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BIOGRAPHY

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Christophe Macabiau graduated as an electronics engineer in 1992 from the ENAC (Ecole Nationale de l'Aviation Civile) in Toulouse, France. Since 1994, he has been working on the application of satellite navigation techniques to civil aviation. He received his Ph.D. in 1997 and has been in charge of the signal processing lab of the ENAC since 2000.

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ABSTRACT

The European satellite navigation system GALILEO will provide radionavigation signals for a variety of applications. Safety Of Life users will get a safe navigation service through ranging signals carrying integrity information.

The Galileo Integrity Baseline algorithm includes the transmission of three parameters allowing users to monitor their integrity level. These parameters are the Signal-In-Space Accuracy (SISA: prediction of the minimum standard deviation of a Gaussian distribution overbounding the Signal-In-Space error in the fault-free case), the Signal-In-Space Monitoring Accuracy (SISMA: minimum standard deviation of a Gaussian distribution overbounding the difference between Signal-In-Space error and its estimation by ground control stations) and the Integrity Flag, which accounts for satellite status (it can be set to “OK”, “DON’T USE” or “NOT MONITORED”). These parameters are part of the input of the user integrity algorithm, which computes user integrity risk at the alert limit and compares it to the Integrity Risk requirement corresponding to user’s phase of flight.

The work presented in this paper studies the influence of the algorithm used for computation of SISMA on user integrity and system availability. The algorithms used to compute SISMA are the reference Least-Squares and several robust methods, designed to reject wrong measurements and decrease ground system False Alarm rate (fault-free satellites flagged “DON’T USE”).

I INTRODUCTION

This article presents results on the influence of the computation of SISMA over user integrity. Indeed, the GALILEO integrity concept consists in providing users with information concerning the system contribution to the final user position error, in order to allow them to autonomously check their integrity level. That information is in fact a quantification of the quality of the SIS (Signal In Space). The SIS is the signal emitted by a satellite in the constellation as received by a fault-free receiver. In the present Integrity Check algorithm, propagation errors are not considered, only the contribution of the satellite payload will be included in the broadcast information.

The error induced by the ground and space segments on the determination of the user’s position and clock bias is assumed to be essentially due to the difference between the satellite true position and clock bias and the values provided by the OD&TS (Orbit Determination and Time Synchronisation) through the broadcast ephemeris data. The projection of these position and clock errors on the satellite user axis is called the SISE (Signal In Space Error): it is in fact the equivalent measurement error due to OD&TS errors. Thus, the integrity information to be provided by GALILEO to the users only concerns OD&TS errors: propagation errors or local errors such as multipath or jamming are not taken into account in the broadcast integrity parameters.

The integrity information broadcast in the navigation message is composed of three parameters:

Signal-In-Space accuracy (SISA): predicted smallest standard deviation of a Gaussian distribution overbounding the SISE distribution in fault-free mode for any user in the service area.

Signal-In-Space Monitoring Accuracy (SISMA): smallest standard deviation of a Gaussian distribution overbounding the difference between SISE and $SISE_{est}$, its estimation by the ground segment (GALILEO satellites are monitored by a network of 40 GALILEO Sensor Stations (GSS), whose pseudorange measurements are used to compute $SISE_{est}$).

Integrity Flag (IF): depending on the results of the monitoring system, this flag can be set to one of the three following values: "USE", "DON'T USE" or "NOT MONITORED"

These parameters are then used in the User Integrity algorithm, which computes directly computes user's current Integrity Risk (as a function of the phase of flight). System availability is obtained by comparing the output to the Integrity Risk (IR) requirement of the current phase of flight.

However, the use, in the process of computing SISMA and $SISE_{est}$, of a least-squares algorithm implies that all input residuals are considered fault-free and Gaussian. In the case of a transient propagation problem or of an undetected GSS malfunction, biased measurements may be included in a fault-free satellite's set of pseudorange measurements, thus leading to the overestimation of $SISE_{est}$ and possibly causing a fault-free satellite to be flagged "DON'T USE".

Therefore a previous study was performed (presented in [6]), in order to assess the possibility of replacing the LS algorithm by a robust one.

Thus, the present paper studies the possibility of replacing the Least-Squares by a robust estimation algorithm in the Integrity check performed by Galileo ground segment in the baseline. The term "robust" refers here to the ability of such algorithms to estimate correct statistical parameters in the presence of some corrupted samples: while a single biased sample (called an outlier, a measurement not belonging to the same distribution as the other samples from the set) may lead a non-robust algorithm (e.g. Least-Squares) to provide an estimate far away from the underlying distribution of the sample space, a robust estimator (M-estimator, Least-Trimmed-Squares…) will resist such a small change and be able to provide an estimate close to the true distribution. Typically, one biased measurement in a set of otherwise Gaussian distributed ones will be either discarded (in the case of the LTS, for instance) or weighted down so as not to influence the estimation process (e.g. in the case of the M-estimator). The cost of this ability to detect and discard outliers is twofold: first, the loss of the optimality in the case of a truly Gaussian distributed sample set, and second, an additional computational cost (most robust algorithms are based on iterative residual computation ranking).

The performance analysis of the robust algorithm on the computation of the SISMA is performed in this paper with simulated data (with and without satellite or station failure), taking into account the steps of the pre-processing that impact most on the shape of the input signal (carrier-phase smoothing, ionospheric and tropospheric corrections).

Thus the paper will be structured as follows: the first section describes the input signal simulator, the second section presents the Integrity Check algorithm. Section 3 gives the definition of overbounding used throughout the article. Then sections 4 to 8 introduce the notion of statistical robustness, the methods that were used, how they were optimized and the gains that could be expected from them. Section 9 presents the results obtained through simulation and finally conclusions are derived on the feasibility of introducing a robust algorithm in the Integrity Check.

II INTEGRITY CHECK ALGORITM

At a given instant, each satellite i is seen by N GSS, which all perform pseudorange measurements based on satellite i signal. Let $(x_s, y_s, z_s, \Delta t_s)$ be the satellite true position in the WGS-84 reference frame and clock bias: these 4 values are the unknowns of the problem. Let (x_r^n, y_r^n, z_r^n) be the coordinates of the GSS of index n. The GSS coordinates are precisely known. The GSS clock biases will be determined through common view techniques, using a specific GSS as a reference station connected to the Galileo Time. Therefore, the relation between the residuals for GSS n and the coordinates of satellite i can be expressed as follows:

$$r = H \cdot \Delta X + E_i$$

where H is a (3xN) matrix, defined as follows

$$H = \begin{pmatrix} \frac{\hat{x}_s - x_r^1}{\rho^1} & \frac{\hat{y}_s - y_r^1}{\rho^1} & \frac{\hat{z}_s - z_r^1}{\rho^1} \\ \vdots & \vdots & \vdots \\ \frac{\hat{x}_s - x_r^N}{\rho^N} & \frac{\hat{y}_s - y_r^N}{\rho^N} & \frac{\hat{z}_s - z_r^N}{\rho^N} \end{pmatrix},$$

$r = Y - h(\hat{X})$ is the residual vector, ΔX is the satellite ephemeris error. Indeed, in the present case, a 3-parameter model is used rather than the usual 4-parameter one. This can be justified by projecting the measurement equation in the local referential of the GSS (north east down), in which the down component of the observation matrix is almost equal to 1. Thus, it is the sum of the down and clock parameters that are estimated. This causes no problem since ΔX is estimated only to be projected on the Worst User Location axis (see [6]).

Let $\Delta\hat{X}$ be the estimation of ΔX and $\operatorname{cov}(\Delta\hat{X})$ its covariance matrix. To obtain an estimation of SISE, the current method consists in forming a user grid on the surface of the Earth. $\Delta\hat{X}$ and $\operatorname{cov}(\Delta\hat{X})$ are projected on each satellite-user axis. Let $X_0 = (x_0, y_0, z_0)$ be the OD&TS coordinates of satellite i and $X_g = (x_g, y_g, z_g)$ the coordinates of a user on the grid.

The relation between the pseudorange measurement performed by the ground user, user position and satellite position is the same as the relation between the pseudorange measurement performed by the GSS, GSS position and satellite position. Therefore, the projection vector h_u for the $(X_0 X_g)$ axis is obtained by replacing the coordinates of the GSS by the user's in a line vector of the observation matrix H:

$$h_u = \begin{pmatrix} \dfrac{x_0 - x_g}{\sqrt{(x_0 - x_g)^2 + (y_0 - y_g)^2 + (z_0 - z_g)^2}} \\ \dfrac{y_0 - y_g}{\sqrt{(x_0 - x_g)^2 + (y_0 - y_g)^2 + (z_0 - z_g)^2}} \\ \dfrac{z_0 - z_g}{\sqrt{(x_0 - x_g)^2 + (y_0 - y_g)^2 + (z_0 - z_g)^2}} \end{pmatrix}$$

Thus, for a given user u, the estimations of the SISE and its standard deviation are:

$$SISE_{est,u} = h_u^t \cdot \Delta\hat{X}$$

$$\sigma_{check,u} = h_u^t \cdot \operatorname{cov}(\Delta\hat{X}) \cdot h_u = h_u^t \cdot \left[H^t \cdot R^{-1} \cdot H\right] \cdot h_u$$

where $R = \operatorname{cov}(E_i)$

The maximum values of $SISE_{est,u}$ and $\sigma_{check,u}$ are called $SISE_{est}$ and SISMA:

$$SISE_{est} = \max_u \left(SISE_{est,u}\right)$$

$$SISMA = \max_u \left(\sigma_{chack,u}\right)$$

It appears that the standard deviation of $SISE_{est}$ depends on the observation time-span. Indeed, if $SISE_{est}$ is observed for a short time, SISE (projection of ΔX) will be considered a constant. $SISE_{est}$ will therefore follow a normal distribution centred on SISE, with a standard deviation equal to SISMA. If, on the other hand, $SISE_{est}$ is observed for a long time, the variations of SISE will have to be taken into account: $SISE_{est}$ will follow a centred normal distribution, with standard deviation equal to $\sqrt{SISA^2 + SISMA^2}$.

The integrity flag is raised when the observed $SISE_{est}$ departs significantly from this statistics. The decision threshold is tuned using the specified false alarm rate, so the integrity flag is raised when

$$\left|SISE_{est}\right| > k_{FA} \times \sqrt{SISA^2 + SISMA^2}$$

where k_{FA} =4.34 as per the design false alarm rate 1.5 x 10^{-5} per independent sample.

Thus, $SISE_{est}$ is computed from the residuals provided by the pre-processing algorithms, while SISMA is only computed from assumptions on the standard deviation of the measurement noise and geometrical data. If one of the input residuals does not respect the assumption on its standard deviation (because of a propagation problem or a reception problem in the vicinity of the GSS), then there is risk that $\left|SISE_{est}\right|$ will exceed the decision threshold despite the fact that the satellite is not malfunctioning. In this case, a robust algorithm, able to reject input data that do not respect the assumptions made on them, might help in not flagging unduly a satellite.

III USER INTEGRITY ALGORITHM

The user integrity algorithm which is planned to be used for users of the Safety-of-Life service of GALILEO is significantly different from usual integrity monitoring methods (such as RAIM), in particular because it will not provide users with horizontal or vertical protection levels (HPL and VPL) to be compared with corresponding alert limits. The method proposed in [5] consists in directly computing the user integrity risk at the alert limit (as a function of HAL, VAL, P_{FA}, P_{MD}, SISA, SISMA, etc.), and comparing it to the Integrity Risk value.

$$\begin{aligned} &P_{HMI}(VAL, HAL) = \\ &1 - erf\left(\frac{VAL}{\sqrt{2}\cdot\sigma_{u,V,FF}}\right) + e^{-\frac{HAL^2}{2\xi_{FF}^2}} \\ &+ \frac{1}{2}\cdot\sum_{j=1}^{N} P_{fail,sat_j}\cdot\left(\left(1 - erf\left(\frac{VAL + \mu_{u,V}}{\sqrt{2}\cdot\sigma_{u,V,FM}}\right)\right) + \left(1 - erf\left(\frac{VAL - \mu_{u,V}}{\sqrt{2}\cdot\sigma_{u,V,FM}}\right)\right)\right) \\ &+ \sum_{j=1}^{N} P_{fail,sat_j}\cdot\left(1 - \chi^2_{2,\delta_{u,H}}cdf\left(\frac{HAL^2}{\xi_{FM}^2}\right)\right) \end{aligned}$$

The parameters $\sigma_{u,V,FF}$ and $\sigma_{u,V,FM}$ are the standard deviations of the vertical position error respectively in fault free and failure mode. The parameters ξ_{FF} and ξ_{FM} are the radius of the circle bounding the error ellipse in the horizontal mode respectively in fault-free and failure mode.

$\sigma_{u,V,FF}$ and ξ_{FF} depend on satellite SISA, on the standard deviation of the propagation errors, and on the constellation geometry.

$\sigma_{u,V,FM}$ and ξ_{FM} depend on the same parameters and on the SISMA that helps determining the size of the maximum unflagged error. $\mu_{u,V}$ is precisely the maximum unflagged vertical position error.

IV PRINCIPLE OF ROBUST ALGORITHMS

The aim of robust statistics is to construct estimation and regression algorithms able to provide reliable results when all the assumptions made on the observation data are not met in full. Indeed, it is generally assumed that all variables are normally distributed: it is the case for which the classical Least-Squares algorithm is optimal. When the underlying Gaussian model does not hold for every sample, for instance when a feared event causes one pseudorange measurement to deviate (such a measurement is called an ***outlier***), the results provided by the Least-Squares may actually be far away from the true data distribution. The whole point of robustness can be expressed in terms of continuity of the estimator: a small variation in the sample space (either a small change on the whole sample space, or great change on a small fraction of the sample space) must bring only small variations to the estimated distribution.

In order to underline the differences between Least-Squares and robust algorithms, we will use the example of linear regression (which is the type of problem solved to obtain a position with satellite positioning systems):
$y_i = \beta_0 + \beta_1 x_{i1} + \ldots + \beta_p x_{ip} + e_i$ for $i = 1, \ldots, N$,
where y_i is the response variable (e.g. a pseudorange measurement), $x_{i1}, \ldots, x_{ip}$ are the regressors (e.g. elements of the user-satellite direction vector) and e_i is a zero-mean normal noise with σ standard deviation. The aim is to obtain $\left(\hat{\beta}_0, \ldots, \hat{\beta}_p\right)$, estimate of the set $\left(\beta_0, \ldots, \beta_p\right)$ of regression coefficients (which are user coordinates and clock bias in the case of satellite positioning). The regression residuals may be expressed as follows:
$r_i = y_i - \left(\hat{\beta}_0 + \hat{\beta}_1 x_{1i} + \ldots + \hat{\beta}_p x_{ip}\right)$.
The estimate $\left(\hat{\beta}_0, \ldots, \hat{\beta}_p\right)$ computed by LS is the one which minimizes the sum of squared residuals:

$$\min_{\left(\hat{\beta}_0, \ldots, \hat{\beta}_p\right)} \sum_{i=1}^{N} r_i^2$$

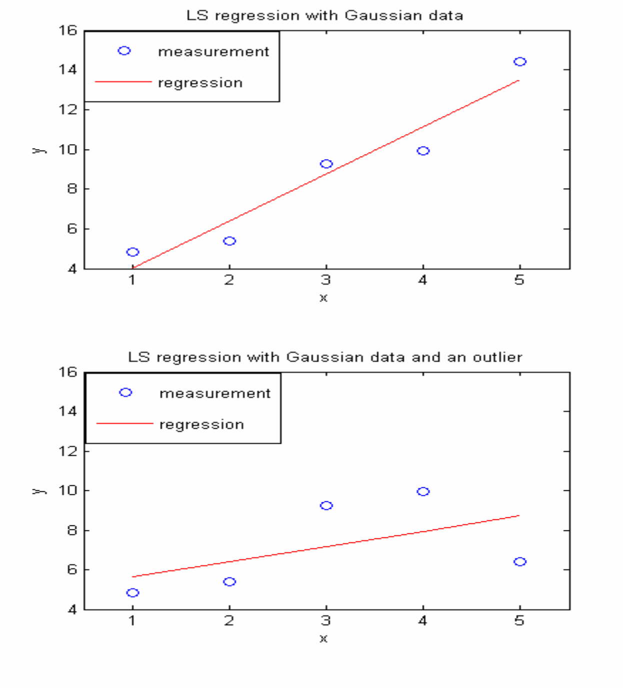

Figure 1: Example of the influence of a single outlier

The LS criterion brings optimal results when e_i is Gaussian. But there is no guarantee of reliability of the algorithm's results when e_i is not Gaussian (which is generally the case). Indeed, figure 1 illustrates the fact that one single outlying sample may cause a two-dimensional LS regression to break down.

This example may be used to introduce the notions of *breakdown value* and *breakdown bound*, as described by Huber in [1]: the breakdown value is the smallest fraction of contamination in the sample space that can cause the regression method to run arbitrarily far from the distribution of the majority of the samples. For instance, it can be seen from the preceding figure that the breakdown value of the LS algorithm is $1/N$. The breakdown bound is the limiting value (for $N \to \infty$) of the breakdown value. It is thus equal to 0 for the LS. Estimators with a breakdown bound strictly superior to 0 are called positive-breakdown methods (as will be seen in the following, robust methods do not systematically have positive breakdown bounds).

The methods that were used for this study are M-estimation, Least-Trimmed-Squares (LTS) and Minimum Covariance Determinant (MCD). M-estimation consists in minimizing $\sum_{i=1}^{N} \rho(r_i)$ rather than $\sum_{i=1}^{N} r_i^2$, where ρ is a symmetric, positive definite function with a unique minimum located at zero, so as to limit the influence of high-residual samples on the final output. Three different ρ functions were used: the Huber and Tukey functions, and a custom function, designed to combine the qualities of the two preceding functions. Both MCD and LTS rely on the selection of a sample subset from which they compute the final output; they differ on the type of criterion to apply to select the subset: LTS selects the h samples for which $\sum_{i=1}^{h} r_i^2$ is minimum, while MCD selects the h input y_i for which the vector $[y_1, \ldots, y_h]$ has the smallest standard deviation. More information will be found in [6].

V RESULTS ON SISMA COMPUTATION

We briefly recall here the results of the preceding study, concerning the computation of SISMA. Results are presented in three ways.

In the fault-free case: LS is optimal when inputs are truly Gaussian. Thus using other (robust) methods implies a degradation of the overall results. The degradation is quantized in table 1, as the percentage of SISMA values inferior to 0.70 m (the SISMA maximum value in fault-free mode)

In the case where each sample set contains a biased measurement: in this test, there is no faulty satellite (and thus no "DON'T USE" must be issued), however, a bias (ranging from 10cm to 10m) has been added to one of the measurements corresponding to each satellite. Robust methods are expected to detect this bias and reject the corresponding measurement, and thus to raise false alarms (a "DON'T USE" flag set on a fault-free SV) less often than the LS. The false alarm rate is illustrated in the top figure of figure 9.

In the case of a true failure, robust methods are expected not to increase the missed detection probability: the detection rate in the true failure case (simulated by an additional bias on all PR measurements of a given satellite) is illustrated in the bottom figure of figure 9.

	Max SISMA (m)	Percentage of SISMA < 0.7m
LS	0.9388	77.9278
M Huber	0.9760	74.9155
M Tukey	1.2638	70.4186
M custom	0.9931	77.5716
LTS	1.7262	64.5722
MCD	1.9752	63.9085

Table 1: SISMA distribution for all estimation methods

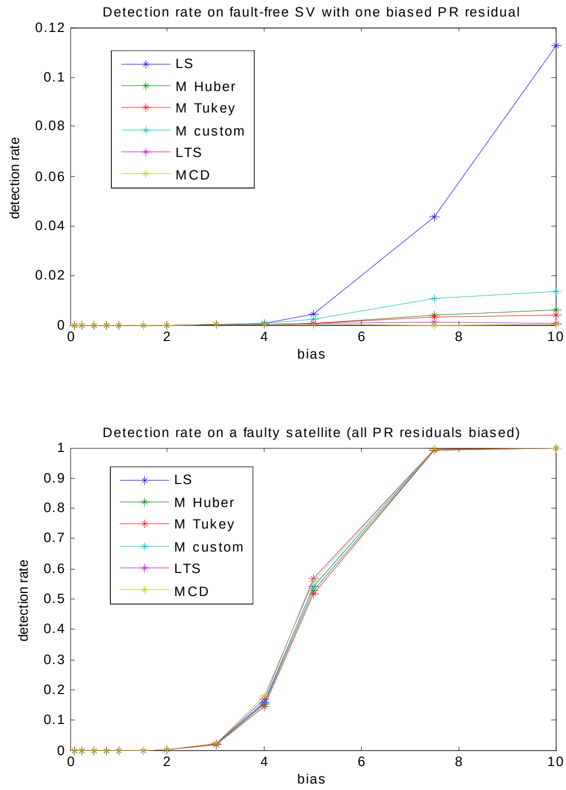

Figure 2: False alarm and true fault detection rates

The final results are summarized in table 2.

	M-estimators	LTS	MCD
False alarm rate	Good	Excellent	Excellent
SISMA distribution	Slightly Degraded	Degraded	Degraded
True fault detection rate	Lowered	Increased	Increased
Overbounding capability	Comparable to LS	Degraded	Increased

Table 2: Results of robust methods

VI RESULTS IN USER INTEGRITY

The main test in user integrity consisted in creating a world user map (composed of 1369 points) and computing user integrity over 24 hours using the SISMA data obtained with the different methods ***in fault-free mode***. The considered navigation requirements were those of APV-II, which are recalled in table 3. For each user position, the comparison of the computed Integrity Risk values with the IR requirement resulted in a system availability percentage over 24 hours. These availability results are illustrated in the following maps. However, readers should be warned against interpreting the figures as the illustrations of the future performances of GALILEO, since the simulated data that was used cannot be considered as a realistic test-bed. On the other hand, they show the expected degradation in system availability in fault-free mode due to the use of robust methods.

Accuracy 95%		H 16 m V 8 m
Integrity	Time to alert	6 s
	Alert limit	H 40 m V 20 m
	Integrity risk	2.10^{-7}/app
Availability		0.99 to 0.99999
Continuity		8.10^{-6} / 15s
P_{FA}		1/15000
P_{MD}		$1.\ 10^{-3}$

Table 3: Operational requirements in APV-II

Figure 3 represents the map of system availability over 24 hours, computed with the Least-Squares. It will be used as a reference. The availability holes are due to the geometry of the constellation and of the GSS network. In most cases however, the availability is 100% in our simulation. Figure 4, 5 and 6 represent the availability map obtained with SISMA computed with M-estimation methods. Finally, figure 7 and 8 correspond to LTS and MCD, the methods which performed best in terms of false alarm and false detection rates and worst in terms of SISMA distribution.

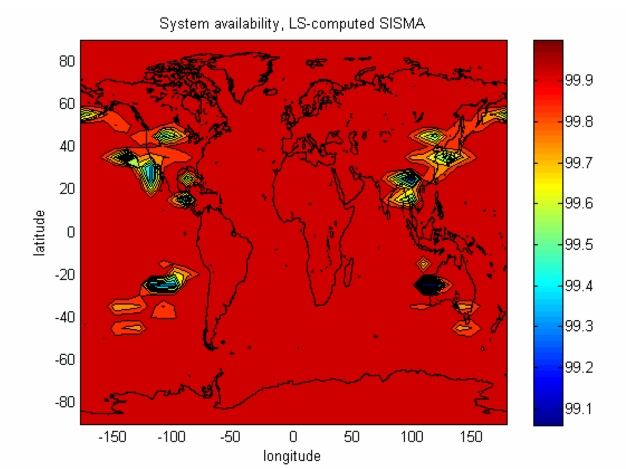

Figure 3: System availability over 24 hours, with LS algorithm

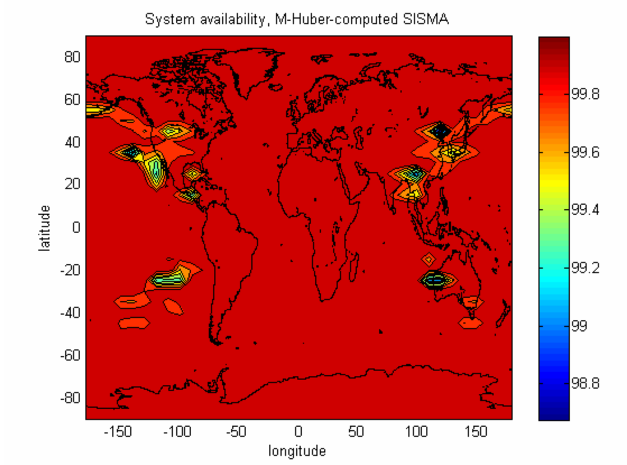

Figure 4: System availability over 24 hours, with M-Huber algorithm

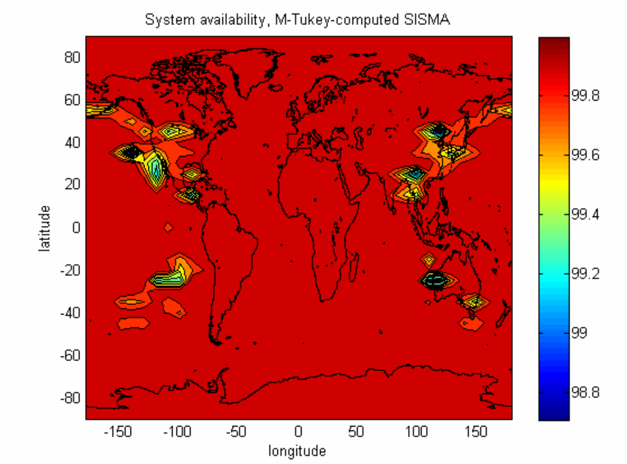

Figure 5: System availability over 24 hours, with M-Tukey algorithm

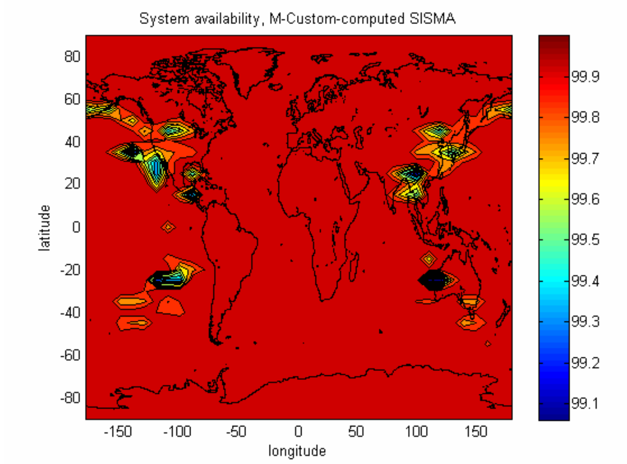

Figure 6: System availability over 24 hours, with M-custom algorithm

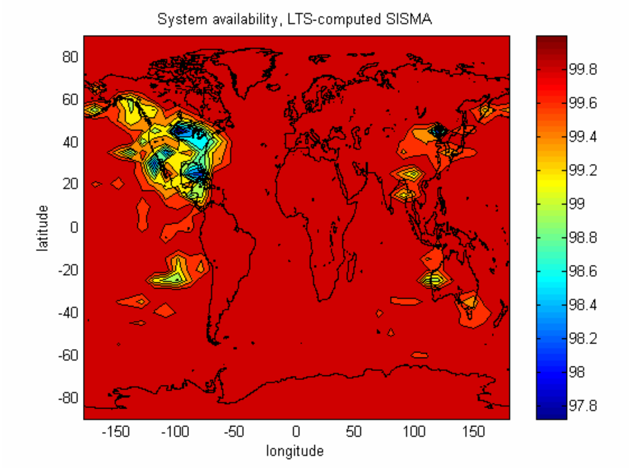

Figure 7: System availability over 24 hours, with LTS algorithm

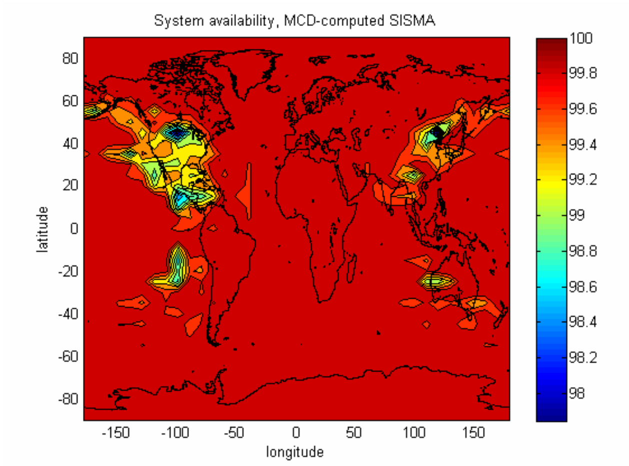

Figure 8: System availability over 24 hours, with MCD algorithm

The preceding availability maps can be summarized by figure 9, which gives, for each method, the Cumulative Density Function of system availability. For clarity, the probability values corresponding to a 100% availability, being much higher than the others, have been omitted.

It appears from the final results that the degradation in availability is moderate. However, it corresponds to the results in SISMA computation: M-estimation, which induces small degradation of the SISMA distribution, provides availability results close to the LS output; in particular, when performed by the custom function, it has a minimum availability that is not lower than that of LS. On the other hand, the methods that did not perform well in SISMA distribution (LTS and MCD) cause a more serious degradation, thus providing an illustration of the direct impact of SISMA on user integrity.

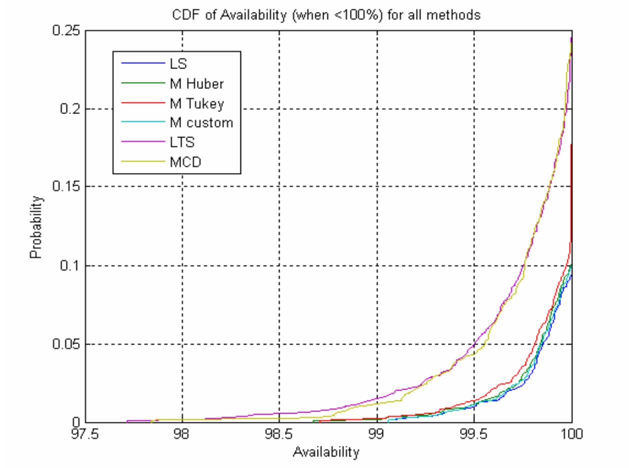

Figure 9: Availability Cumulative Density Function for all methods

VII CONCLUSION

The following table compares the performance in system availability achieved through LS with the other procedures:

	Minimum availability (%)	Percentage of positions with availability inferior to 100 %	Percentage of positions with availability inferior to LS
LS	99.06	9.42	N/A
M Huber	98.68	10.08	8.54
M Tukey	98.71	17.75	17.02
M custom	99.06	10.01	6.14
LTS	97.72	24.54	23.88
MCD	97.84	24.25	23.81

Table 4: Results in system availability

This demonstrates the feasibility of replacing LS by robust methods. The gain in false alarm rates and SISMA values reliability would cause little degradation since minimum availability remains correct.

However, there clearly is a trade-off to make between false detection rate and user availability.

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