sqrt{2.261^2\sigma^2_{\text{multipath}\ L1} + 1.261^2\sigma^2_{\text{multipath}\ L5}}$$

$$\sigma_{\text{multipath}\ E1-E5b} = \sqrt{2.422^2\sigma^2_{\text{multipath}\ E1} + 1.422^2\sigma^2_{\text{multipath}\ E5b}}$$

User Equivalent Range Error

Finally the different User Equivalent Range Error components are represented on the following figure:

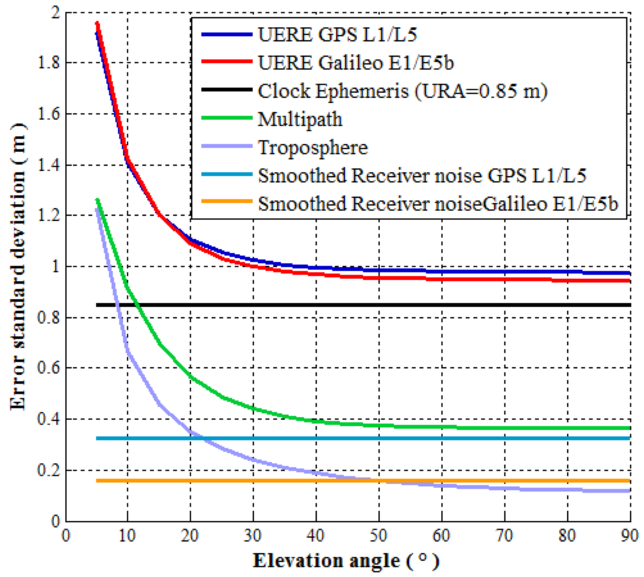

Figure 1 – User equivalent range error components

For $URA = 0.85\,m$:

	5	10	15	20	30	50	60	90
GPS L1/L5	1.92	1.43	1.20	1.11	1.02	0.99	0.98	0.98
Galileo E1/E5b	1.96	1.43	1.20	1.09	1.00	0.96	0.95	0.95

Table 4- GPS L1C/A /L5 and Galileo E1/E5b smoothed iono-free UERE values

It can be seen that the high quality of the future GNSS measurements (dual frequency measurements, better clock and ephemeris information, better ranging signals) significantly decreases the UERE variance compared to GPS L1 C/A only. Considering that UERE is the major parameter of position estimation and autonomous integrity monitoring performance, great RAIM availability could be expected from an UERE standard deviation of approximately one meter.

Nominal biases

In order to be more realistic, pseudorange measurement models can take into account a bias that bounds errors that may appear random but that affect user in the same way repeatedly [Walter et al., 2008]. For example, GPS Evolutionary Architectural Study (GEAS) has agreed to consider explicitly the presence of biases in range measurement under non faulted conditions and has assumed a level of bias magnitude under fault - free condition called maximum bias magnitude such as [Lee and McLaughlin, 2007]:

$$50\ cm \leq B_{max} \leq 2\ m$$

Correlation time

The correlation time will depend on the source of the measurement error:

- Receiver and multipath will be driven by the smoothing time constant of the receiver noise which is assumed to be on the order of two minutes.

- Tropospheric error will be modelled using this first order Gauss Markov process with a 30 minutes correlation time [RTCA, 2006].

- [RTCA, 2006] states that the satellite clock and ephemeris error shall be modelled using a first-order Gauss Markov process with a 2 hour correlation time. But a correlation time of approximately one hour, based on the average period of time satellites are visible to the user can be used.

SMALLEST BIAS THAT LEADS TO A POSITIONING FAILURE

The integrity monitoring requires that the navigation system detects the presence of an unacceptably large position error for a given mode of flight, and if possible, isolates and removes the source of unacceptably large position error from the navigation solution, thereby allowing navigation to return to normal performance without an interruption in service.

Therefore, only faults that lead to a positioning failure (horizontal or vertical) need to be detected. The goal of this subsection is to propose a method to identify pseudorange biases that lead to a positioning failure, that is say to compute for each available pseudorange, the smallest bias on this pseudorange that will lead to a positioning failure. These smallest biases correspond to the worst case detection/exclusion situation, they can be used to design RAIM algorithm and/or to estimate their statistical properties.

This concept has been introduced in [Nikiforov, 2005] as an input parameter of the constrained GLR test.

A pseudorange error γ is considered as a horizontal positioning failure if its impact violates the integrity risk, that is to say if:

$$(1 - P_f)P_0\left(\|X_H - \hat{X}_H\| > HAL\right) + P_f P_\gamma\left(\|X_H - \hat{X}_H\| > HAL\right) > P_{Int}$$

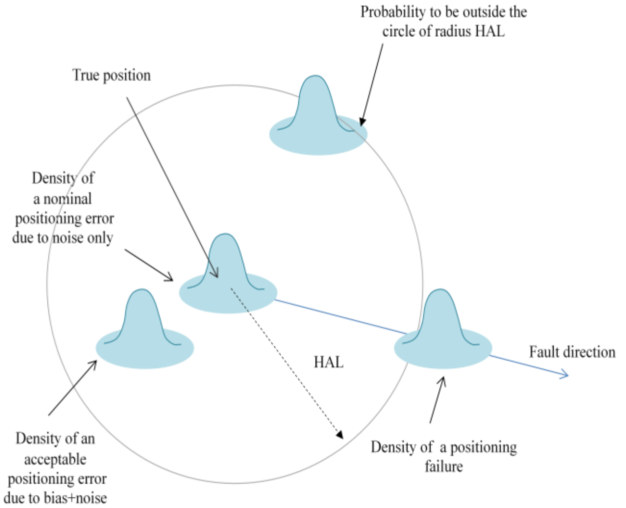

Figure 2 - Horizontal positioning failure

As it is depicted on the previous figure, each individual satellite fault (additional pseudorange bias) produces a fault direction in the horizontal plane. The main question is: how far from the true position the ellipse's centre can be moved along the corresponding fault direction in order to consider that this bias leads to a positioning failure? That will depend, among other things, on the mutual orientation of this "ellipse of uncertainty" and the fault direction. Other important factors are the allowed integrity risk, the probability of failure, the probability to exceed the xAL in fault free mode. The computation, which has to be done for each pseudorange, is detailed in appendix.

A pseudorange error γ is considered as a vertical positioning failure if its impact violates the integrity risk such as:

$$(1-P_f)P_0(|X_V-\hat{X}_V|>VAL)$$
$$+P_fP_\gamma(|X_V-\hat{X}_V|>VAL)>P_{Int}$$

where P_f is the probability of failure of one satellite
P_0 corresponds to the fault free case
P_γ corresponds to the faulty case

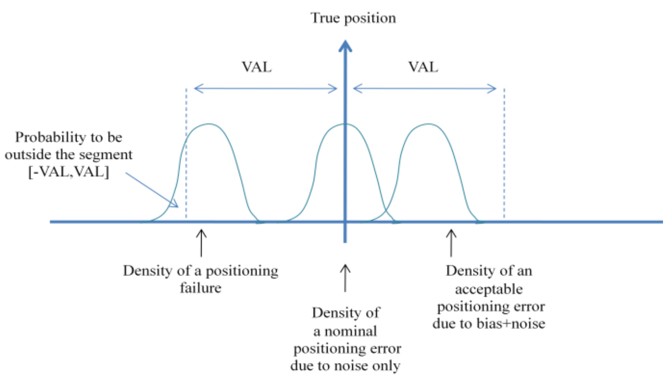

Figure 3 - Vertical positioning failure

These critical biases values are to be computed for a given user position at a given moment by (for a given sample):

- Computing the probability to exceed the alert limit in the fault free case $P_0(\|X_H-\hat{X}_H\|>HAL)$ and $P_0(|X_V-\hat{X}_V|>VAL)$

- For each available pseudorange measurement, computing the smallest additional bias b_i that leads to a probability $P_{b_i}(\|X_H-\hat{X}_H\|>HAL)$ or $P_{b_i}(|X_V-\hat{X}_V|>VAL)$ such as:

$$(1-P_f)P_0(\|X_H-\hat{X}_H\|>HAL)$$
$$+P_fP_{b_i}(\|X_H-\hat{X}_H\|>HAL)=P_{Int}$$

$$(1-P_f)P_0(|X_V-\hat{X}_V|>VAL)$$
$$+P_fP_{b_i}(|X_V-\hat{X}_V|>VAL)=P_{Int}$$

The computations of the probabilities P_0 and P_{b_i} do not depend on any detection algorithm. But it can be seen that they depend on the failure probability of occurrence, and also on the algorithm used for positioning computation.

The amplitude of the smallest bias on a given pseudorange that will lead to positioning failure will depend on:

- the intended operation: size of the alert limit, integrity risk
- the quality of measurement
- the geometry: impact of one pseudorange bias on the global position solution (number of available measurement, position of the faulty satellite)
- its rate of occurrence: the lower a failure occurrence rate is, the higher its amplitude can be

With a very poor satellite geometry, noisy measurements and stringent requirement, the amplitude of the smallest bias that would lead to a positioning failure could be zero. This would correspond to an integrity failure in fault free conditions

It seems to us that knowing the amplitude of critical biases is fundamental in RAIM design as it defines the limit of threat model. It is particularly true for the computation of the required probability of missed detection

REQUIRED PROBABILITY OF MISSED DETECTION

The targeted probability of missed detection constitutes a major input of RAIM algorithm. This parameter derives from the integrity risk but also depends on the probability of satellite failure. Thus it refers to the threat model and particularly needs to be detailed.

Only considering the single failure case, the probability of missed detection P_{md} shall be lower than the integrity risk requirement divided by the probability of failure of one satellite among the all satellites in view.

$$P_{md} = \frac{P_{int}}{P_{\text{satellite failure},N,1}}$$

We will consider the multiple failure case in a dedicated subsection of this paper.

The computation can be described for En-route, terminal and NPA operations. For these phases of flight the required integrity risk is 1×10^{-7}/h and the only feared events used to be the major service failure because of the wide acceptance regions. This probability is approximated by:

$$P_{\text{major satellite failure}} \cong 10^{-4}/h$$

And the probability of missed detection is the result of:

$$P_{md} = \frac{P_{int}}{P_{\text{major satellite failure}}} = \frac{10^{-7}/h}{10^{-4}/h} = 10^{-3}$$

This result is only valid for En-route to NPA operations, only assuming single satellite failure from major service failure category and only considering GPS constellation.

To correctly address our issue, it is first necessary to know the probability of occurrence of feared events. Indeed, numerous events can cause a fault event. The main ones are satellite failures but there are also environmental effects such as multipath or interference.

Concerning satellite failure, the most monitored error, the one that corresponds to NPA threat model, is GPS major service failure.

In [GPS SPS, 2001], a major service failure is defined to be a condition over a time interval during which a healthy GPS satellite's ranging signal error exceeds the range error limit. This range error limit is the larger of:

- 30 m
- 4.42 times the URA

The probability of occurrence of such an event is 3 per year for a 24- satellite GPS constellation

In [GPS SPS, 2008], the tolerance value is set to 4.42 times the upper bound on the URA value currently broadcast by the satellite. This parameter can take values between two meters and several thousand meters.

Three major service failures per year for a 24- satellite GPS constellation leads to:

$$p \cong 1.43 \times 10^{-5}/h$$

where p is the individual major satellite failure probability

Denoting N the number of satellite in view from the user, then the probability of having k simultaneous failures among N satellites in view is:

$$P_{\text{major satellite failure},N,k} = C_N^k p^k (1-p)^{N-k}$$

The same rate of occurrence of major service failure, plus the same induced range error size will be considered for Galileo in this study.

However, for navigation with much tighter position protection limits, even small errors would be considered significant:

- Signal deformation and distortion
- Ephemeris error

But their rate of occurrence is not clearly defined.

The effects of the interference are taken into account in the total standard deviation as the sigma noise is computed at the lowest C/N_0 possible for nominal conditions.

The targeted probability of missed detection derives from the integrity risk but also depends on the probability of occurrence of the failure. To know the probability of occurrence of a single failure, it is useful to evaluate the minimal amplitude of a single pseudorange failure that leads to an unacceptable positioning error for the intended operation. This is defined as the critical bias. This will corresponds to the minimal bias amplitude that needs to be detected by RAIM algorithms.

Every critical bias has been computed according to the following assumptions.

User grid

A worldwide evaluation is conducted and thus a user grid needs to be defined. It has been decided for this study to use a grid with a latitude step of 5° and a longitude step of 5°. This represents a total amount of 2520 user positions.

Simulation period

Some simulations will imply both Galileo and GPS satellites. In order to have representative satellite geometries, the simulation period has to correspond to both constellation orbital periods.

According to ESA, Galileo satellites will have orbit altitude of 23 222 kilometers resulting in a ground track repeat cycle of ten days during which each satellite has completed seventeen revolutions. Nevertheless, each Galileo satellite has an approximate orbit revolution period of 14 hours and 7 minutes which corresponds to five revolutions in three days.

The nominal orbital period of all vehicles in the GPS constellation is 12 sidereal hours that is to say that each GPS satellite has an orbital period of 11 hours and 58 minutes, at an altitude of 20 183 kilometers. Therefore, three days also approximately correspond to six GPS satellites periods.

This is why a simulation time of three days has been chosen for dual constellation whereas a simulation time of one day has been chosen for single constellation studies.

For three days simulation, values will be evaluated every 4 minutes which correspond to 1080 values for each user point. Thus, that will provide an amount of 2 721 600 values for the 2520 points of the user grid. For one day simulation, values will be evaluated every minute which correspond duration to 1440 values for each user point. That will provide an amount of 3 628 800 values for the 2520 points the user grid.

Mask angle

As indicated in the Galileo Integrity Concept [ESA, 2005], the user elevation angle above which SISA is guaranteed is 10°. Even if this study does not concern Galileo ground integrity channel, this SISA value has been consider for UERE provision. This is why a 10° degree mask will be used for Galileo satellite visibility computation.

As specified in [GPS SPS, 2008], GPS performance are given for a receiver which tracks all satellites in view above a 5° mask angle. This is why a 5° degree mask will be used for GPS satellite visibility computation.

Constellations

A 27 satellites Galileo constellation [Eurocae, 2006] and an optimized 24 satellites GPS constellation [RTCA, 2006] has been considered for these simulations and the satellite position computation will be made thanks to corresponding almanac data.

Available satellites

Those assumptions lead to a given set of visible satellites. Actually a greater number of GPS satellites can be expected in future and therefore, the following results are quite conservative.

It can be seen on the following figure that, considering the GPS and Galileo constellations described above, an average number of 17 satellites will be available.

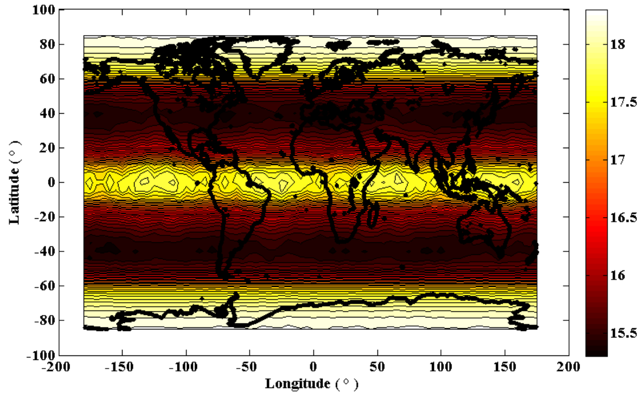

Figure 4 - Average number visible satellites over 3 days considering 24 sat GPS and 27 sat Galileo constellations

The same computation has been conducted only considering 24 satellite GPS constellation, then only considering 27 Galileo constellation.

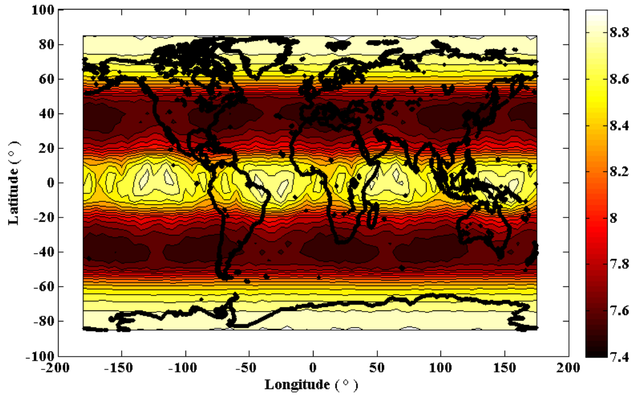

Figure 5 - Average number of satellites over 1 day considering 24 satellites GPS constellation

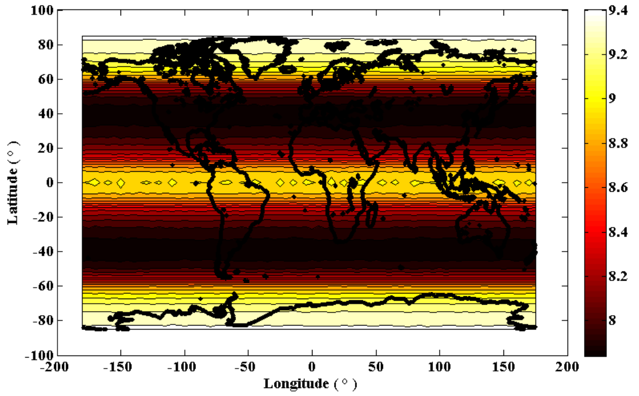

Figure 6 - Average number of satellites over 1 day considering 27 satellites Galileo constellation (mask angle 10°)

Critical biases

The objective of this preliminary study is to evaluate the amplitude of every critical bias as a function of their occurrence rate.

The following requirements have been considered:

	Horizontal Alert Limit	Vertical Alert Limit	Integrity risk
APV I	40 m	50 m	2×10^{-7} in any approach
LPV 200	40 m	35 m	1×10^{-7} in any approach

Table 5 – Requirements

LPV 200 is a new concept of aircraft instrument approach procedure in which guidance is provided down to a minimum decision altitude as low as 200 feet height above touchdown. This category of approach is not included yet in the Annex 10 but some proposed requirements to support this new operation can be presented [DeCleene, 2007]

It is to be noticed that the 35 m VAL satisfies obstacle clearance requirement but does not necessarily set up the aircraft to land safely on the runway. This is why a system that is intended to provide LPV 200 must also have a very low probability of 10 meter positioning errors. This is why we are also going to evaluate the amplitude of biases that would lead to 10 meter positioning error.

We compute for each pseudorange the smallest bias that leads to a horizontal positioning failure and the smallest bias that leads to a vertical positioning failure. For each pseudorange, the minimum of these two biases is the smallest bias that leads to a positioning failure. In most of the cases, this smallest bias leads to a vertical positioning failure.

The two following figures represent the average and minimal values of smallest biases that lead to a positioning failure for APVI and LPV 200 requirements. These values have been computed for different satellite failure probabilities of occurrence and are represented as a function of the integrity risk-probability of satellite failure occurrence ratio.

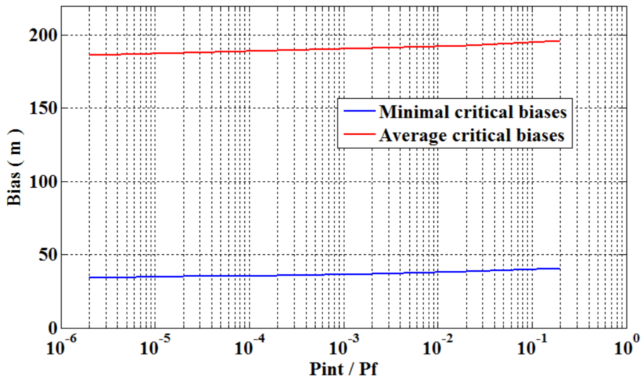

Figure 7 - Smallest bias that leads to a positioning failure - APVI operations (HAL=40m,VAL=50m) – dual constellation (GPS + Galileo)

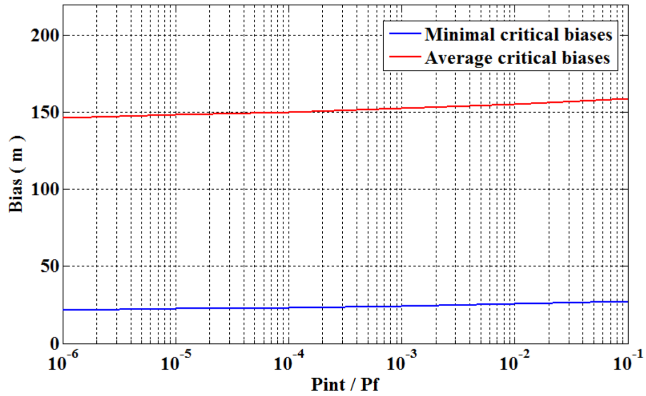

Figure 8 - Smallest bias that leads to a positioning failure - LPV200 operations (HAL=40m,VAL=35m)– dual constellation (GPS + Galileo)

It can be seen that for APVI operations and for a large scale of probability of occurrence, the amplitude of smallest critical biases is systematically larger than 35 meters.

For LPV 200 operations, the amplitude of smallest critical biases is systematically larger than 22 meters.

We can conclude that in both cases (APVI and LPV 200 operations) and for a large scale of probability of occurrence, the amplitude of smallest critical biases belongs to the GPS major service failure category and so to the extended Galileo major service failure category.

It can be noticed that there is very minimal sensitivity of the smallest bias to the probability of occurrence. This is due to the small UERE standard deviation compared to the alert limit.

We may also have to detect errors that would lead to a 10 meters positioning error for LPV200 operation. The smallest biases that would lead to such errors have also been computed:

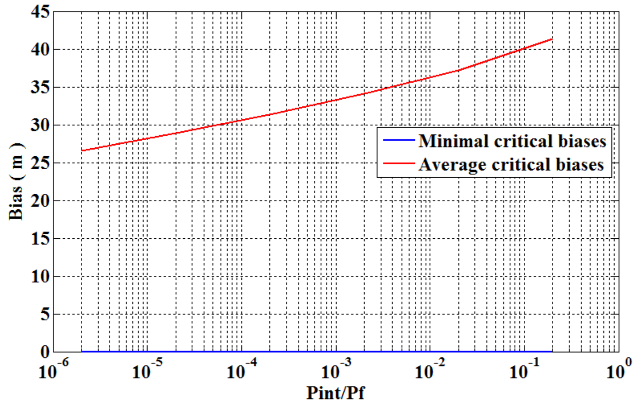

Figure 9 - Smallest bias that leads to a positioning failure - LPV200 operations (horizontal threshold=10m, vertical threshold=10m)– dual constellation (GPS + Galileo)

In some situation, the smallest bias that would lead to such a positioning failure has an amplitude of 0 meter that would corresponds to the fault free case.

Therefore, with 10 meters threshold, it will mandatory to know the probability of occurrence of very small failures to correctly design RAIM algorithms.

We may have to detect errors that would lead to a 15 meters positioning error with a probability of 1×10^{-5}. The smallest biases that would lead to such errors have also been computed:

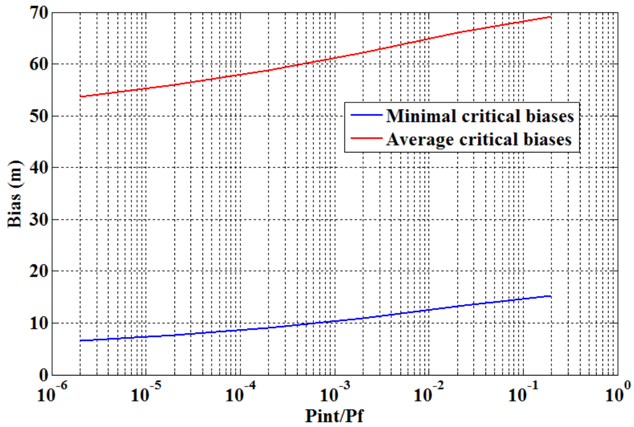

Figure 10 - Smallest bias that leads to a positioning failure - LPV200 operations (horizontal threshold=15m,vertical threshold=15m) – dual constellation (GPS + Galileo)

It can be seen that in the worst situation a bias of 7 meter on a given pseudorange can lead to 15 meters positioning failure. In this case, considering only major service failure in the threat model is more questionable.

In the coming section,the required probability of missed detection computation are made only considerating major service failure in the threat model. Indeed , a vertical alert limit of 35 meters is considered for LPV200 operations.

Outage duration time

As the integrity risk is specified for an approach duration for APV and LPV operation, one method to obtain the probability of one satellite failure per approach could have been to divide the hourly rate by 24 (as $24 \times 150 = 3600$). But in the determination of the probability of encountering a major failure, the outage duration time is a major parameter.

The GPS specified time to remove the faulty satellite when a major service failure has occurred is 6 hours and actual performance is typically one hour. Even if shorter delays can be expected from GPS III and Galileo systems, it may be lowered but likely not below 1 hour.

For this study, it will be considered that a failure duration is one hour which leads to convert this integrity failure rate for one satellite $p \cong 1.43 \times 10^{-5}/h$ in $p \cong 1.43 \times 10^{-5}$ per approach. As if an approach duration was artificially set to one hour, because a failure that had occurred one hour before could still have an impact.

Required probability of missed detection computation: single failure

Only considering the single failure case, the probability of missed detection P_{md} shall be lower than the integrity risk requirement divided by the probability of failure of one satellite among the all satellites in view.

$$P_{md} = \frac{P_{int}}{P_{\text{major satellite failure},N,1}}$$

Required probability of missed detection computation: multiple failure

It is proposed to not to try to detect multiple failures because of their low probability of occurrence and therefore to set the probability of detecting an integrity failure cause by multiple faults to zero (corresponding P_{md} equal to one) such as:

$$P_{int} = P_{\text{satellite failure},N,1} \times P_{md} + P_{\text{multiple satellite failures},N}$$

Therefore, the multiple failure case is derived from the single failure case by allocating a smaller integrity risk.

$$P_{md} = \frac{P_{int} - P_{\text{multiple satellite failures},N}}{P_{\text{major satellite failure},N,1}}$$

The multiple failure case is derived from the single failure case by allowing a smaller integrity risk.

Corresponding results are gathered in the following table.

Dual constellation (17 satellites in view)	APV	LPV 200
Single failure case	$P_{md} = 8.23 \times 10^{-4}$	$P_{md} = 4.12 \times 10^{-4}$
Multiple failures case (single failure detection)	$P_{md} = 6.56 \times 10^{-4}$	$P_{md} = 2.43 \times 10^{-4}$

Table 6 – Required probability of missed detection

PROBABILITY OF FALSE ALERT

In absence of Selective Availability SA the correlation time is driven by the receiver noise whose smoothing time constant is assumed to be on the order of two minutes.

For this study, a correlation time of two minutes will be considered. As it is the maximum allowable false alert rate per sample that constitutes an input for RAIM algorithms threshold computation, the following values will be used:

Operations	Maximum Allowable False Alert rate	Maximum Allowable False Alert rate (per sample)
En-route to Non precision approach	10^{-5}per hour [RTCA, 2001]	3.33×10^{-7} per test [RTCA, 2001], [RTCA, 2006]
Approach with Vertical Guidance	2×10^{-5}per approach [RTCA, 2001]	1.6×10^{-5}per test [RTCA, 2006]
LPV 200	2×10^{-5}per approach	1.6×10^{-5}per test

Table 7 - Maximum allowable false alert rate

CONCLUSION

A set of major RAIM assumptions has been detailed for RAIM design for vertically guided approaches APV and LPV200 using dual frequency GPS/GALILEO.

Signals used were L1C/A /L5 for GPS and E1/E5b for Galileo. An assumption of URA=0.85 m has been made for both constellations.

For the critical biases evaluation, computations have been perform considering the following assumptions:

- A user grid with a latitude step and a longitude step of 5° representing a total amount of 2520 positions has been considered.
- Values have been evaluated every 4 minutes over 3 days simulations for dual constellation simulations and every minute over 1 day simulation for single constellation simulations.
- A 10° mask angle has been used for Galileo satellites and a 5° mask angle has been used for GPS ones.
- A 24 satellites GPS constellation and a 27 Galileo satellite have been considered.

Then we have demonstrated that, for the single failure case using GPS + Galileo constellations, the amplitude of pseudo range additional biases that lead to a positioning failure systematically belongs to the major service failure category for both APV I and LPV 200 (VAL=35 m) operations. Therefore even if the targeted phases of flight are characterized by smaller horizontal and vertical tolerable position errors compared to NPA, this effect is mitigated by the great number of available measurements that reduces the impact a of single satellite bias on the global positioning error. Thus only Major Service Failures need be taken into account for single failure case consideration using GPS + Galileo constellations.

This assumption is more questionable if a monitoring threshold is set to 10 meters or 15 meters for LPV200 operations. Some biases, smaller than major service failures, could lead in some worst case situations to dangerous positioning failure. Unfortunately, the rate of occurrence of such biases is not currently known due to a lack of monitoring. This constitutes a major issue for the use of RAIM for approaches with vertical guidance.

A failure duration of 1 hour is proposed for the design of RAIM for vertically guided approaches

Multiple failures have been into account by allocating a smaller integrity risk to the required probability of missed detection computation. This operation leads to more stringent required probability of missed detection for single failure but allows the use of various detection algorithms that have been designed assuming only one pseudorange failure at the same time.

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APPENDIX

The aim of this section is to detail the computation for each pseudo range of the bias b_i that will lead to a horizontal positioning failure with a given probability.

Let us consider the case where there is a bias on the pseudo range i,

The error in the position domain is:

$$\Delta X_{\text{WGS }84} = \left[H^t \Sigma^{-1} \mathrm{H}\right]^{-1} H^t \Sigma^{-1} E$$

where

$$E \sim N(B, \Sigma)$$

$$\Sigma = \begin{bmatrix} \sigma_1^2 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \sigma_N^2 \end{bmatrix}, B = \begin{bmatrix} 0 \\ \vdots \\ b_i \\ \vdots \\ 0 \end{bmatrix}$$

If the matrix H is expressed in the local geographic frame such as:

$$H = \begin{bmatrix} \cos E_1 \cos A_1 & \cos E_1 \sin A_1 & \sin E_1 & 1 \\ \vdots & \vdots & \vdots & \vdots \\ \cos E_N \cos A_N & \cos E_N \sin A_N & \sin E_N & 1 \end{bmatrix}$$

Then the positioning error is directly expressed in the local geographic frame

$$\Delta X_{\text{local}} = [H^t \Sigma^{-1} \mathrm{H}]^{-1} H^t \Sigma^{-1} E$$

The covariance matrix C of the error is such as:

$$C = E\left[\Delta X_{\text{local}} . \Delta X_{\text{local}}{}^{\mathrm{t}}\right]$$
$$= \left(\left[H^t \Sigma^{-1} \mathrm{H}\right]^{-1} H^t \Sigma^{-1}\right) \Sigma \left(\left[H^t \Sigma^{-1} \mathrm{H}\right]^{-1} H^t \Sigma^{-1}\right)^t$$

$$C = [H^t \Sigma^{-1} \mathrm{H}]^{-1}$$

The horizontal positioning error is a two dimensions vector which follows a gaussian bi-dimensional law of mean $b_{i,H,\text{local}}$ the projection of b_i in the horizontal plane and of covariance matrix C_H, such as $C_H = C(1{:}2,1{:}2)$, $b_{i,\text{local}} = \left[H^t \Sigma^{-1} \mathrm{H}\right]^{-1} H^t \Sigma^{-1} \mathrm{B}$ and $b_{i,H,\text{local}} = b_{i,\text{local}}(1{:}2)$

Its density function is:

$$f_{\Delta X_{\mathrm{H,local}}}(X) = \frac{1}{2\pi\sqrt{det(C_H)}} \times \exp\left(-\frac{1}{2}\left(\mathrm{X} - b_{i,H,\text{local}}\right)^{\mathrm{t}} . C_H{}^{-1} . \left(\mathrm{X} - b_{i,H,\text{local}}\right)\right)$$

where X is expressed in the Nord East local frame such as $X = \begin{bmatrix} x_N \\ x_E \end{bmatrix}$

Since C_H is a covariance matrix, C_H is a positive definite matrix, it is diagonalizable and its eigenvalues are all positive. In particular we can find an orthonormal basis $B = \left(\vec{e}_1, \overrightarrow{e_2}\right)$ that is composed of eigenvectors $\vec{e}_{1,i}, \vec{e}_{2,i}$ corresponding to the eigenvalues λ_1 and λ_2 and such as:

$$C_H = \mathrm{P}_\perp . \Delta . \mathrm{P}_\perp{}^{\mathrm{t}}$$

where

$\Delta = \mathrm{diag}(\lambda_1, \lambda_2)$ is the diagonal matrix whose elements are the eigenvalues of C_H

$\mathrm{P}_\perp$ is the projection matrix whose columns are the eigenvectors $\vec{e}_1, \vec{e}_2$. In particular $\mathrm{P}_\perp$ is orthogonal: $\mathrm{P}_\perp{}^{-1} = \mathrm{P}_\perp{}^{\mathrm{t}}$

Then, $\det(C_H) = \lambda_1 \lambda_2$ and $C_H{}^{-1} = \mathrm{P}_\perp . \Delta^{-1} . \mathrm{P}_\perp{}^{\mathrm{t}}$

$$\left(\mathrm{X} - b_{i,H,\text{local}}\right)^{\mathrm{t}} . C_H{}^{-1} . \left(\mathrm{X} - b_{i,H,\text{local}}\right)$$
$$= \left(\mathrm{X} - b_{i,H,\text{local}}\right)^{\mathrm{t}} . \mathrm{P}_\perp . \Delta^{-1} . \mathrm{P}_\perp{}^{\mathrm{t}} . \left(\mathrm{X} - b_{i,H,\text{local}}\right)$$

$$= \left[\mathrm{P}_\perp{}^t\left(\mathrm{X} - b_{i,H,\text{local}}\right)\right]^t.\Delta^{-1}.\left[\mathrm{P}_\perp{}^t.\left(\mathrm{X} - b_{i,H,\text{local}}\right)\right]$$

Denoting $X_\perp = \mathrm{P}_\perp{}^t.\mathrm{X}$ and $\Omega = \mathrm{P}_\perp{}^t.b_{i,H,\text{local}}$, $X_\perp$ is the vector X expressed in the new local frame and Ω is the vector $b_{i,H,\text{local}}$ in the new local frame.

$$f_{\Delta X_{\text{H,local}},\text{b}}(X) = \frac{1}{2\pi\sqrt{\lambda_1\,\lambda_2}}\exp\left(-\frac{1}{2}\left(\frac{(\mathrm{x}_\perp - \Omega_1)^2}{\lambda_1} + \frac{(\mathrm{y}_\perp - \Omega_2)^2}{\lambda_2}\right)\right)$$

The probability that a couple (x, y) be such that $x^2 + y^2 \leq HAL^2$ is the probability that $x_\perp{}^2 + y_\perp{}^2 \leq HAL^2$ and considering the distribution of the horizontal positioning error, this probability is:

$$P(\Delta X_{\text{H,local}} \in \mathrm{D}) = \iint_D \frac{1}{2\pi\sqrt{\lambda_1\,\lambda_2}}\exp\left(-\frac{1}{2}\left(\frac{(x_\perp - \Omega_1)^2}{\lambda_1} + \frac{(y_\perp - \Omega_2)^2}{\lambda_2}\right)\right)\mathrm{d}x\,\mathrm{d}y$$

denoting D the domain such as $x_\perp{}^2 + y_\perp{}^2 \leq HAL^2$.

Let's make a change of coordinates such as we could have:

$$\frac{(x_\perp - \Omega_1)^2}{\lambda_1} + \frac{(y_\perp - \Omega_2)^2}{\lambda_2} = \mathrm{r}^2$$

$(x_\perp, y_\perp)$ re-written this way:

$$\begin{cases} x_\perp = \Omega_1 + \mathrm{r}\cos\theta\sqrt{\lambda_1} \\ y_\perp = \Omega_2 + \mathrm{r}\sin\theta\sqrt{\lambda_2} \end{cases}$$

The equation $x_\perp{}^2 + y_\perp{}^2 = HAL^2$ that defines the boundaries of the integration domain becomes:

$$x_\perp{}^2 + y_\perp{}^2 = \left(\Omega_1 + \mathrm{r}\cos\theta\sqrt{\lambda_1}\right)^2 + \left(\Omega_2 + \mathrm{r}\sin\theta\sqrt{\lambda_2}\right)^2$$
$$= \Omega_1{}^2 + \mathrm{r}^2\lambda_1\cos^2\theta + 2\Omega_1\mathrm{r}\sqrt{\lambda_1}\cos\theta + \Omega_2{}^2 + \mathrm{r}^2\lambda_2\sin^2\theta + 2\Omega_2\mathrm{r}\sqrt{\lambda_2}\sin\theta = HAL^2$$

$$\mathrm{r}^2(\lambda_1\cos^2\theta + \lambda_2\sin^2\theta) + r\left(2\Omega_1\sqrt{\lambda_1}\cos\theta + 2\Omega_2\sqrt{\lambda_2}\sin\theta\right) + \left(\Omega_1{}^2 + \Omega_2{}^2 - HAL^2\right) = 0$$

Finally, denoting

$$a(\theta) = (\lambda_1\cos^2\theta + \lambda_2\sin^2\theta)$$
$$b(\theta) = \left(2\Omega_1\sqrt{\lambda_1}\cos\theta + 2\Omega_2\sqrt{\lambda_2}\sin\theta\right)$$
$$c(\theta) = \left(\Omega_1{}^2 + \Omega_2{}^2 - HAL^2\right)$$

Solving this equation, two roots $r_1(\theta)$ and $r_2(\theta)$ for $\theta\epsilon[0, \pi]$ are obtained such as:

$$\begin{cases} x_\perp = \Omega_1 + r_1(\theta)\cos\theta\sqrt{\lambda_1} \\ y_\perp = \Omega_2 + r_1(\theta)\sin\theta\sqrt{\lambda_2} \end{cases}, \theta\epsilon[0, \pi]$$

and

$$\begin{cases} x_\perp = \Omega_1 + r_2(\theta)\cos\theta\sqrt{\lambda_1} \\ y_\perp = \Omega_2 + r_2(\theta)\sin\theta\sqrt{\lambda_2} \end{cases}, \theta\epsilon[0, \pi]$$

define the boundaries of the integration domain.

The jacobian of this transformation is computed to make the change of coordinates $J = |r|\sqrt{\lambda_1\lambda_2}$, and:

$$P(\Delta X_{\text{H,local}} \in \mathrm{D}) = \iint_{D'} \frac{|r|}{2\pi}\exp\left(-r^2/2\right)\mathrm{d}r\mathrm{d}\theta$$

where the new domain D' is defined by:

$$\begin{cases} (r - r_1(\theta))(r - r_2(\theta)) \leq 0 \\ \theta\epsilon[0, \pi] \end{cases}.$$

Considering properties of second order polynomials:

$$P(\Delta X_{\text{H,local}} \in \mathrm{D}) = \frac{1}{2\pi}\int_{\theta=0}^{\theta=\pi}\int_{\mathrm{r}=r_1(\theta)}^{\mathrm{r}=r_2(\theta)} |\mathrm{r}|\exp\left(-r^2/2\right)\mathrm{d}r\mathrm{d}\theta$$

Assuming for example that $r_1(\theta) \leq 0 \leq r_2(\theta)$,

$$P(\Delta X_{\text{H,local}} \in \mathrm{D}) = \frac{1}{2\pi}\int_{\theta=0}^{\theta=\pi}\left[-\int_{\mathrm{r}=r_1(\theta)}^{\mathrm{r}=0}\mathrm{r}\exp\left(-r^2/2\right)\mathrm{d}r + \int_{\mathrm{r}=0}^{\mathrm{r}=r_2(\theta)}\mathrm{r}\exp\left(-r^2/2\right)\mathrm{d}r\right]\mathrm{d}\theta$$

$$P(\Delta X_{\text{H,local}} \in \mathrm{D}) = 1 - \frac{1}{2\pi}\int_{\theta=0}^{\theta=\pi}\left[\exp\left(-r_2(\theta)^2/2\right) + \exp\left(-r_1(\theta)^2/2\right)\right]\mathrm{d}\theta$$

This last integral is computed numerically.

Thus the probability that the point (x, y) representing the horizontal position error is out of the circle of radius HAL is:

$$P(\Delta X_{\text{H,local}} \notin \mathrm{D}) = \frac{1}{2\pi}\int_{\theta=0}^{\theta=\pi}\left[\exp\left(-r_2(\theta)^2/2\right) + \exp\left(-r_1(\theta)^2/2\right)\right]\mathrm{d}\theta$$