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▶ **To cite this version:**

Emilie Rebeyrol, Christophe Macabiau, Lionel Ries, Jean-Luc Issler, Michel Bousquet, et al.. Interplex modulation for navigation systems at the L1 band. ION NTM 2006, National Technical Meeting of The Institute of Navigation, Jan 2006, Monterey, United States. pp 100-111. hal-01021777

**HAL Id: hal-01021777**

**https://enac.hal.science/hal-01021777**

Submitted on 27 Oct 2014

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# Interplex Modulation for Navigation Systems at the L1 band

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## BIOGRAPHY

Emilie Rebeyrol graduated as a telecommunications engineer from the INT (Institut National des Télécommunications) in 2003. She is now a Ph.D student at the satellite navigation lab of the ENAC. Currently she carries out research on Galileo signals and their generation in the satellite payload in collaboration with the CNES (Centre National d'Etudes Spatiales), in Toulouse, France.

Christophe Macabiau graduated as an electronics engineer in 1992 from the ENAC (Ecole Nationale de l'Aviation Civile) in Toulouse, France. Since 1994, he has been working on the application of satellite navigation techniques to civil aviation. He received his Ph.D. in 1997 and has been in charge of the signal processing lab of the ENAC since 2000.

Lionel Ries is a navigation engineer in the "Transmission Techniques and Signal Processing Department", at CNES since June 2000. He is responsible of research activities on GNSS2 signals, including BOC modulations and modernised GPS signals (L2C & L5). He graduated from the Ecole Polytechnique de Bruxelles, at Brussels Free University (Belgium) and then specialized in space telecommunications systems at Supaero (ENSAE), in Toulouse (France).

Jean-Luc Issler is head of the Transmission Techniques and signal processing department of CNES, whose main tasks are signal processing, air interfaces and equipments in Radionavigation, Telecommunication, TT&C, High Data Rate TeleMetry, propagation and spectrum survey. He is involved in the development of several spaceborne receivers in Europe, as well as in studies on the European RadioNavigation projects, like GALILEO and the Pseudolite Network. With DRAST and DGA, he represents France in the GALILEO Signal Task Force of the European Commission. With Lionel Ries and Laurent Lestarquit , he received the astronautic prize 2004 of the French Aeronautical and Astronautical Association (AAAF) for his work on Galileo signal definition.

Michel Bousquet is a Professor at SUPAERO (French Aerospace Engineering Institute of Higher Education), in charge of graduate and post-graduate programs in aerospace electronics and communications. He has over twenty five years of teaching and research experience, related to many aspects of satellite systems (modulation and coding, access techniques, onboard processing, system studies...). He has authored or co-authored many papers in the areas of digital communications and satellite communications and navigation systems, and textbooks, such as "Satellite Communications Systems" published by Wiley.

Marie-Laure Boucheret graduated from the ENST Bretagne in 1985 (Engineering degree in Electrical Engineering) and from Telecom Paris in 1997 (PhD degree). She worked as an engineer in Alcatel Space from 1986 to 1991 then moved to ENST as an Associated Professor then a Professor. Her fields of interest are digital communications (modulation/coding, digital receivers, multicarrier communications …), satellite on-board processing (filter banks, DBFN …) and navigation system.

## ABSTRACT

Because of the limited availability of the spectrum allocated for navigation systems, the numerous navigation signals broadcast by Modernized GPS and Galileo system will have to be combined and employ bandwidth-efficient modulations.

Indeed, the GPS modernization scheme entails the addition of the new military signal (M-code) to the established C/A and P(Y) codes at the same carrier frequency. One of the most important questions is how to combine this new signal with the legacy ones at the

payload level, while maintaining good performance at reception. This problem also exists for Galileo since:

- in the L1 band, the Open Service (OS) signal (two channels) and the Public Regulated Service (PRS) signal must be transmitted on the same carrier.
- in the E6 band, the Commercial Service (CS) signal must be transmitted with the PRS signal.

The Interplex modulation, a particular phase-shifted-keyed/phase modulation (PSK/PM), was chosen to transmit all these signals because it is a constant-envelope modulation, thereby allowing the use of saturated power amplifiers with limited signal distortion ([STF, 2002; Rajan and Irvine, 2005]).

The main objective of this paper is to study the Interplex modulation, as it is used for the GPS and Galileo signals. In a first part we will present the Interplex modulation, its general formulation and its application for the multiplexing of three signals. Then, we will be interested in the application of this modulation to the GPS L1 signals and the Galileo L1 signals. We will give the general expression of the GPS L1 Interplex signal and show the modification which must be made on the general expression of the Interplex modulation to apply it to the case of the combination of the C/A and P(Y) codes with the new M-code. With regards to the Galileo signals we will study two different cases. In the first case we will consider that the OS signal is a classical BOC(1,1). In the second case we will make the study, assuming that the OS signal is the new signal called Composite Binary Coded Symbol (CBCS), recently published ([Hein et al., 2005]). To conclude, the theoretical formula of the power spectrum densities of the GPS L1 Interplex signals and of the Galileo L1 Interplex signal are given.

## I. INTRODUCTION

The modernization of GPS and the development of the GALILEO system have led to the study of different modulation techniques in the L1 band in order to obtain the best performance at the reception level. Several techniques were proposed to solve this problem: (1) the Coherent Adaptive Subcarrier Modulation (CASM), which is mathematically equivalent to the Interplex modulation, presented in [Butman and Timor, 1972], was proposed by Dafesh et al (1999); (2) the Quadrature Product Subcarrier Modulation (QPSM) method which was developed for general quadrature-multiplexed communication systems [Dafesh, 1999]; and (3) the so-called majority vote logic technique explored by Spilker and Orr (1998).

The Interplex modulation was eventually preferred ([Wang et al., 2004; STF, 2002]) because it provides the best overall satellite power efficiencies by combining multiple signals into a phase modulated composite signal that keeps a constant envelope. Thanks to this modulation, the satellite's high-power amplifier may be operated into saturation with limited undesirable Amplitude-Modulation to Amplitude-Modulation (AM/AM) and Amplitude-Modulation to Phase-Modulation (AM/PM) distortions. However its main disadvantage is that it implies intermodulation (IM) terms in order to obtain a constant envelope, and thus wastes part of the transmitted power through this IM component.

For GNSS, this waste of useful power should be carefully analyzed because it is an element for the system optimization. The Interplex modulation, proposed for each "new signals", should be studied because the IM product could consume more or less power and therefore induce worse or better performance.

This paper proposes a review of the Interplex modulation. First a general formulation of this modulating technique will be presented. Then, we will show which modulation index may be chosen in the case of the GPS system and the Galileo system at the L1 band. For the Galileo system we will study two different cases, whether the OS signal is a classical BOC(1,1) or the OS signal is a CBCS (Composite Binary Coded Symbol). For both navigation systems, the phase diagram of the Interplex modulation will be presented. Finally we will give the expression of the power spectrum densities for the GPS L1 Interplex signal, and for the Galileo L1 Interplex signal.

## II. FORMULATION

As already mentioned, the Interplex modulation is a particular phase-shifted-keyed/phase modulation (PSK/PM), combining multiple signals into a phase modulated composite signal.

The general form of the Interplex phase-modulated signal, as presented in [Butman and Timor, 1972], is:

$$s(t) = \sqrt{2P}\cos\left(2\pi f_c \cdot t + \theta(t) + \varphi\right) \qquad (1)$$

where:

- P is the total average power
- fc is the carrier frequency
- $\theta(t)$ is the phase modulation
- $\varphi$ is a random phase

In the case of GNSS applications the phase modulation can be defined as:

$$\theta(t) = \beta_1 s_1(t) + \sum_{n=2}^{N} \beta_n \cdot s_1(t) \cdot s_n(t) \qquad (2)$$

with:

- $s_n(t) = c_n(t) \cdot d_n(t) \cdot sq(2\pi f_n t) = \pm 1$

where

sq(t) is a square-wave sub-carrier,
$d_n(t)$ is the materialization of the data message
$c_n(t)$ is the materialization of the spreading code

- N is the number of components, and
- $\beta_n$ is the modulation angle or modulation index which choice determines the power allocation for each signal component.

The most common case for future GNSS signal configuration is the transmission of three signals on the same carrier:

- one signal in the quadrature channel: $s_1$
- two signals in the in-phase channel: $s_2$ and $s_3$

The Interplex signal can then be expressed as:

$$s(t) = \sqrt{2P}\cos\left(\begin{array}{l} 2\pi f_c \cdot t - \dfrac{\pi}{2} \cdot s_1(t) + \beta_2 \cdot s_1(t) \cdot s_2(t) \\ + \beta_3 \cdot s_1(t) \cdot s_3(t) + \varphi \end{array}\right) \qquad (3)$$

Note that $\beta_1$ is taken equal to $-\pi/2$ because the signal $s_1$ is in quadrature with the two others signals.

Such a signal can be generated thanks to the following scheme, presented in Figure 1 [US Patent, 2002].

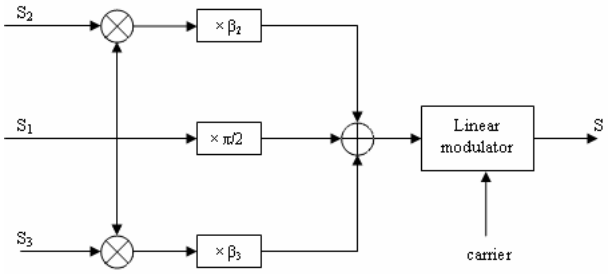



*Figure 1: Interplex generator scheme*

By developing Equation 3, it can be shown that:

$$s(t) = \sqrt{2P}\left[\begin{array}{l} \cos\left(2\pi f_c \cdot t - \dfrac{\pi}{2} \cdot s_1(t) + \varphi\right)\cos\left(\begin{array}{l}\beta_2 \cdot s_1(t) \cdot s_2(t) \\ + \beta_3 \cdot s_1(t) \cdot s_3(t)\end{array}\right) \\ -\sin\left(2\pi f_c \cdot t - \dfrac{\pi}{2} \cdot s_1(t) + \varphi\right)\sin\left(\begin{array}{l}\beta_2 \cdot s_1(t) \cdot s_2(t) \\ + \beta_3 \cdot s_1(t) \cdot s_3(t)\end{array}\right) \end{array}\right] \qquad (4)$$

$$s(t) = \sqrt{2P}\left[\begin{array}{l} s_1(t) \cdot \sin(2\pi f_c \cdot t + \varphi) \cdot \left(\begin{array}{l}\cos(\beta_2)\cos(\beta_3) \\ - s_2(t) \cdot s_3(t) \cdot \sin(\beta_2)\sin(\beta_3)\end{array}\right) \\ + s_1(t) \cdot \cos(2\pi f_c \cdot t + \varphi) \cdot \left(\begin{array}{l} s_2(t) \cdot s_1(t) \cdot \sin(\beta_2)\cos(\beta_3) \\ + s_1(t) \cdot s_3(t) \cdot \cos(\beta_2)\sin(\beta_3)\end{array}\right) \end{array}\right] \qquad (5)$$

and finally,

$$s(t) = \sqrt{2P}\left[\begin{array}{l} \left(\begin{array}{l} s_2(t) \cdot \sin(\beta_2)\cos(\beta_3) \\ + s_3(t) \cdot \cos(\beta_2)\sin(\beta_3)\end{array}\right) \cdot \cos(2\pi f_c \cdot t + \varphi) \\ + \left(\begin{array}{l} s_1(t) \cdot \cos(\beta_2)\cos(\beta_3) \\ - s_1(t) \cdot s_2(t) \cdot s_3(t) \cdot \sin(\beta_2)\sin(\beta_3)\end{array}\right) \cdot \sin(2\pi f_c \cdot t + \varphi) \end{array}\right] \qquad (6)$$

Thanks to Equation 6, it can be noticed that the first three terms correspond to the desired useful signal terms $s_1$, $s_2$, $s_3$; the fourth term is the undesired intermodulation term. This IM term is equal to the product of the three desired signals balanced by the modulation indexes $\beta_2$ and $\beta_3$. It consumes some of the total transmitted power that could be available for the three desired signals. Indeed, with the Interplex modulation, the power of each component is equal to:

$$\begin{aligned} P_1 &= P \cdot \cos^2(\beta_2) \cdot \cos^2(\beta_3) \\ P_2 &= P \cdot \sin^2(\beta_2) \cdot \cos^2(\beta_3) \\ P_3 &= P \cdot \cos^2(\beta_2) \cdot \sin^2(\beta_3) \\ P_{IM} &= P \cdot \sin^2(\beta_2) \cdot \sin^2(\beta_3) \end{aligned} \qquad (7)$$

Equation 7 shows that the power of each signal component only depends on two variables $\beta_2$ and $\beta_3$. Thus the expression of the equivalent baseband Interplex signal can be re-written as:

$$\hat{s}(t) = \sqrt{P_2}s_2(t) + \sqrt{P_3}s_3(t) + j\left[\begin{array}{l} \sqrt{P_1}s_1(t) \\ -\sqrt{\dfrac{P_2 \cdot P_3}{P_1}}s_1(t) \cdot s_2(t) \cdot s_3(t) \end{array}\right] \qquad (8)$$

where

$$s(t) = \mathrm{Re}\{\hat{s}(t) \cdot \exp(2j\pi f_c t + \varphi)\}$$

Figure 2 represents the power of the different signals as a function of the values of the modulation indexes. The first graph represents the power of each signal component in function of $\beta_2$ with $\beta_3=\pi/3$ and the second graph represents the power of each signal component in function of $\beta_3$ with $\beta_2=\pi/3$.

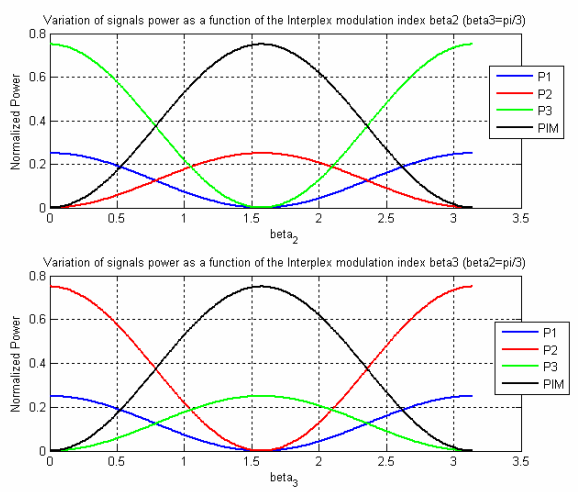
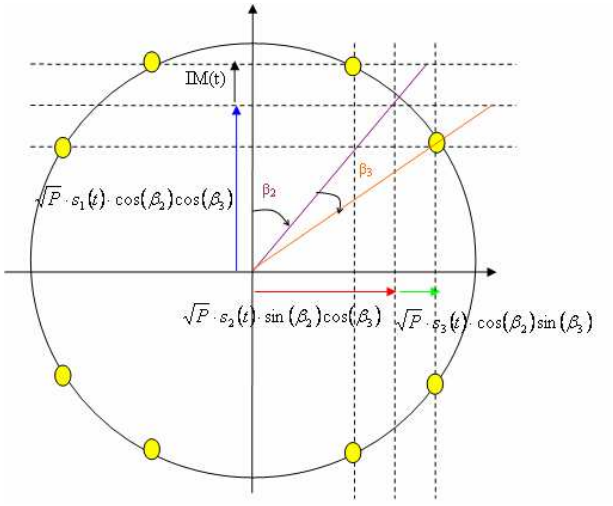



*Figure 2: Variation of signals powers as a function of Interplex modulation indexes*

The Figure 2 shows that the choice of $\beta_2$ and $\beta_3$ depends on the power that we want to give to each signal. A trade-off must be made to have sufficient power on the desired signals and non-disadvantageous power on the IM signal. An accurate study must be made to find the most suitable values for $\beta_2$ and $\beta_3$.

The different states of the Interplex signal can be represented on a phase diagram whose x-axis is the in-phase component and whose y-axis is the quadrature component. For the present case, the diagram of the modulation constellation is shown in Figure 3:



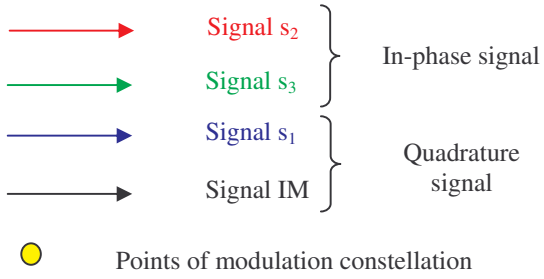




*Figure 3:Interplex modulation constellation*

Note that if $\varphi = \pi/2$, we have the same figure but with the complementary angles.

This diagram shows that even if the introduction of the IM product consumes some of the available power, the Interplex modulation keeps the magnitude of the composite signal envelope constant, which facilitates the use of saturated amplifier in the payload.

## III. APPLICATION TO THE NAVIGATION SYSTEM AT THE L1 BAND

### GPS Case

In the L1 band, modernized GPS satellites will transmit three signals:

- the C/A code signal component, $s_1$.
- the P code signal component, $s_2$.
- the new military signal M-code, $s_3$.

The signal $s_1$ is a C/A-code at 1.023 Mchips per second code chipping rate in Non-Return to Zero (NRZ) format. The signal $s_2$ is a P-code at 10.23 Mchips/s code chipping rate in NRZ format. The M-code signal $s_3$ is the product of a code formed with NRZ symbols running at 5*1.023 Mchips/s and a square-wave sub-carrier running at 10*1.023 MHz. It is a Binary Offset Carrier, a BOC(10,5).

In [Dafesh et al., 1999; Dafesh et al., 2000], the Coherent Adaptive Subcarrier Modulation (CASM) is proposed as a specific solution for transmitting the three GPS signals present in the L1 band. This modulation could be considered as a three components Interplex modulation with a particular and optimal choice of the modulation indexes. The CASM proposes to put the P-code signal and the M-code in the in-phase component and the C/A code in the quadrature component with the IM product. We will take this signals layout as reference but others cases have been studied, particularly in [Wang et al., 2004], where the P-code and the C/A code signals are in the in-phase component and the M-code in the quadrature component.

In the present case, $\beta_2$ is defined thanks to the two following equations, proposed in [Dafesh et al., 2000]:

$$\cos(\beta_2)=\sqrt{\frac{P_Q}{P}} \qquad \sin(\beta_2)=\sqrt{\frac{P_I}{P}} \tag{9}$$

where $P_I$ is the power of the initial signal in phase
$P_Q$ is the power of the initial signal in quadrature
P is the total average power.

The only modulation index, which is not set, is $\beta_3$. It is renamed m. Consequently, the power of each signal depends only on m and is equal to:

$$\begin{aligned} P_1 &= P_Q \cdot \cos^2(m) \\ P_2 &= P_I \cdot \cos^2(m) \\ P_3 &= P_Q \cdot \sin^2(m) \\ P_{IM} &= P_I \cdot \sin^2(m) \end{aligned} \tag{10}$$

Note that the total average power is maintained constant:

$$P = P_1 + P_2 + P_3 + P_{IM} = P_I + P_Q \tag{11}$$

The GPS signal transmitted with the CASM modulation can be written as:

$$s(t)=\sqrt{2P}\cos\left(\begin{array}{l} 2\pi f_c \cdot t+\frac{\pi}{2}\cdot s_1(t)-\beta_2 \cdot s_1(t)\cdot s_2(t) \\ +m\cdot s_1(t)\cdot s_3(t)+\varphi \end{array}\right) \tag{12}$$

$$s(t)=\sqrt{2P}\left[\begin{array}{l} \cos\left(2\pi f_c \cdot t+\frac{\pi}{2}\cdot s_1(t)+\varphi\right)\cos\left(\begin{array}{l}-\beta_2 \cdot s_1(t)\cdot s_2(t) \\ +m\cdot s_1(t)\cdot s_3(t)\end{array}\right) \\ -\sin\left(2\pi f_c \cdot t+\frac{\pi}{2}\cdot s_1(t)+\varphi\right)\sin\left(\begin{array}{l}-\beta_2 \cdot s_1(t)\cdot s_2(t) \\ +m\cdot s_1(t)\cdot s_3(t)\end{array}\right) \end{array}\right] \tag{13}$$

$$s(t)=\sqrt{2P}\left[\begin{array}{l} -s_1(t)\cdot\sin(2\pi f_c\cdot t+\varphi)\cdot\left(\begin{array}{l}\cos(-\beta_2)\cos(m) \\ -s_2(t)\cdot s_3(t)\cdot\sin(-\beta_2)\sin(m)\end{array}\right) \\ -s_1(t)\cdot\cos(2\pi f_c\cdot t+\varphi)\cdot\left(\begin{array}{l}s_2(t)\cdot s_1(t)\cdot\sin(-\beta_2)\cos(m) \\ +s_1(t)\cdot s_3(t)\cdot\cos(-\beta_2)\sin(m)\end{array}\right) \end{array}\right] \tag{14}$$

$$s(t)=\sqrt{2}\left[\begin{array}{l} \left(\begin{array}{l}\sqrt{P_I}\cdot s_2(t)\cdot\cos(m) \\ -\sqrt{P_Q}\cdot s_3(t)\cdot\sin(m)\end{array}\right)\cdot\cos(2\pi f_c\cdot t+\varphi) \\ -\left(\begin{array}{l}\sqrt{P_Q}\cdot s_1(t)\cdot\cos(m) \\ +\sqrt{P_I}\cdot s_1(t)\cdot s_2(t)\cdot s_3(t)\cdot\sin(m)\end{array}\right)\cdot\sin(2\pi f_c\cdot t+\varphi) \end{array}\right] \tag{15}$$

As already seen in the previous section, the intermodulation product is the product of the signals $s_1$, $s_2$ and $s_3$. In this case, the product of the three GPS signals is a BOC(10,10) sub-carrier, so the IM product is a BOC(10,10).

The modulation constellation of the GPS Interplex modulation is presented in Figure 4:

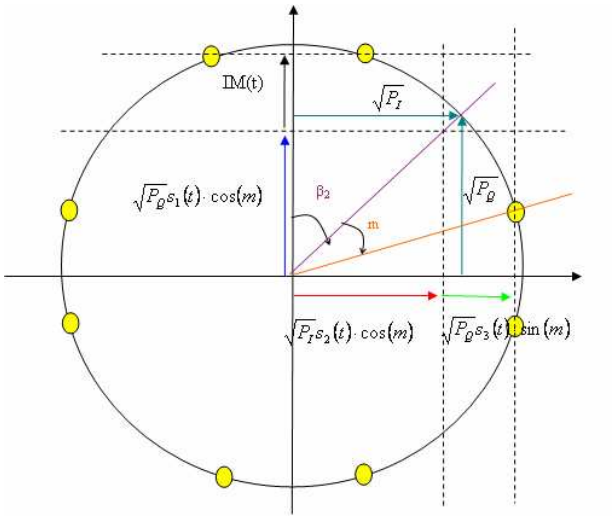



*Figure 4:GPS Interplex modulation constellation*

As previously the envelope of the signal is constant even if some of the available power is wasted in the IM product.

## GALILEO System

The modulation scheme used to transmit the L1 Galileo signal and proposed in [GJU, 2005] is similar to the modulation scheme proposed, previously, for the GPS case.

Let's assume that in the case of the Galileo system the signals that will be broadcast in the L1 band are:

- the PRS signal. It is a cosine-phased BOC(15,2.5) signal, $s_1$.
- the data OS signal. It is a BOC(1,1) signal $s_2$.
- the pilot OS signal. It is a BOC(1,1) signal $s_3$.

The only difference between the signals $s_2$ and $s_3$ is their codes, which don't have the same values even if they have the same code rate.

In the present case, as referred in [GJU, 2005], the total power should be equally divided into the in-phase component and the quadrature component. Moreover the power of the data OS component should be equal to the power of the pilot OS component. Consequently, the parameters $\beta_2$ and $\beta_3$ are set by the following relationships:

$$\begin{cases} P_1 = P \cdot \cos^2(\beta_2) \cdot \cos^2(\beta_3) = 2 \cdot P \cdot \sin^2(\beta_2) \cdot \cos^2(\beta_3) \\ P_2 = P_3 = P \cdot \sin^2(\beta_2) \cdot \cos^2(\beta_3) \end{cases} \quad (16)$$

This system leads to $\beta_2=-\beta_3=m=0.6155$ rad.

Consequently, the expression of the signal transmitted is:

$$s(t) = \sqrt{2P} \cos \begin{pmatrix} 2\pi f_c \cdot t - \frac{\pi}{2} \cdot s_1(t) + m \cdot s_1(t) \cdot s_2(t) \\ -m \cdot s_1(t) \cdot s_3(t) + \varphi \end{pmatrix} \quad (17)$$

$$s(t) = \sqrt{2P} \left[ \begin{pmatrix} s_2(t) \cdot \sin(m)\cos(m) \\ -s_3(t) \cdot \cos(m)\sin(m) \end{pmatrix} \cdot \cos(2\pi f_c \cdot t + \varphi) + \begin{pmatrix} s_1(t) \cdot \cos^2(m) \\ +s_1(t) \cdot s_2(t) \cdot s_3(t) \cdot \sin^2(m) \end{pmatrix} \cdot \sin(2\pi f_c \cdot t + \varphi) \right] \quad (18)$$

In this case, the power of each component is equal to:

$$\begin{aligned} P_1 &= P \cdot \cos^4(m) \\ P_2 &= P_3 = P \cdot \cos^2(m) \cdot \sin^2(m) \\ P_{IM} &= P \cdot \sin^4(m) \end{aligned} \quad (19)$$

The diagram of the modulation constellation is shown in Figure 5:

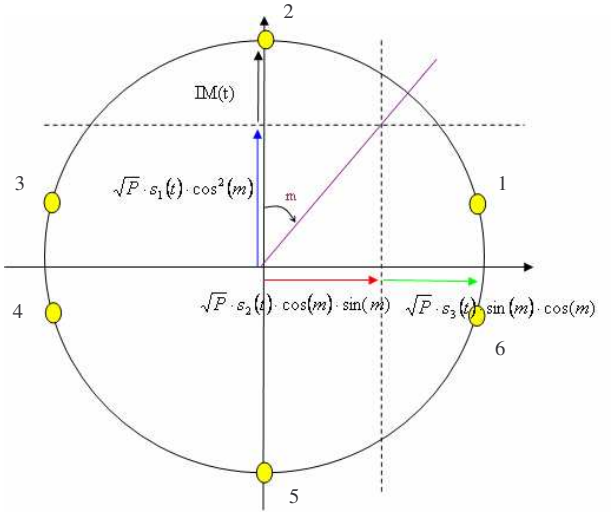



*Figure 5: Galileo Interplex modulation constellation*

The Figure 5 shows that the modulation constellation is only composed of 6 plots. This is due to the fact that the signals $s_2$ and $s_3$ are both BOC(1,1) sub-carrier and by the way the constellation goes through the points 2 and 5 twice.

Currently, other studies are made to transmit a Galileo L1 signal which performance is better than the one obtained with a BOC(1,1). [Hein et al., 2005] proposes to transmit a linear combination of a BOC(1,1) sub-carrier and a Binary Coded Signal (BCS) sub-carrier instead of a classical BOC(1,1) sub-carrier. Following these assumptions, the signals transmitted on Galileo L1 would be:

- A data OS signal that can be represented as: $\cos(\theta_1) \cdot BOC(1,1) + \cos(\theta_2) \cdot BCS(n,1)$
- A pilot OS signal that can be represented as: $\cos(\theta_1) \cdot BOC(1,1) - \cos(\theta_2) \cdot BCS(n,1)$
- the PRS signal already described earlier.

To transmit these signals with the Interplex modulation we must, in fact, consider separately the BOC(1,1) and the BCS. So the signals transmitted with the Interplex modulation are:

- $s_1$, the BOC(15,2.5) (including the PRS PRN code).
- $s_2$, the BOC(1,1) sub-carrier only.
- $s_3$, the BCS sub-carrier only.

The expression of the Interplex signal transmitted is then:

$$s(t) = \sqrt{2P} \cos \begin{pmatrix} 2\pi f_c \cdot t - \frac{\pi}{2} \cdot s_1(t) \\ + \left( \frac{\pi}{4} - \frac{\theta_1}{2} \right) \cdot s_1(t) \cdot s_2(t) \cdot (c_A(t) + c_B(t)) \\ + \left( \frac{\pi}{4} - \frac{\theta_2}{2} \right) \cdot s_1(t) \cdot s_3(t) \cdot (c_A(t) - c_B(t)) + \varphi \end{pmatrix} \quad (20)$$

where $C_A$ is the OS data channel code (spreading code and data) and $C_B$ is the OS pilot channel code (spreading code only).

Equation 20 shows that the values of the modulation indexes are expressed as a function of the angles $\theta_1$ and $\theta_2$, which depend on the percentage of power that is put on the BCS component.

It can be noticed that the model used for the optimized Galileo L1 signal is in fact an Interplex modulation model with 5 signal components and not only three as the previous ones.

Developing the equation (20) gives:

$$s(t) = \sqrt{2P} \left( \begin{bmatrix} \frac{(c_A(t) + c_B(t))}{2} \cdot \cos(\theta_1) \cdot s_2(t) \cdot \\ + \frac{(c_A(t) - c_B(t))}{2} \cdot \cos(\theta_2) \cdot s_3(t) \end{bmatrix} \cdot \cos(2\pi f_c t + \varphi) + \begin{bmatrix} s_1(t) \cdot \frac{\sin(\theta_1) + \sin(\theta_2)}{2} \\ - \frac{s_1(t) \cdot c_A(t) \cdot c_B(t)}{2} (\sin(\theta_2) - \sin(\theta_1)) \end{bmatrix} \cdot \sin(2\pi f_c t + \varphi) \right) \quad (21)$$

This expression is the expression of the optimized signal which is proposed to transmit the OS signal and the PRS signal in the L1 band and presented in [Hein et al., 2005].

In this case the IM product signal shape depends only on the PRS signal, $C_A$ and $C_B$. It does not depend on the BOC(1,1) sub-carrier or on the BCS sub-carrier.

As confirmed in [Hein et al., 2005], the power of each component is equal to:

$$\begin{aligned} P_{OS} &= P_2 + P_3 = P \cdot \frac{\cos^2(\theta_1) + \cos^2(\theta_2)}{2} \\ P_{PRS} &= P_1 = P \cdot \left( \frac{\sin(\theta_1) + \sin(\theta_2)}{2} \right)^2 \\ P_{IM} &= P \cdot \left( \frac{\sin(\theta_2) - \sin(\theta_1)}{2} \right)^2 \end{aligned} \qquad (22)$$

We can also notice that:

$$P_{BOC(1,1)} = P \cdot \frac{\cos^2(\theta_1)}{2} \quad \text{and} \quad P_{BCS} = P \cdot \frac{\cos^2(\theta_2)}{2} \qquad (23)$$

Consequently the values of $\theta_1$ and $\theta_2$ allow to set the percentage of desired BCS or BOC(1,1) powers.

The phase diagram of the modulation constellation is similar, as the other examples, to the phase diagram of a 8-PSK modulation:

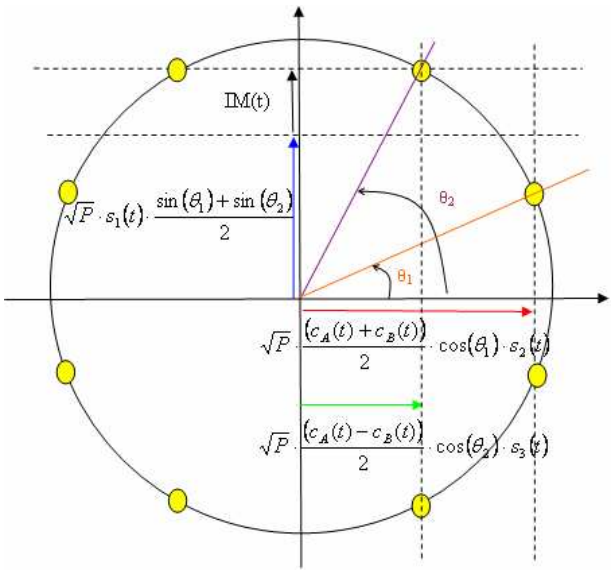



*Figure 6: Optimized Galileo Interplex modulation constellation*

If we compare the power of the IM product of the BOC(1,1) signal and of the CBCS signal:

$$P_{IM\_CBCS} = P \cdot \left( \frac{\sin(\theta_2) - \sin(\theta_1)}{2} \right)^2 \quad , P_{IM\_BOC(1,1)} = P \cdot \sin^4(m)$$

we notice that for the case of the BOC(1,1) signal the power depends on only one modulation index, whereas for the case of the CBCS signal the IM power depends on two modulation indexes. So the setting of the IM power seems to be easier with the optimized signal. The comparison of both IM products will be more precisely analyzed in the next part with the study of their power spectrum densities.

## IV. POWER SPECTRUM DENSITIES

In order to study the impact of the IM signal on the spectrum of the L1 GPS and Galileo signals, we will give, in this part, the theoretical expression of the power spectrum densities of the Interplex signals presented previously.

### GPS Case

The GPS signal in the L1 band could be written as:

$$s(t) = \mathrm{Re}\{\hat{s}(t) \cdot \exp(2j\pi f_c t + \varphi)\}$$

where

$$\hat{s}(t) = \sqrt{P_2} s_2(t) - \sqrt{P_3} s_3(t) - j \left[ \begin{array}{l} \sqrt{P_1} s_1(t) \\ + \sqrt{\frac{P_2 \cdot P_3}{P_1}} s_1(t) \cdot s_2(t) \cdot s_3(t) \end{array} \right] \qquad (24)$$

$$\begin{aligned} \hat{s}(t) = \sqrt{P_2} s_2(t) - \sqrt{P_3} s_3(t) - j \cdot \sqrt{P_1} s_1(t) \\ - j \cdot \sqrt{P_{IM}} \cdot IM(t) \end{aligned} \qquad (25)$$

So the autocorrelation function of such a signal is:

$$\Re_s(\tau) = \Re_{\hat{s}}(\tau) \cdot \frac{1}{2} \cos(2\pi f_c t) \qquad (26)$$

with

$$\Re_{\hat{s}}(\tau) = E\left[\hat{s}(t) \cdot \hat{s}(t-\tau)\right]$$

$$\Re_s(\tau) = E\left[ \begin{array}{l} \left( \begin{array}{l} \sqrt{P_2} s_2(t) - \sqrt{P_3} s_3(t) - j \cdot \sqrt{P_1} s_1(t) - \\ j \cdot \sqrt{P_{IM}} s_1(t) \cdot s_2(t) \cdot s_3(t) \end{array} \right) \cdot \\ \left( \begin{array}{l} \sqrt{P_2} s_2(t-\tau) - \sqrt{P_3} s_3(t-\tau) - j \cdot \sqrt{P_1} s_1(t-\tau) \\ - j \cdot \sqrt{P_{IM}} s_1(t-\tau) \cdot s_2(t-\tau) \cdot s_3(t-\tau) \end{array} \right) \end{array} \right] \qquad (27)$$

The different codes which compose the signals $s_1$, $s_2$ and $s_3$ have a very low cross-correlation, so the cross-correlation between the different codes is herein assumed to be equal to zero. Consequently,

$$\Re_{\hat{s}}(\tau) = P_2 \cdot \Re_{S_2}(\tau) + P_3 \cdot \Re_{S_3}(\tau) + P_1 \cdot \Re_{S_1}(\tau) + P_{IM} \cdot \Re_{IM}(\tau) \quad (28)$$

The power spectrum density of the GPS Interplex signal is the Fourier Transform of the autocorrelation function:

$$S_s(f) = \frac{1}{4} S_{\hat{s}}(f - f_c) + \frac{1}{4} S_{\hat{s}}(f + f_c) \quad (29)$$

with

$$S_{\hat{s}}(f) = TF[\Re_{\hat{s}}(\tau)] = TF\begin{bmatrix} P_2 \cdot \Re_{S_2}(\tau) + P_3 \cdot \Re_{S_3}(\tau) \\ + P_1 \cdot \Re_{S_1}(\tau) + P_{IM} \cdot \Re_{IM}(\tau) \end{bmatrix} \quad (30)$$

$$S_{\hat{s}}(f) = P_2 \cdot S_{S_2}(f) + P_3 \cdot S_{S_3}(f) + P_1 \cdot S_{S_1}(f) + P_{IM} \cdot S_{IM}(f) \quad (31)$$

If n=2p/q is even and if the sub-carrier is sine-phased, the power spectrum density of a BOC(p,q) signal, is equal to ([Betz, 2001]):

$$S_{BOC}(f) = \frac{1}{T_c}\left( \frac{\sin(\frac{\pi f T_c}{n}) \sin(\pi f T_c)}{\pi f \cos(\frac{\pi f T_c}{n})} \right)^2 \quad (32)$$

with $T_c$ the code period.

So the power spectrum density of the BOC(10,5) M-code signal is :

$$S_{S_3}(f) = \frac{5}{T_c}\left( \frac{\sin(\frac{\pi f T_c}{20}) \sin(\pi f \frac{T_c}{5})}{\pi f \cos(\frac{\pi f T_c}{20})} \right)^2 \quad (33)$$

with $T_c$=1/1.023e6 s.

And in the case of the BOC(10,10) IM signal:

$$S_{IM}(f) = \frac{10}{T_c}\left( \frac{\sin(\frac{\pi f T_c}{20}) \sin(\pi f \frac{T_c}{10})}{\pi f \cos(\frac{\pi f T_c}{20})} \right)^2 \quad (34)$$

The power spectrum density of a NRZ modulation is:

$$S(f) = T_c\left( \frac{\sin(\pi f T_c)}{\pi f T_c} \right)^2 \quad (35)$$

with $T_c$ the code period.

Consequently, the power spectrum density of the C/A code signal component is:

$$S_{s_1}(f) = T_c\left( \frac{\sin(\pi f T_c)}{\pi f T_c} \right)^2 \quad (36)$$

and the power spectrum density of the P-code signal component is :

$$S_{s_2}(f) = \frac{T_c}{10}\left( \frac{\sin(\pi f \frac{T_c}{10})}{\pi f \frac{T_c}{10}} \right)^2 \quad (37)$$

Finally,

$$S_{\hat{S}}(f) = \frac{1}{\pi^2 f^2 T_c}\left\{ \tan^2\left(\frac{\pi f T_c}{20}\right) \cdot \begin{bmatrix} 5 \cdot P_3 \cdot \sin^2(\pi f \frac{T_c}{5}) \\ + 10 \cdot P_{IM} \cdot \sin^2(\pi f \frac{T_c}{10}) \end{bmatrix} + P_1 \cdot \sin^2(\pi f T_c) + 10 \cdot P_2 \cdot \sin^2(\pi f \frac{T_c}{10}) \right\} \quad (38)$$

with $T_c$=1/1.023e6 s.

The next graph represents the curve obtained with simulations and the curve obtained thanks to the theoretical expression of the power spectrum density. We consider that $P_1$=0.5 dB, $P_2$=0 dB and $P_3$=-3 dB [Fan et al., 2005] and the curve which correspond to the simulation case is normalized by its sample number.

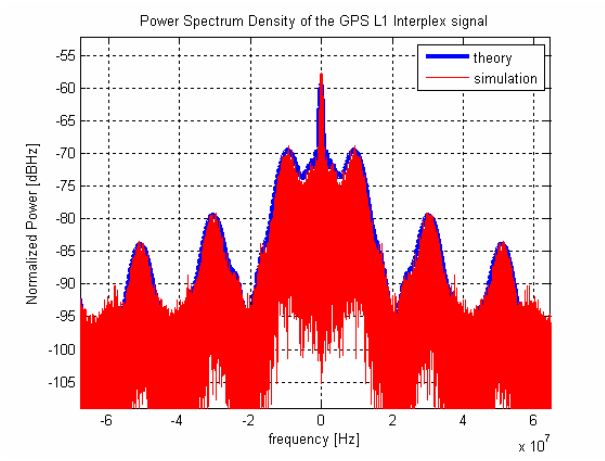



*Figure 7: Power Spectrum Density of the L1 GPS Signal*

## GALILEO System

If the case of a BOC(1,1) signal is considered, the Galileo L1 signal could be written :

$$s(t) = \mathrm{Re}\{\hat{s}(t) \cdot \exp(2j\pi f_c t + \varphi)\}$$

where

$$\hat{s}(t) = \sqrt{P_2}\, s_2(t) - \sqrt{P_3}\, s_3(t) + j \cdot \sqrt{P_1}\, s_1(t) + j \cdot \sqrt{P_{IM}} \cdot IM(t) \quad (39)$$

The calculation of the power spectrum densities of this signal is similar to the calculation of the power spectrum densities of the GPS signal, so:

$$S_s(f) = \frac{1}{4} S_{\hat{s}}(f - f_c) + \frac{1}{4} S_{\hat{s}}(f + f_c) \quad (40)$$

with

$$S_{\hat{s}}(f) = P_2 \cdot S_{S_2}(f) + P_3 \cdot S_{S_3}(f) + P_1 \cdot S_{S_1}(f) + P_{IM} \cdot S_{IM}(f) \quad (41)$$

The power spectrum density of the signal $s_2$ and the signal $s_3$ are equal. Their expression is:

$$S_{S_2}(f) = S_{S_3}(f) = \frac{1}{T_c} \left( \frac{\sin(\frac{\pi f T_c}{2}) \sin(\pi f T_c)}{\pi f \cos(\frac{\pi f T_c}{2})} \right)^2 \quad (42)$$

The signal $s_1$ is a cosine-phased BOC(15,2.5), so the power spectrum density of this signal is:

$$S_{S_1}(f) = \frac{2.5}{T_c} \cdot \left( \frac{\sin\left(\pi f \frac{T_c}{2.5}\right)}{\pi f \cos\left(\pi f \frac{T_c}{30}\right)} \left\{ \cos\left(\pi f \frac{T_c}{30}\right) - 1 \right\} \right)^2 \quad (43)$$

The IM term is the product of the signals $s_1$, $s_2$ and $s_3$, so it is a BOC(15,2.5) as the PRS signal. Its power spectrum density is therefore similar to the equation (43).

So, the power spectrum density of the Galileo L1 signal is:

$$S_{\hat{s}}(f) = 2P \cdot \cos^2(m) \cdot \sin^2(m) \cdot S_{S_2}(f) + P \cdot \begin{pmatrix} \cos^4(m) \\ +\sin^4(m) \end{pmatrix} \cdot S_{S_1}(f) \quad (44)$$

Now the case of the Galileo L1 optimized signal is considered, the expression of the signal transmitted in the Galileo L1 band is:

$$s(t) = \mathrm{Re}\{\hat{s}(t) \cdot \exp(2j\pi f_c t + \varphi)\}$$

with

$$\hat{s}(t) = \sqrt{P} \left( \begin{bmatrix} \frac{c_A(t)}{2} \cdot (\cos(\theta_1) \cdot s_2(t) + \cos(\theta_2) \cdot s_3(t)) \cdot \\ + \frac{c_B(t)}{2} \cdot (\cos(\theta_1) \cdot s_2(t) - \cos(\theta_2) \cdot s_3(t)) \end{bmatrix} + j \cdot \begin{bmatrix} s_1(t) \cdot \frac{\sin(\theta_1) + \sin(\theta_2)}{2} \\ - \frac{IM(t)}{2} (\sin(\theta_2) - \sin(\theta_1)) \end{bmatrix} \right) \quad (45)$$

The signal ŝ could also be written:

$$\hat{s}(t) = \sqrt{P} \left( \left[ \frac{S_{OSA}(t)}{2} + \frac{S_{OSB}(t)}{2} \right] + j \cdot \begin{bmatrix} s_1(t) \cdot \frac{\sin(\theta_1) + \sin(\theta_2)}{2} \\ - \frac{IM(t)}{2} (\sin(\theta_2) - \sin(\theta_1)) \end{bmatrix} \right) \quad (46)$$

where $S_{OSA}$ and $S_{OSB}$ represent the data Open Service signal and the pilot Open Service signal, including respectively the code A and the code B and the weighted factor depending on $\theta_1$ and $\theta_2$.

As previously, we have:

$$\Re_s(\tau) = \Re_{\hat{s}}(\tau) \cdot \frac{1}{2} \cos(2\pi f_c t)$$

with

$$\Re_{\hat{s}}(\tau) = E[\hat{s}(t) \cdot \hat{s}(t - \tau)]$$

The crosscorrelation between the different codes is again assumed to be equal to zero. Consequently,

$$\begin{aligned}\Re_{\hat{s}}(\tau) &= \frac{P}{4}\cdot\Re_{S_{OSA}}(\tau)+\frac{P}{4}\cdot\Re_{S_{OSB}}(\tau)\\ &+\frac{P}{4}\cdot(\sin(\theta_1)+\sin(\theta_2))^2\cdot\Re_{S_1}(\tau)\\ &+\frac{P}{4}\cdot(\sin(\theta_2)-\sin(\theta_1))^2\cdot\Re_{IM}(\tau)\end{aligned} \quad (47)$$

The power spectrum densities of the optimized Galileo signal is the Fourier Transform of the autocorrelation function:

$$S_s(f)=\frac{1}{4}S_{\hat{s}}(f-f_c)+\frac{1}{4}S_{\hat{s}}(f+f_c)$$

with

$$S_{\hat{s}}(f)=TF[\Re_{\hat{s}}(\tau)]$$

$$S_{\hat{s}}(f)=\frac{P}{4}\cdot\begin{bmatrix}S_{S_{OSA}}(f)+S_{S_{OSB}}(f)+\begin{pmatrix}\sin(\theta_1)\\+\sin(\theta_2)\end{pmatrix}^2\cdot S_{S_1}(f)\\+(\sin(\theta_2)-\sin(\theta_1))^2\cdot S_{IM}(f)\end{bmatrix} \quad (48)$$

The calculation of the power spectrum densities of the OSA and OSB signals are presented in [Hein et al., 2005].

So we have:

$$\begin{aligned}S_{OSA}(f) &= \cos^2(\theta_1)\cdot S_{BOC(1,1)}(f)+\cos^2(\theta_2)\cdot S_{BCS}(f)\\ &+\frac{2}{T_C}\cdot\cos(\theta_1)\cdot\cos(\theta_2)\cdot\mathrm{Re}\{FT(p_{BOC(1,1)})\cdot FT^*(p_{BCS})\}\end{aligned} \quad (49)$$

$$\begin{aligned}S_{OSB}(f) &= \cos^2(\theta_1)\cdot S_{BOC(1,1)}(f)+\cos^2(\theta_2)\cdot S_{BCS}(f)\\ &-\frac{2}{T_C}\cdot\cos(\theta_1)\cdot\cos(\theta_2)\cdot\mathrm{Re}\{FT(p_{BOC(1,1)})\cdot FT^*(p_{BCS})\}\end{aligned} \quad (50)$$

with $S_{BOC(1,1)}$ and $S_{BCS}$ the power spectrum densities of the BOC(1,1) and the BCS modulations.

The signals $s_1$ and the IM product are both cosine-phased BOC(15,2.5) modulation. So, the power spectrum density of the signal ŝ is:

$$S_{\hat{s}}(f)=\frac{P}{2}\cdot\begin{bmatrix}\cos^2(\theta_1)\cdot S_{BOC(1,1)}(f)+\cos^2(\theta_2)\cdot S_{BCS}(f)\\+(\sin^2(\theta_1)+\sin^2(\theta_2))\cdot S_{BOC(15,2.5)}(f)\end{bmatrix} \quad (51)$$

with

$$S_{BOC(15,2.5)}(f)=\frac{2.5}{T_c}\cdot\left(\frac{\sin\left(\pi f\frac{T_c}{2.5}\right)}{\pi f\cos\left(\pi f\frac{T_c}{30}\right)}\left\{\cos\left(\pi f\frac{T_c}{30}\right)-1\right\}\right)^2 \quad (52)$$

$$S_{BOC(1,1)}(f)=\frac{1}{T_c}\left(\frac{\sin(\frac{\pi fT_c}{2})\sin(\pi fT_c)}{\pi f\cos(\frac{\pi fT_c}{2})}\right)^2 \quad (53)$$

and

$$S_{BCS}(f)=f_c\frac{\sin^2\left(\frac{\pi f}{nf_c}\right)}{(\pi f)^2}\sum_{i=1}^{n}\sum_{j=i}^{n}2s_is_j\cos\left[(j-i)\frac{2\pi f}{nf_c}\right] \quad (54)$$

where n refers to the number of symbols in one chip. The power spectrum density of a BCS([$s_1$ … $s_n$],$f_c$) was presented in [Hein et al., 2005].

The Galileo optimized signal the BCS, investigated by [Hein et al., 2005], is the BCS([1 -1 1 -1 1 -1 1 -1 1 1],1). It is proposed that the percentage of BCS power represents 20% of the total OS power. This condition involves $\theta_1$=0.51 rad and $\theta_2$=1.12 rad, considering that the OS power should be equal to the PRS power.

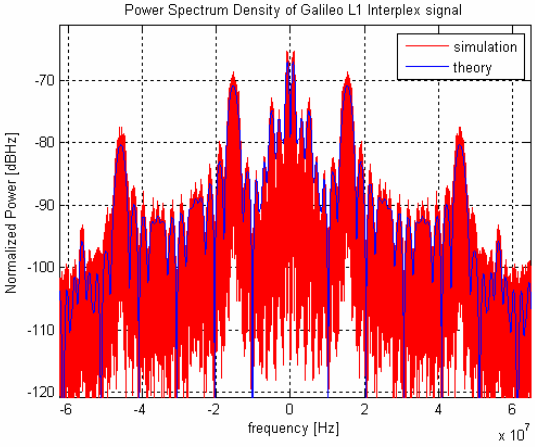



*Figure 8: Power Spectrum Density of the optimized L1 Galileo Signal*

Now that the power spectrum densities of the IM product for the two Galileo signal cases were calculated, we can notice that in both cases the IM product is a BOC(15,2.5) sub-carrier, only the power of the two IM product is different. As already mentioned, we have:

$$P_{IM\_CBCS}=P\cdot\left(\frac{\sin(\theta_2)-\sin(\theta_1)}{2}\right)^2,\ P_{IM\_BOC(1,1)}=P\cdot\sin^4(m)$$

Considering the optimal value for the modulation indexes in each case and that the total power is equal to 1, we have:

- for the BOC(1,1) case, [STF, 2002] proposes m=0.6155 so $P_{IM}$ = -9.54 dB.
- For the CBCS case, [Hein et al., 2005] proposes 20% of BCS, so $P_{IM}$ = -13.72 dB.

Therefore the power wasted in the IM product is more important in the case of the classical BOC(1,1) signal.

To conclude, in the next graph the GPS signal and the optimized Galileo signal power spectrum densities envelopes have been plotted:

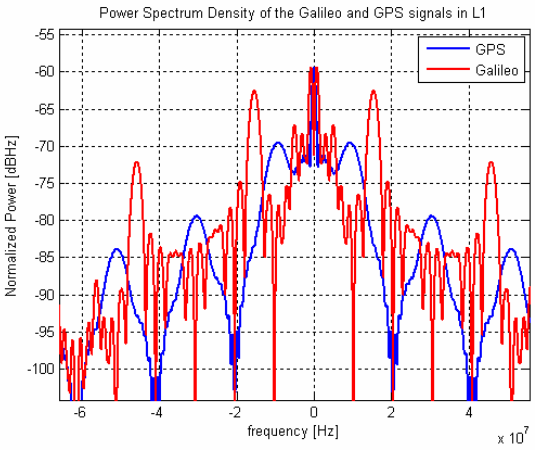



*Figure 9:Power Spectrum Densities of the L1 band signals*

## V. CONCLUSION

This paper has provided two main points. First it has been shown that the expression of the combined L1 GPS signal and the expression of the combined L1 Galileo signal could be linked to a unique formula considered as the definition of the Interplex modulation. It was also shown that the optimized Galileo L1 signal could be linked to this formula. For the different cases the phase diagram of the modulation was presented.

Secondly the expression of the power spectrum densities of all the navigation L1 signals have been theoretically calculated and a particular attention is made on the intermodulation term. Indeed the Interplex modulation guarantees a constant envelope for the navigation signals by creating an intermodulation term which is useless for the navigation but must be taken into account for a signal power optimization. Besides it has been shown that the IM product power of the optimized Galileo signal is weaker than the IM product power of the classical Galileo signal.

## ACKNOWLEDGMENT

The authors are thankful to Olivier JULIEN for his valuable remarks and suggestions.

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