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# TV on Mobility: a Two-Sided Market Analysis

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## 1 Introduction

The opportunity to watch TV on a mobile phone is taking more and more importance as mobile operators are investing in new technologies allowing a better quality of the broadcasting. According to US consultants of Visiongain, 270 Millions of new subscribers, which represents 10% of the mobile users, are expected by 2009.

This paper analyzes how the market of the TV on mobile phone works. We model this market as a two-sided market, where the customers from one side of the market, wants to watch TV on their mobile, and, TV producers/advertisers, on the other side of the market, want to advertise on a new media. Customers and advertisers are not directly related: The mobile operator plays the role of a platform connecting the two sides of the market. With respect to the classical TV market (TV at home), this market is larger: Customers can consume TV on their mobile while in mobility. This feature translates in concrete terms to the fact that consumers are willing to pay more to receive the TV on their mobile. It means as well that the advertisers are willing to pay more to acquire advertising space because the audience is larger. From now on, we will use indifferently "mobile TV" for TV on mobility and TV on mobile phone. There are externalities on the mobile TV market: Customers want to consume broadcasts and not advertisements, whereas broadcasts are made because of the revenues coming from advertising. The mobile operator trades off the two sources of profits: Increasing the space for advertising increases the profit made on the market of advertisers. However there exists a side effect on the other side of the market as customers incur a disutility of watching advertising and decrease consequently their

demand for mobile TV, which, in turn, decreases the profit made on TV broadcasting. In our model, the mobile operator is a monopolist. The mobile TV market is an emerging market and mobile operators introduce the possibility to watch TV firstly to their own consumers of mobile phone. They are thus considered to be in a dominant position on this part of the demand.

We show in this paper that a wise mobile operator, i.e. an operator which is aware of the two-sidedness of the mobile TV market, leads to a higher social welfare than a myopic monopolist. The wise monopolist generally produces closer to the social optimum. We also show that the wise monopolist uses two-part tariffs on both side of the market, to reach higher profits. The wise monopolist broadcasts less advertising and less TV on mobile than a myopic monopolist when the markets are really linked up. It broadcasts less advertising and more TV when the externality of advertising is high. We then show that the wise monopolist always sets a lower price of one minute of TV broadcasted while generally the price paid by advertisers is higher than the myopic monopolist : Because of the existence of the externality, the advertisers bear part of its cost to compensate the consumers.

The literature on two-sided markets is becoming more and more important with a certain number of seminal papers. We use a classical framework as presented in Rochet and Tirole [2004] with usage externalities on both sides of the market as advertisers demand depend on audience and demand for TV depends on advertising. To the difference to Rochet and Tirole [2003] or Armstrong [2002], we do not model competition between platforms in this paper, as consumers are considered to be captive, but fully model the consumers side as in Crampes et alii [2004], adding however one supplementary ingredient: TV can be consumed while in mobility. This modifies the utility of consumers and the equilibrium prices the mobile operator sets. Besides we use two-part tariffs which allows us to derive interesting results in terms of welfare. The welfare analysis gives results which are close to those found by Anderson and Coate [2005] in another context, in which two-part tariffs are absent and TV broadcasting is considered as a public good.

The paper is organized as follows: The second Section presents the model and the different assumptions. The equilibrium of the market is presented in Section 3. A comparison with a benchmark situation, in which the mobile operator is myopic, i.e. when

it does not consider the two sides of the market while taking its decisions, is proposed in Subsection 3.3. In Section 4, we propose a normative analysis of the problem. The optimal solutions are then compared with the equilibrium derived in Section 3. Section 5 concludes.

# 2 A Two-sided Market Model

## 2.1 Market Characteristics

### 2.1.1 Mobile operators

The market is a two-sided market in which mobile operators are platforms linking the two markets of advertising and TV consumption, i.e. advertisers and final consumers.

The market is larger than the market of classical TV (TV at home) in the sense that people can consume while in mobility. Consumers are indeed able to watch TV out of their home, when commuting to their working places. Potentially, the frequency of watching TV is thus higher, creating new prospects for advertisers.

Consider a situation in which the mobile operator is a monopolist. This assumption avoids modeling the competition between operators to increase their demand for mobile services via the offer of a new service, i.e., TV on their mobile. In other words, the addressed demand for phone services is captive and the fact that consumers could not switch from one operator to another in response to changes in the features of competitors' offer of mobile TV should not affect our result.

The monopolist has to set together the price of advertising and the price of mobile TV. The prices are two-part tariffs: consumers (resp. advertisers) pay a certain fee for accessing TV to the TV on their mobile (resp. for being able to advertise on mobile phone) and pay as well a price per each minute of TV watched (resp. per each minute of advertising broadcasted). The monopolist chooses this system of prices in order to maximize its profit subject to the following constraints. Firstly, customers (resp. advertisers) are willing to pay the access to the TV on mobile service (resp. to the space on the mobile). This defines the individual rationality constraints. Second, the monopolist faces the demands of each sides of the market: one for advertisers and one for customers.

We denote by $(p, T)$ the tariff offered to consumers, where $T$ is the fixed part of the tariff, corresponding to the access cost to the TV service, $p$ corresponds to the variable part of the tariff and is defined as a price per minute of TV watched on the mobile. Equally, $(r, A)$ stands for the tariff offered to advertisers, where $A$ is an access fee and $r$ is the price per minute of advertisement broadcasted.

Assume that it costs $c$ per minute of mobile TV consumed and $d$ per minute of advertisement broadcasted. Fixed costs are denoted $C_A$ and $C_N$ and stand respectively for the fixed cost related to the advertising activity and the broadcasting activity.

### 2.1.2 TV Producers

The TV producers (and/or advertisers) build up TV broadcasts and advertisements. These packages are supplied to TV mobile operators. The broadcasts are largely subsidized by the advertisements. That is the main reason why we restrict our attention to the market of advertisers.

Advertisers profits are composed of two parts: the first part is the profit generated by the advertising activity. It is composed of the gains coming from advertising minus the costs to produce advertisements. This profit is increasing in the audience: the more the audience, the higher the impact of advertising. The second part of the profits is the costs to broadcast the advertisements to the mobile TV watchers. Advertisers pay an access price $A$ and a price per minute of advertising broadcasted $r$. The profit is thus equal to

$$\Pi(a; n) = \pi(a; n) - A - ra,$$

where $\pi(a; n)$ is increasing and concave. This part of the profit represents together the gains from advertising, and the cost to produce advertisements. Demand for advertising space $a_D$ is thus defined by the following equation:

$$r = \frac{\delta \pi(a; n)}{\delta a}.$$

The advertising space demand is increasing with the audience $n$, i.e. the price per minute of advertisement broadcasted $r$ is higher, the higher the audience.

### 2.1.3 Customers

Customers derive a satisfaction $U$ of the consumption of a composite good $z$ and of watching $n$ minutes of TV on their mobile. Assume that the utility function is separable in the two arguments. Besides, customers incur a disutility because of the presence of advertisements in the TV programs they watch. This disutility is assumed to be proportional to the amount of advertising broadcasted. The existence of this negative externality links the two markets of the customers and of the advertisers, as the audience. The utility writes:

$$U(z, n) = \alpha z + u(n; m) - \lambda a,$$

where $\alpha$ and $\lambda$ are positive parameters and u(.) is strictly increasing and concave in the audience. Parameter $m$, stands for the possibility to consume the good "TV" more often than the composite good $z$ (to take into account the positive impact of mobility on consumption). As a consequence, the more $m$, the higher the utility for a given quantity of mobile TV consumed and the higher the marginal utility derived for the consumption of one more unit of TV.

Customers have an exogenous income $I$ affected to the consumption of the two goods. Without loss of generality, the price of good $z$ is set to 1. The customers pay an access fee $T$ to receive television on their mobile and a price $p$ per minute of TV watched. The budget constraint can thus be written as follows:

$$I \geq pn + z + T$$

.

The demand for TV, denoted $n_D$ is obtained by saturating the budget constraint and eluding $z$. The inverse demand function is then

$$\alpha p(n) = u'(n; m)^1.$$

Because of the possibility to consume TV while in mobility, the price of one minute of TV, $p$, is higher than in the case of classical TV. The price is increasing in the marginal utility for TV but decreases with the quantity of TV consumed.

[1] As $m$ is an exogenous parameter in our model, we denote $u'(n; m)$ the first derivative of $u(n; m)$ with respect to $n$.

Besides, we assume that customers have a strictly positive reservation utility, corresponding to the utility of the income $\alpha I$. We denote $V(n)$ the indirect utility function, which writes:

$$V(n) = \alpha(I - pn - T) + u(n; m) - \lambda a.$$

# 3 Equilibrium of the Market

## 3.1 Benchmark: a Myopic Monopolist (M)

Let us first analyze the situation of an operator which does not take into account the link between the two markets. The monopolist sets the prices $r$ and $A$ to maximize its profit on the advertising market and, independently, $p$ and $T$ to maximize its profit on the final market.

On the market for advertising, the program of the monopolist is the following:

$$\begin{array}{llll} \underset{\{r,A\}}{Max} & \Pi = & ra & +A \quad -da \\ s.t. & \pi(a;n) - A - ra & \geqslant & \overline{\pi}_a, \\ & r & = & \frac{\delta\pi(a;n)}{\delta a} \quad . \end{array}$$

The participation constraint just says that the advertisers make no loss when not posting any advertisement. $\overline{\pi}_a$ represents the reservation profit for advertisers. Then, the monopolist sets the fixed part of the tariff $A$ in order to saturate this constraint. When replacing in the operator profit, the first order condition gives:

$$\frac{\delta\pi(a;n)}{\delta a} = d.$$

Price $r$ is thus set to cover the marginal cost of broadcasting advertisements on a mobile phone. The myopic monopolist makes all its profit using the fixed part of the tariff.

But, the myopic monopolist takes as given the level of audience $n$ when optimizing its profits. It thus leads to an underestimation of the price advertisers are willing to pay.

The choice of prices $p$ and $T$ is similar to the choice of the advertising tariff: The monopolist sets up the prices to maximize the profit from broadcasting TV on mobile phones, taking as given the amount of advertisements. The program is thus:

$$\begin{array}{llll} \underset{\{p,T\}}{Max} & \Pi = & pn & +T \qquad -cn \quad -C_N \\ s.t. & V(n) & \geqslant & \alpha I, \\ & p & = & \frac{u'(n;m)}{\alpha}. \end{array}$$

The participation constraint is saturated: The monopolist extracts as much as possible from the customers, provided that they want to consume TV on their mobile phone. The monopolist takes as given the amount of advertisements broadcasted while optimizing its profit on this side of the market. This leads to an overestimation of the demand for TV.

$$\begin{array}{lll} V(n) & = & \alpha I \\ & \Longleftrightarrow & \\ T & = & \frac{u(n;m)}{\alpha} \quad -pn \quad -\frac{\lambda}{\alpha}a \end{array}$$

Replacing $T$ and $p$ as their expression as a function of $n$ and solving for $n$ yields the solution. The first order condition gives

$$\frac{u'(n;m)}{\alpha} \quad -c \quad = 0.$$

It means that the monopolist sets the price of one minute of TV, $p$, exactly at the marginal cost, $c$, of broadcasting one minute of TV on the mobile.

When the marginal utility of income $\alpha$ is higher, the monopolist prices a lower fee $T^M$. As the price $p^M$ equals the marginal production cost $c$ at equilibrium, the higher $c$, the lower the subscription fee. On the contrary, the higher the opportunity to consume TV while in mobility, the higher the ability for customers to pay the fee.

At the equilibrium, customers utility derived from the consumption of TV is just equal to zero. Indeed the fixed transfer $T^M$ takes all the surplus from customers. The firm makes exactly $T^M - C_N$ profits as the price $p^M$ equals marginal production cost $c$.

We thus have to check that the mobile operator recoups at least its fixed cost $C_N$ by ensuring

$$T^M \geq 0.$$

This constraint is easier to fulfill the higher the satisfaction. Given a certain set of parameters, the mobile operator may then find it not profitable to provide customers with TV, because it does not take into account the interaction between the two sides of the market.

### 3.2 A Wise Monopolist (W)

A well-advised operator is aware of the existence of the negative externality advertisements exert on customers as well as the impact of audience on advertising space demand. The monopolist will trade off the profits made on each side of the market: If the price of advertising space decreases, everything equals, demand for advertising increases and potentially profits from the advertising activity for the monopolist. However, demand for TV decreases and the monopolist makes less profits on this side of the market.

The program of maximization is thus:

$$\begin{array}{llcl} \underset{\{p,r,A,T\}}{Max} & \Pi = & ra \quad +A \quad +pn \quad +T \quad -cn - da \quad -C_N - C_A & \\ s.t. & V(n) & \geqslant & \alpha I \\ & \pi(a;n) & \geqslant & \overline{\pi}_a \\ & \frac{\delta\pi(a;n)}{\delta a} & = & r \\ & \frac{u'(n;m)}{\alpha} & = & p \end{array}$$

The demand for TV on mobile is determined by utility maximizing, customers taken the prices, $p$ and $T$, as given. The wise monopolist now knows that there is a negative externality exerted by the advertisements: The higher the demand for TV, the higher the demand for advertisements but the lower the utility, everything equals. Taking into account the externality of advertising, the demand for TV $n_D$ is then solution of:

$$p = \frac{u'(n;m)}{\alpha} - \frac{\lambda}{\alpha}\frac{\delta a_D}{\delta n},$$

where $a_D$ is the demand from advertisers.

The price of one minute of advertising is increasing in the mobility parameter: The higher the valuation of the capacity to consume while in mobility or the time spent in mobility, the higher the price customers are ready to pay for one minute of advertising. The price is also increasing with the marginal utility brought by the consumption of TV. However, the existence of an externality decreases the price that can be imposed for one minute of TV. To reach the same level of demand for TV, the wise monopolist has to price lower. The demand for TV is, at equilibrium, lower than the demand in the myopic situation since the necessary condition evaluated at the myopic solution is negative.

The demand for advertising space is determined by the first order condition of the advertisers program, but taking into account the effect of the advertising on the audience:

$$r(a,n) = \frac{\delta\pi(a,n)}{\delta a} + \frac{\delta\pi(a,n)}{\delta n}\frac{\delta n_D}{\delta a},$$

where $n_D$ is the demand for mobile TV.

The monopolist will anticipate both the positive externality of the audience on advertising demand and the negative externality exerted by the advertisements on the audience. To reach the same demand as the myopic monopolist, it has to price lower.

Let us now solve the program. The participation constraints are both binding because the mobile operator profit is increasing in transfers $T$ and $A$. The monopolist thus wants to increase them while making sure both customers and advertisers participate to the market.

$$\begin{cases} A^W &= \pi(a;n) - \overline{\pi}_a & -r(a,n)a \\ T^W &= \frac{u(n;m)}{\alpha} & -p(n,a)n \quad -\frac{\lambda}{\alpha}a \end{cases}$$

Substituting these values into the profit function yields

$$\Pi = \pi(a;n) - \overline{\pi}_a + \frac{u(n;m)}{\alpha} - \frac{\lambda}{\alpha}a.$$

The first order conditions associated with the maximization with respect to $a$ and $n$ are

$$\begin{cases} \frac{\delta\Pi}{\delta a} = 0 \quad \Leftrightarrow \quad \frac{\delta\pi(a,n)}{\delta a} \quad -\frac{\lambda}{\alpha} \quad -d \quad = 0 \\ \frac{\delta\Pi}{\delta n} = 0 \quad \Leftrightarrow \quad \frac{\delta\pi(a,n)}{\delta n} \quad +\frac{u'(n;m)}{\alpha} \quad -c \quad = 0 \end{cases}$$

Recall that at equilibrium, the myopic monopolist sets the price of one minute of TV at the efficient level, i.e., the marginal cost of production. Here, the existence of a negative externality created by advertising decreases the price chosen by the wise monopolist at equilibrium. Everything happens as if the wise monopolist is pooling the cost to provide the TV on mobile between the customers and advertisers. The existence of the externality makes the wise monopolist to set a lower price to customers. This is reinforced by the positive impact of the audience on the demand for advertising space: the advertisers are more willing to pay to have access to space of advertisements if there is a higher audience, which is possible if the price of one minute of TV is lower. The number of minutes of TV watched at equilibrium is not clearly higher because decreasing the price increases the

quantity of TV watched, which ceteris paribus increases the demand for advertising space, which in turn decreases the demand for watching TV.

Let us now specify both the utility function and the profit of advertisers to realize numerical simulations. Suppose that $u(n;m) = ln(n) + \alpha mn$, increasing and concave and $\pi(a,n) = \kappa na - \frac{a^2}{2} + \overline{\pi_a}$, with $\kappa > 0$ representing the impact of the externality of the audience on the advertising profits.

The relevant parameters to take into account the links between the two sides of the market are thus $\lambda$, which stands for the negative impact of advertising on mobile TV consumption and $\kappa$ which measures the positive impact of audience on the advertising demand.

Results are summarized in the following proposition:

**Proposition 1** *The well-advised monopolist uses two-part tariffs such that fees are used to set up the advertisers and customers to their reservation profit and utility.*

*The equilibrium level of consumption of TV and of advertising space are, for $d^2\alpha < 1$ and $\lambda < \widehat{\lambda}$, $\underline{\kappa} \leq \kappa \leq \overline{\kappa}$:*

$$\begin{cases} n^W = & \frac{1}{2\alpha\kappa^2} \quad (\alpha(c-m) \quad +\kappa(\lambda+d\alpha) \quad -\sqrt{-4\alpha\kappa^2 + (\alpha(c-m)+\kappa(\lambda+d\alpha)^2)} \\ a^W = & \frac{1}{2\alpha\kappa} \quad (\alpha(c-m) \quad -\kappa(\lambda+d\alpha) \quad -\sqrt{-4\alpha\kappa^2 + (\alpha(c-m)+\kappa(\lambda+d\alpha))^2}), \end{cases}$$

*where* $\widehat{\lambda} = \sqrt{\alpha} - d\alpha$, $\underline{\kappa} = (c-m)(\lambda+d\alpha)$, $\overline{\kappa} = \frac{(c-m)\alpha}{2\sqrt{\alpha}-(\lambda+d\alpha)}$.

**Proof.**

See in appendix. ■

Both existence and positiveness conditions reflect the trade-off faced by the monopolist: if the externality of advertising is really too high, the monopolist cannot ensure the provision of the market because it would mean negative provision of advertisements. This goes together with a not too low positive externality of audience on the advertising side of the market in order to compensate the negative externality of advertisement.

Making some comparative statics on the equilibrium solution lead us to the following conclusions: both the consumption of TV and the consumption of advertising space are reacting in the same direction with $\lambda$ and $\alpha$. Things are more complicated with parameter $\kappa$. For instance, the number of minutes of TV broadcasted and the number of minutes of

advertisements are decreasing in parameter $\lambda$. The higher the externality of advertising, the lower the consumption of TV and advertising space. The externality thus plays a negative role on the equilibrium values of consumption. The first effect of the externality seems to dominate: When $\lambda$ increases, the demand for TV decreases, ceteris paribus, leading to a decrease of the demand for advertising space (through the parameter $\kappa$). However, as the demand for advertising space decreases, the demand for TV increases because the externality is less important. Now the demand for advertising then increases. Hence the first effect should dominate at last.

The number of minutes of TV broadcasted is decreasing in parameter $\alpha$, i.e., the consumption of TV is decreasing with the marginal utility of income. The richer the customers, i.e., the lower parameter $\alpha$, the higher the consumption of TV. The consumption of advertising space is equally decreasing in $\alpha$. Richer customers are potentially more likely to address a strong demand for TV, which in turn has a positive impact through the audience on the advertising space demand.

The impact of parameter $\kappa$ is positive on both advertisements and TV broadcasted. Indeed the demand for advertising space is increasing with the audience through parameter $\kappa$. Thus as $\kappa$ increases, the equilibrium value of advertisements increases. The number of minutes of TV is increasing with the impact of audience on advertising demand for low values of $\kappa$, and then decreasing. When the advertising space demand is really sensitive to the audience, then the potential increase in the demand makes the demand for TV decreasing. Thus when $\kappa$ is high, $n^W$ decreases with $\kappa$. On the contrary, when the impact of the audience on advertising demand is low, one need to increase much the number of minutes of TV broadcasted to be of interest for the other side of the market. Thus when $\kappa$ is low, $n^W$ increases with $\kappa$.

### 3.3 Comparison myopic/wise monopolist

To analyze the solutions, we set the marginal utility of income to be equal to unity, i.e. $\alpha = 1$. Results are not qualitatively changed for different strictly positive and bounded values of $\alpha$. The constraints of existence impose

$$\frac{2(2\kappa - 1)}{\kappa} \leq \lambda \leq 2\kappa$$

**Proposition 2** *The number of minutes of TV broadcasted at equilibrium is higher in the wise monopolist case when both the externality of advertising and the sensitivity of advertising space demand to audience are low. The number of minutes of advertising in the wise monopolist case is generally lower except when the sensitivity to audience is low so as the externality of advertising.*

$$n^W > n^M \quad \textit{if and only if} \quad \kappa > 1$$
$$a^W > a^M \quad \textit{if and only if} \quad \kappa < \frac{\sqrt{5}-1}{2} \quad \textit{for} \quad \lambda \leq \frac{2\kappa^3}{1-\kappa^2}$$

**Proof.**

See in appendix. ■

Results are illustrated in Figure 1.

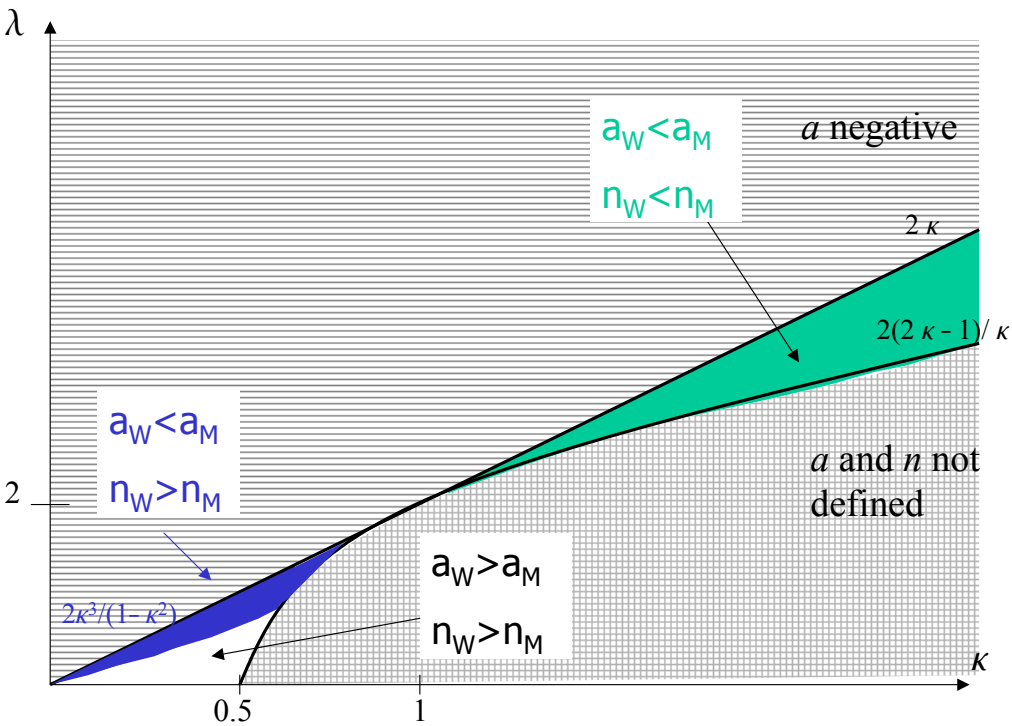


Figure 1: Comparison of quantities

According to Figure 1, three area have to be distinguished. The first area corresponds to high values of both the externality of advertising and the sensitivity of advertising to

audience. In this case, the wise monopolist broadcasts both less minutes of TV on mobile and less advertisements than the myopic monopolist. A high $\lambda$ and a high $\kappa$ means that the myopic monopolist has largely underestimated the two-sided aspect of the market. There is a strong effect of advertising on the demand for TV on mobile and a strong effect of the audience on the demand for advertising space. When $\lambda$ is high, the demand for TV on mobile is lower in the wise monopolist case. As a consequence, demand for advertising is reduced because $\kappa$ is high. The induced effect is an increase in the demand for TV on mobile, and in turn an increase of the advertising space demand. Finally, it seems that the first order effect measured by the fact that the myopic monopolist underestimates the links between the two sides of the market dominates for $\lambda$ and $\kappa$ sufficiently high.

When the demand for advertising space is less to sensitive to audience, the wise monopolist can provide more demand for TV on mobile. When $\kappa$ is low, the number of minutes of TV broadcasted is higher than in the myopic monopolist situation. However the number of minutes of advertising depends on the externality of advertising: When the externality is high, the number of minutes of advertising is lower in the case of the wise monopolist. This reflects the fact that the wise monopolist takes into account the impact of the externality on the demand for TV.

**Proposition 3** *The price of one minute of TV broadcasted on mobile is always lower in the case of the wise monopolist. The price of one minute of advertising on mobile is generally higher except when the sensitivity of the advertising demand to audience becomes high, while the impact of advertising externality is not too important.*

$$\begin{array}{lllll} p^W < p^M & \textit{for all} & \kappa > 0 & \textit{and} & \lambda > 0 \\ r^W > r^M & \textit{if and only if} & \lambda > \frac{-1+3\kappa^2+\sqrt{\kappa^4-6\kappa^2+1}}{2\kappa} & \textit{for} & \kappa \geq 3+\sqrt{8} \end{array}$$

**Proof.**

See in appendix. ■

Results are illustrated Figure 2.

The price set by the wise monopolist is always lower than the marginal cost to broadcast one minute of TV on mobile. This is possible and profitable because the monopolist consider simultaneously the two sides of the market: The cost is compensated with the revenues both raised from customers and advertisers. The price paid by customers of the

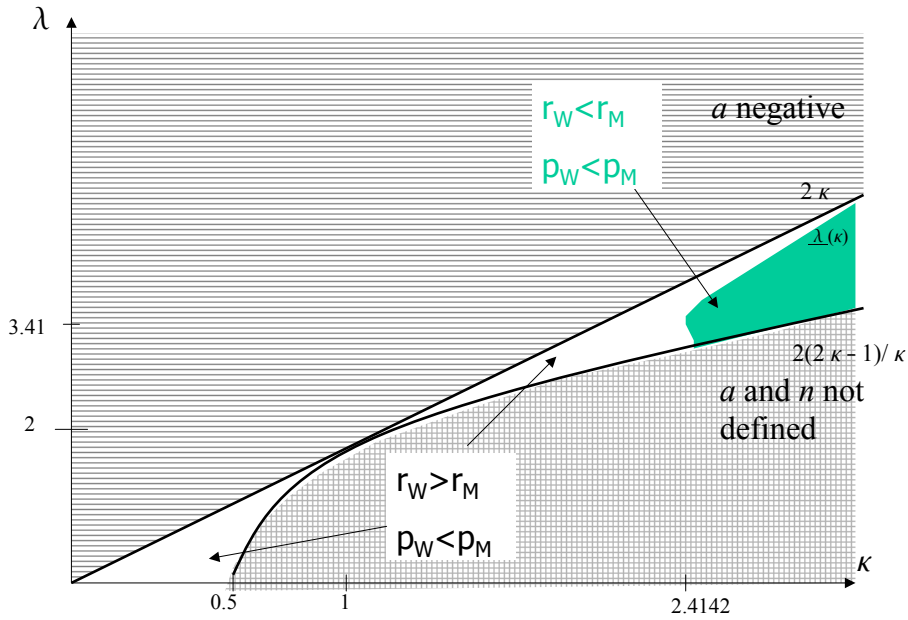


Figure 2: Comparison of prices

wise monopolist is lower than the one of the myopic monopolist because of the existence of the externality which advertisements exert on customers. This externality has to be taken into account by the one who produce the externality, i.e., the advertisers. The price of one minute of TV is lower to attract more customers, even if, as shown in proposition 2, it may not be sufficient, especially when both $\kappa$ and $\lambda$ are high, i.e. when both the externality of advertising and the sensitivity of advertising space demand are important.

The price paid by advertisers is higher in the case of a wise monopolist when both $\kappa$ and $\lambda$ are not too high. When the sensitivity of the advertising demand to audience is not too high, the advertisers are sanctioned for the externality they cause on customers. Indeed when $\kappa$ and $\lambda$ are small, it is more profitable for the wise monopolist to increase the number of minutes of TV broadcasted. Thus $p^W < p^M$ leading to a higher $n^W$. However, because of the existence of $\kappa$, the audience being higher, the demand for advertising space will be higher. The wise monopolist will increase the price paid by advertisers to limit the

increase of advertisements at equilibrium.

When $\kappa$ becomes higher, it is only when $\lambda$ is high enough that the wise monopolist sets a higher price of one minute of advertising to limit the increase of the advertising space demand. When the sensitivity to the audience increases, while the externality of advertising is not too high, it becomes profitable to lower the price of one minute of advertising in order to maintain a sufficient demand of advertising space (even if at equilibrium, the number of minutes of advertising is lower with a wise monopolist).

# 4 Normative Analysis

One interesting question is whether to know what would have proposed a benevolent planner, knowing the characteristics of the market. Indeed a benevolent agent internalizes both the negative externality of the advertising and the impact of the audience on advertising space demand, as a wise monopolist. However the objective function of a benevolent is to maximize the sum of consumer surplus and profits of the advertisers and of the mobile operator.

A benevolent planner will choose the amount of advertisements such that the social cost of advertising, i.e., the impact of the externality on customers utility $\lambda$, equals the social benefits, i.e., the marginal profit made by the advertisers. The benevolent agent chooses the amount of TV broadcasted in order to equalize the marginal cost of production to the marginal utility of customers plus the marginal profit realized by advertisers through the increase of audience.

The use of the two-part tariffs lead us to make an important remark: the wise monopolist achieves a higher level of Social Welfare than the myopic monopolist. This is due to the fact that the fixed part of the two-part tariffs are used by both monopolists to set the customers and the advertisers down to their reservation utility and profit. The wise monopolist, which knows both the impact of the advertising on audience and the impact of audience on advertising, i.e. which knows more than the myopic monopolist, achieves a higher profit than the myopic monopolist. Thus the Social Welfare is higher in the case of a wise monopolist. These arguments are independent of the shape of the utility function or of the different values of the parameters. It holds because of the structure of the prices,

i.e. the two-part tariffs. This result is summed up in the following proposition.

**Proposition 4** *The wise monopolist achieves a higher social welfare than the myopic monopolist.*

**Proof.**

See in appendix. ■

Another interesting question is whether to know if the wise and the myopic monopolist rations or not the broadcasting of TV so as the advertising on mobile. In order to explicit the solutions, we make the same assumptions for the analysis of the wise and myopic monopolists.

The Social Welfare is composed of the utility of the customers plus the benefits from advertisers reduced by the total costs to produce the advertisements and to broadcast TV on mobile. We do not have modeled till now the market of the advertisers, by considering only the demand for advertising space. Nonetheless it is easy to reconstitute the profit of the advertisers from the simple assumption on the form of the demand. Demand is

$$a_D = -\delta r + \kappa n$$

This demand can be issued from the maximization of the following profit function (defined up to a constant):

$$\pi_a = \underbrace{\frac{\kappa n a}{\delta} + \overline{\pi}_a}_{\text{revenues}} - \underbrace{\frac{a^2}{2\delta}}_{\text{cost to produce a}} - \underbrace{ra}_{\text{cost to broadcast a}}$$

Then the social welfare is equal to

$$SW = \alpha I + u(n) + \alpha m n - \lambda a + \overline{\pi}_a + \frac{\kappa n a}{\delta} - cn - C - \frac{a^2}{2\delta}$$

The social planner chooses the quantities of TV and advertisements to broadcast in order to maximize the social welfare. The first order conditions on $a$ and $n$ then write:

$$\begin{cases} \frac{\delta SW}{\delta a} = 0 \quad \Leftrightarrow \quad -\lambda + \frac{\kappa n}{\delta} - \frac{a}{\delta} = 0 \\ \frac{\delta SW}{\delta n} = 0 \quad \Leftrightarrow \quad u'(n) + \alpha m - c + \frac{\kappa a}{\delta} = 0 \end{cases}$$

The quantity of advertisements to broadcast is such that the marginal social benefit, i.e., the marginal gain of the advertisers linked to the audience $\frac{\kappa n}{\delta}$, equals the marginal

social cost, including the cost to produce advertisements $a$, i.e., $\frac{a}{\delta}$ and the cost of the externality of advertising $\lambda$.

The quantity of TV broadcasted is defined in the same way: the social benefits composed of the marginal utility of customers $u'(n) + \alpha m$ and of the gain of profit of the advertisers $\frac{\kappa a}{\delta}$ equals the cost to broadcast one minute of TV on mobile $c$.

The following proposition gives explicit forms for the optimal number of minutes of TV and advertisements.

**Proposition 5** *The social planner chooses the following quantities of TV and advertisements, defined for* $\lambda \geq \frac{2\sqrt{2}\kappa - 1}{\kappa}$

$$\begin{cases} n^* = & \frac{\kappa\lambda + 1 - \sqrt{(1+\kappa\lambda)^2 - 8\kappa^2}}{4\kappa^2} \\ a^* = & \frac{-\kappa\lambda + 1 - \sqrt{(1+\kappa\lambda)^2 - 8\kappa^2}}{4\kappa} \end{cases}$$

*Both the wise and the myopic monopolist oversupply the broadcasting of TV when both the sensitivity to the audience and the externality of advertising are high. When this sensitivity is small, the wise monopolist undersupplies less the broadcasting of TV than the myopic monopolist:*

$$\begin{cases} \textit{For} \quad \kappa \geq 1 & n^M \geq n^W > n^* \\ \textit{For} \quad \frac{2+\sqrt{2}}{4} \leq \kappa < 1 & n^W > n^M \geq n^* \\ \textit{For} \quad \kappa < \frac{2+\sqrt{2}}{4} & n^* > n^W > n^M \end{cases}$$

**Proof.**

See in appendix. ■

Results are illustrated in Figure 3.

When the sensitivity of the advertising demand to audience is high as well as the externality of advertisements, i.e. when the market is clearly two-sided, the social planner broadcasts less TV than the wise monopolist, which broadcasts less TV than the myopic monopolist.

When there is a low interaction between the two sides of the market, i.e. when both $\kappa$ and $\lambda$ are low, the optimal level of TV broadcasted is higher. The wise monopolist is again closer to the optimal solution.

Finally, for intermediate levels of $\kappa$ and $\lambda$, there is overproduction of TV. The wise monopolist is the one which produces the more.

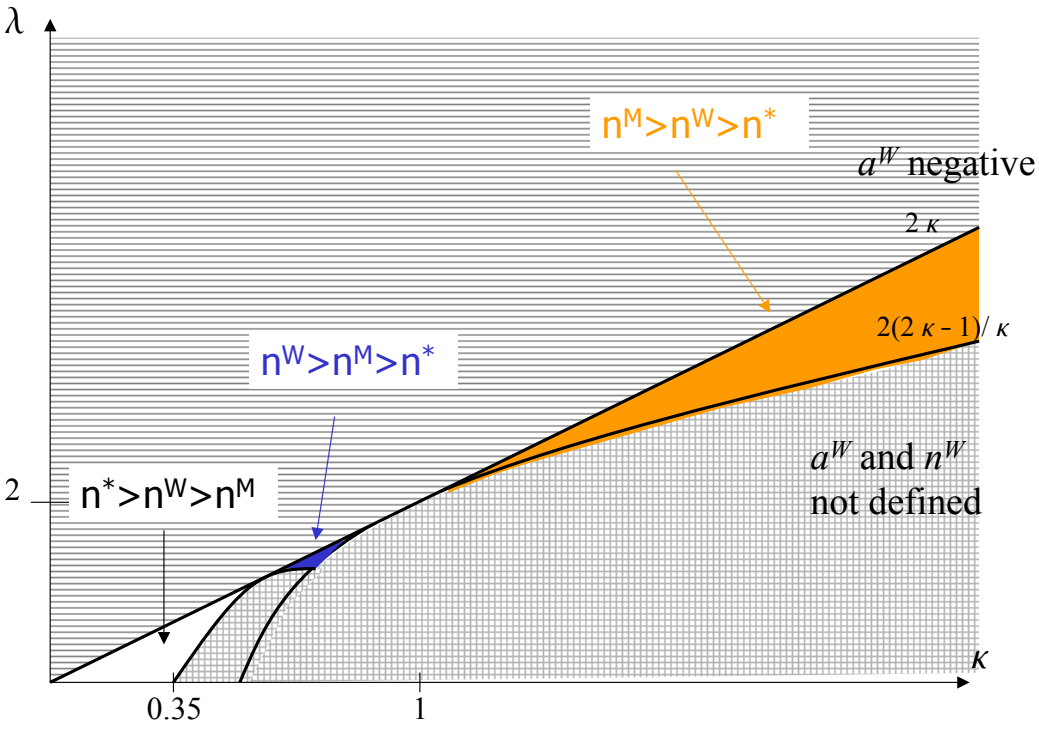


Figure 3: Comparison of minutes of TV broadcasted

In the following proposition, we compare the levels of advertisements proposed.

**Proposition 6** *Both the wise and the myopic monopolists oversupply advertisements when the sensitivity to audience and the externality of advertising are high. Generally the myopic monopolist oversupplies more and more often than the wise monopolist:*

$$\begin{cases} \textit{For} \quad \kappa > \frac{2+\sqrt{2}}{4} & & a^M \geq a^W > a^* \\ \textit{For} \quad \frac{\sqrt{6}-\sqrt{2}}{2} < \kappa < \frac{2+\sqrt{2}}{4} & & a^M > a^* > a^W \\ \textit{For} \quad \kappa \leq \frac{\sqrt{6}-\sqrt{2}}{2} & \textit{and} \quad \lambda < \frac{2\kappa^3}{1-\kappa^2} & a^* > a^W > a^M \\ & \frac{\kappa(1+\kappa^2)}{1-\kappa^2} < \lambda < \frac{2\kappa^3}{1-\kappa^2} & a^* > a^M > a^W \\ & \lambda > \frac{\kappa(1+\kappa^2)}{1-\kappa^2} & a^M > a^* > a^W \end{cases}$$

**Proof.**

See in appendix. ■

Results are illustrated in Figure 4.

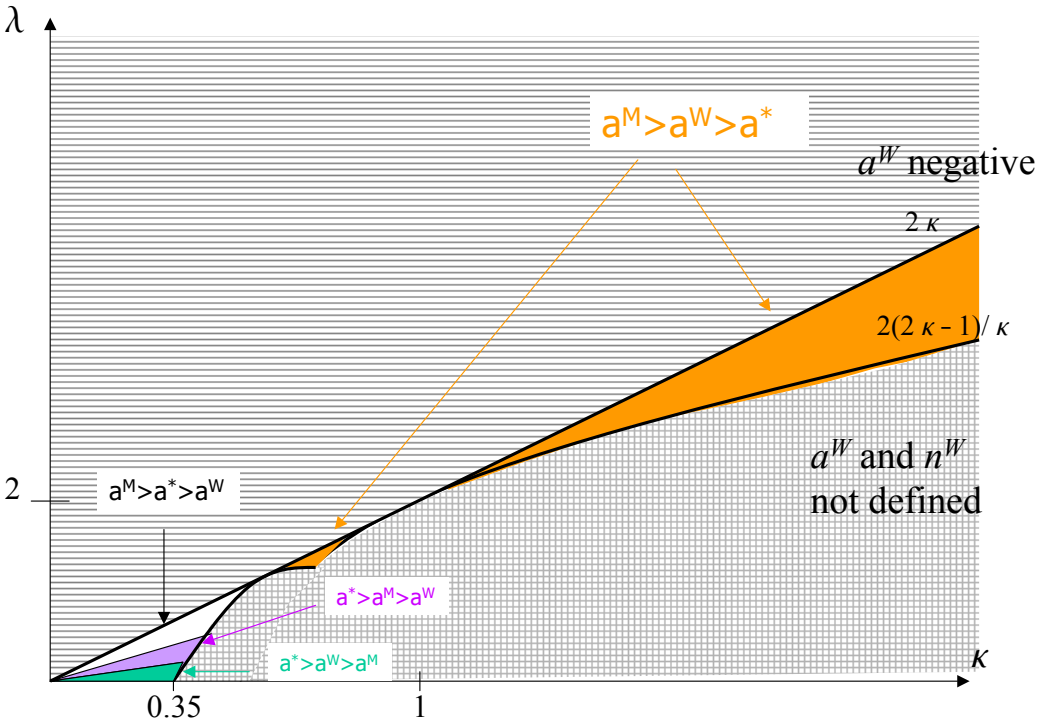


Figure 4: Comparison of minutes of advertisements broadcasted

When the interaction between the two sides of the market is very important, there is overbroacasting of advertisements by the two structures, wise and myopic monopolist.

When the interaction is less important, the level of the externality of advertising is important: when the externality is low, the social planner proposes more advertisements than both the wise and the myopic monopolist. When $\lambda$ is very low, the wise monopolist is closer to the optimal solution. On the contrary for an intermediate value of $\lambda$, the myopic monopolist is closer. When the externality becomes higher, the myopic monopolist overproduces advertisements, while the wise monopolist underproduces.

# 5 Conclusion

Considering the mobile TV market as a two-sided market improves the profits of the mobile operator as well as the economic welfare of the Society. This result holds for different specification of demand or utility functions and relies on the use of two-part tariff to implement the equilibrium. One natural extension of the model is to consider competition among the mobile operators. Results should be a convergence to the social optimum.

# A Proof of proposition 1: Existence and positiveness of the solutions

## A.1 Existence

The solutions of the program for the wise monopolist exist provided that

$$(\kappa(\lambda + d\alpha) + (c - m)\alpha)^2 - 4\alpha\kappa^2 \geq 0.$$

This expression has the same sign as

$$\kappa(\lambda + d\alpha) + (c - m)\alpha) - 2\sqrt{\alpha}\kappa,$$

since by definition $\alpha \geq 0$ and $(c - m) \geq 0$.[2]

- If $\lambda \geq \underline{\lambda} \equiv 2\sqrt{\alpha} - d\alpha$, then there is no other constraint on $\kappa$ needed to ensure existence of the solutions.
- If $\lambda < \underline{\lambda}$, then one needs

  $$\kappa \leq \overline{\kappa}(\lambda) \equiv \frac{(c - m)\alpha}{2\sqrt{\alpha} - (\lambda + d\alpha)},$$

  where $\overline{\kappa}(\lambda)$ is increasing in $\lambda$ at an increasing rate.

Besides, the second order conditions state

$$n^W \leq \frac{1}{\kappa^2\alpha}.$$

## A.2 Positiveness

When $a^W$ and $n^W$ exist, one have to check there positiveness. There is no ambiguity for $n^W$, while there are conditions on the parameters to ensure $a^W$ is positive.

$$a^W \geq 0$$

is equivalent to

$$\alpha(c - m) - \kappa(\lambda + d\alpha) - \sqrt{-4\alpha\kappa^2 + (\alpha(c - m) + \kappa(\lambda + d\alpha))^2} \geq 0.$$

[2] For c¡m, the profit is strictly increasing in $n$ and there is no interior solution. Indeed, if the opportunity to consume TV on mobility is higher than the marginal production cost, utility is increasing in $n$.

One necessary condition for this condition to hold is to ensure that $\kappa$ is not too high:

$$\alpha(c-m) - \kappa(\lambda + d\alpha) \geq 0,$$

which is equivalent to

$$\kappa \leq \tilde{\kappa}(\lambda) \equiv \frac{(c-m)\alpha}{\lambda + d\alpha},$$

where $\tilde{\kappa}(\lambda)$ is decreasing with $\lambda$ at an increasing rate.

Then, $a^W \geq 0$ if

$$\kappa \geq \underline{\kappa}(\lambda) \equiv (c-m)(\lambda + d\alpha),$$

where $\underline{\kappa}(\lambda)$ is increasing with $\lambda$.

The compatibility of the two last conditions is ensured for a certain level of parameter $\lambda$:

$$\underline{\kappa}(\lambda) \leq \tilde{\kappa}(\lambda),$$

only for $\lambda$ such that

$$\lambda^2 + 2d\alpha\lambda + d^2\alpha^2 - \alpha \leq 0.$$

The relevant solution for this second order equation is

$$\hat{\lambda} = \sqrt{\alpha} - d\alpha,$$

which we need to be positive, implying

$$d^2\alpha < 1.$$

The compatibility is then ensured for

$$\lambda \leq \hat{\lambda}.$$

### A.3 Compatibility between existence and positiveness

Notice that

$$\widehat{\lambda} < \underline{\lambda},$$

which means that existence imposes a third constraint on $\kappa$.

Notice as well that

$$\underline{\kappa}(\widehat{\lambda}) = \widetilde{\kappa}(\widehat{\lambda}) = \overline{\kappa}(\widehat{\lambda}) = (c-m)\sqrt{\alpha}.$$

Thus, as $\widetilde{\kappa}(.)$ is increasing and $\overline{\kappa}(.)$ is decreasing, it is immediate that for the range of acceptable parameter $\lambda \leq \widehat{\lambda}$:

- $\widetilde{\kappa}(\lambda) > \overline{\kappa}(\lambda)$
- $\widetilde{\kappa}(\lambda) > \underline{\kappa}(\lambda)$.

Notice then that $\underline{\kappa}(0) = (c-m)d\alpha$ and that $\overline{\kappa}(0) = \frac{(c-m)\alpha}{2\sqrt{\alpha}-d\alpha}$. Then

$$\underline{\kappa}(0) \leq \overline{\kappa}(0)$$

if and only if

$$-\alpha d^2 + 2\sqrt{\alpha}d - 1 \leq 0$$

which is always true for $d > 0$ and $\alpha > 0$. Thus

$$\underline{\kappa}(0) < \overline{\kappa}(0),$$

which means, as $\underline{\kappa}(.)$ is linear and $\overline{\kappa}(.)$ is increasing that

$$\underline{\kappa}(.) \leq \overline{\kappa}(.).$$

As a conclusion, solutions $a^W$ and $n^W$ exist only if

$$\begin{cases} \lambda & \leq & \widehat{\lambda} \\ \underline{\kappa} & \leq & \kappa \leq \quad \overline{\kappa}. \end{cases}$$

# B Proof of Proposition 2

First constraint is the existence of both $n^W$ and $a^W$ which simplifies to

$$\lambda > \frac{2(2\kappa - 1)}{\kappa}$$

for $\alpha = 1$. Second constraint is to be sure that the equilibrium does give positive solutions. There is no problem with the number of minutes of TV broadcasted, however the equilibrium advertising space is positive only if

$$\lambda \leq 2\kappa$$

after simplifications. The solution of the myopic problem is

$$n^M = 1$$

and the wise monopolist solution simplifies to

$$n^W = \frac{2 + \kappa\lambda - \sqrt{-16\kappa^2 + 4 + 4\kappa\lambda + \kappa^2\lambda^2}}{4\kappa^2}$$

The computation of the difference $n^W - n^M$ leads to the following condition:

$$n^W > n^M \quad \text{if and only if} \quad \lambda < \tfrac{2}{\kappa}$$

Adding the existence conditions, the comparison leads to the following condition:

$$n^W > n^M \quad \text{if and only if} \quad \kappa < 1$$

The computation to the difference $a^W - a^M$ leads to the following expression

$$\frac{1}{4\kappa}(1 - \frac{\kappa\lambda}{2} - 2\kappa^2 - \sqrt{(1 + \frac{\kappa\lambda}{2})^2 - 4\kappa^2})$$

which is positive only if

$$\lambda(-1 + \kappa^2) \geq 2\kappa^3$$

Thus this condition is never satisfied for $\kappa > 1$, in this case $a^W < a^M$. For $\kappa \leq 1$, we have then

$$a^W \geq a^M \quad \text{if and only if} \quad \lambda \leq \tfrac{2\kappa^3}{1-\kappa^2} \quad \text{with} \quad \kappa < \tfrac{\sqrt{5}-1}{2}$$

# C Proof of Proposition 3

Firstly, the constraints on existence and positivity of the solutions have to be taken into account. For this reason,

$$\lambda > \frac{2(2\kappa - 1)}{\kappa}$$

for $\alpha = 1$ and

$$\lambda \leq 2\kappa$$

. The equilibrium price in the case of a myopic monopolist is just the marginal cost to broadcast one minute of TV on the mobile. After computations, the equilibrium price in the case of the wise monopolist is

$$p^W = m - \kappa\lambda + \frac{2\kappa^2}{1 + \frac{\kappa\lambda}{2} - \sqrt{(1 + \frac{\kappa\lambda}{2})^2 - 4\kappa^2}}$$

for $\alpha = 1$ and $\delta = \frac{1}{2}$. The computation of the difference of the two prices leads to the following expression, after assuming $c - m = 1$,

$$p^W - p^M = -1 - \kappa\lambda + \frac{2\kappa^2}{1 + \frac{\kappa\lambda}{2} - \sqrt{(1 + \frac{\kappa\lambda}{2})^2 - 4\kappa^2}}$$

This expression is equal to zero for the unique solution

$$\tilde{\kappa} = -\frac{\lambda}{2 + \lambda^2}$$

, which is negative. The derivative of the difference between the two prices is strictly negative. Thus the difference of prices is positive before $\tilde{\kappa}$ and negative after. As we only care about positive values for parameter $\kappa$ and $\lambda$, we can say that the difference $p^W - p^M$ is unambiguously negative for $\kappa > 0$ and $\lambda > 0$.

The equilibrium price of one minute of advertising is equal to $r^M = \kappa$ in the case of a myopic monopolist, while it is equal to $r^W = \frac{1+\frac{3}{2}\kappa\lambda - \sqrt{(1+\frac{\kappa\lambda}{2})^2 - 4\kappa^2}}{2\kappa}$ in the wise monopolist situation. The analysis of the difference leads to the following condition:

$$\left\{ \lambda > \underline{\lambda} \quad \text{is equivalent to} \quad r^W > r^M \right.$$

Where

$$\underline{\lambda} = \frac{-1 + 3\kappa^2 + \sqrt{\kappa^4 - 6\kappa^2 + 1}}{2\kappa}$$

and when $\underline{\lambda}$ is defined, i.e. for $\kappa \geq 3 + \sqrt{8}$.

# D Proof of Proposition 4

The myopic monopolist uses the fixed part of the two-part tariffs to set both the advertisers and the customers to their reservation utility/profit. Thus the social welfare at the equilibrium is equal to

$$SW^M = \alpha I + \overline{\pi}_a + \Pi^M$$

The wise monopolist chooses the fees exactly in the same purpose. Thus the social welfare achieved is thus:

$$SW^W = \alpha I + \overline{\pi}_a + \Pi^W$$

The profit of the wise monopolist is higher than the one of the myopic monopolist because the wise monopolist, as shown in section 3, does differently from the myopic monopolist. This means that even if the wise monopolist could replicate what the myopic monopolist did and thus reach the same level of profit, it is not optimal to do so. As $\Pi^M \leq \Pi^W$, then

$$SW^M \leq SW^W$$

.

# E Proof of Proposition 5

The social planner chooses the quantities of TV and advertisements to broadcast in order to maximize the social welfare which is equal to

$$SW = \alpha I + u(n) + \alpha mn - \lambda a + \overline{\pi}_a + \frac{\kappa na}{\delta} - cn - C - \frac{a^2}{2\delta}$$

The first order conditions on $a$ and $n$ write:

$$\begin{cases} \frac{\delta SW}{\delta a} = 0 & \Leftrightarrow \quad -\lambda + \frac{\kappa n}{\delta} - \frac{a}{\delta} = 0 \\ \frac{\delta SW}{\delta n} = 0 & \Leftrightarrow \quad u'(n) + \alpha m - c + \frac{\kappa a}{\delta} = 0 \end{cases}$$

After simplifying with $c - m = 1$, $\delta = \frac{1}{2}$, $\alpha = 1$ and assuming $u(n) = ln(n)$, we obtain

$$\begin{cases} n^* = & \frac{\kappa\lambda + 1 - \sqrt{(1+\kappa\lambda)^2 - 8\kappa^2}}{4\kappa^2} \\ a^* = & \frac{-\kappa\lambda + 1 - \sqrt{(1+\kappa\lambda)^2 - 8\kappa^2}}{4\kappa} \end{cases}$$

The two solutions are always positive, however they are defined only for

$$\lambda \geq \frac{2\sqrt{2}\kappa - 1}{\kappa}$$

Computing the difference $n^* - n^W$ leads to the following expression:

$$n^* - n^W = \frac{1}{4\kappa^2}(-1 - \sqrt{(1+\kappa\lambda)^2 - 8\kappa^2} + \sqrt{(1 + \frac{\kappa\lambda}{2})^2 - 4\kappa^2})$$

Let us denote $\Delta = (1+\kappa\lambda)^2 - 8\kappa^2$ and $\widetilde{\Delta} = (1+\frac{\kappa\lambda}{2})^2 - 4\kappa^2$. The difference is positive only if

$$(-1 - \sqrt{\Delta} + \sqrt{\widetilde{\Delta}}) \geq 0$$

We show that this is possible for $\kappa \leq \frac{2+\sqrt{2}}{4}$, when $\lambda \leq 2\kappa$, which corresponds to the definition of $a^W \geq 0$. Moreover, for $\kappa > 1$, the difference is always negative, i.e. $n^W > n^*$. As we already shown that for $\kappa > 1$, $n^M > n^W$, hence, $n^M > n^W > n^*$.

For $\kappa \leq 1$, we have to compute $(n^* - n^M)$, which writes:

$$\frac{\kappa\lambda + 1 - 4\kappa^2 - \sqrt{\Delta}}{4\kappa^2}$$

It is positive when the following condition is satisfied, for $\kappa \leq \frac{2+\sqrt{2}}{4}$

$$2\kappa \geq \lambda$$

which is compatible with the definition of $n^*$ since

$$2\kappa > \frac{(2\sqrt{2}\kappa - 1)}{\kappa}$$

thus there is no problem with the existence condition. To sum up:

| | | |
|---|---|---|
| For | $\kappa \geq 1$ | $n^M \geq n^W > n^*$ |
| For | $\frac{2+\sqrt{2}}{4} \leq \kappa < 1$ | $n^W > n^M > n^*$ |
| For | $\kappa < \frac{2+\sqrt{2}}{4}$ | $n^* > n^W > n^M$ |

.

# F Proof of Proposition 6

Using the same notations as for the proof of proposition 5, the computation of the difference $a^* - a^W$ leads to the following expression

$$\frac{1}{4\kappa}(-\frac{\kappa\lambda}{2} - \sqrt{\Delta} + \sqrt{\widetilde{\Delta}})$$

which can be shown to be negative for $\kappa \geq \frac{2+\sqrt{2}}{4}$ and positive when $\kappa < \frac{2+\sqrt{2}}{4}$ for $\lambda \leq 2\kappa$. For $\kappa \geq 1 > \frac{2+\sqrt{2}}{4}$, we have shown that $a^M > a^W$, thus, for $\kappa \geq \frac{2+\sqrt{2}}{4}$,

$$a^M \geq a^W > a^*$$

For $\kappa < 1$, we have to compute $a^* - a^M$, which writes:

$$\frac{1}{4\kappa}(-\frac{\kappa\lambda}{2} - \sqrt{\Delta} + \sqrt{\widetilde{\Delta}})$$

which is positive only if $\lambda \leq \frac{\kappa(1+\kappa^2)}{1-\kappa^2}$. Besides, we know that for $\lambda < \frac{2\kappa^3}{1-\kappa^2}$, $a^W > a^M$. To summarize, we do identify 4 areas:

$$\begin{cases} \text{For} \quad \kappa > \frac{2+\sqrt{2}}{4} & & & a^M \geq a^W > a^* \\ \text{For} \quad \frac{\sqrt{6}-\sqrt{2}}{2} < \kappa < \frac{2+\sqrt{2}}{4} & & & a^M > a^* \geq a^W \\ \text{For} \quad \kappa \leq \frac{\sqrt{6}-\sqrt{2}}{2} & \text{and} & \lambda < \frac{2\kappa^3}{1-\kappa^2} & a^* \geq a^W > a^M \\ & & \frac{\kappa(1+\kappa^2)}{1-\kappa^2} < \lambda < \frac{2\kappa^3}{1-\kappa^2} & a^* \geq a^M > a^W \\ & & \lambda > \frac{\kappa(1+\kappa^2)}{1-\kappa^2} & a^M \geq a^* > a^W \end{cases}$$

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