# Sequential RAIM: Theory and Application to Civil Aviation Needs

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## Biography

**Abdelrazak Younes** works as a Ph.D. **candidate** at the Ecole Nationale de l'Aviation Civile (ENAC, France). As part of his Ph.D. thesis at the CNS Research Laboratory of the ENAC, he is now working on the application of sequential algorithms to GNSS integrity monitoring and to GNSS/INS hybridization techniques.

**Bacem Bakhache** obtained the engineer degree in electrical and electronics from the lebanese university, faculty of engineering, Tripoli, in 1996. He received the D.E.A. (Diplôme d'Etude Approfondie) from the Institut National des Sciences Appliquées (INSA) of Rennes, France. He became a Ph.D. candidate at the University of Technology of Troyes (UTT) in October 1997. His current interests is detection of abrupt changes in signals and systems.

**Igor Nikiforov** received the Ph.D. in automatic control from the Institute of Control Sciences, Moscow, in 1981. He joined the UTT in 1995, where he is currently Professor. His scientific interest is statistical decision theory.

**Abdelahad Benhallam** obtained his Ph.D. in communications from the Institut National Polytechnique de Toulouse in 1988. His areas of research include satellite communications, radionavigation and non-stationary signal processing. He is currently in charge of the signal processing department of the CNS Research Laboratory of the ENAC.

## Abstract

GPS by itself is unsatisfactory as a sole means of navigation for civil aviation users even with a conventional *Receiver Autonomous Integrity Monitoring* (RAIM). The principal limitation of RAIM is its availability. Indeed, even in differential mode (DGPS) or with the Selective Availability turned off, there are periods when the five satellites with sufficiently good geometry required for fault detection are not available. These periods can last several minutes and this is even worse with the fault detection and exclusion function. New sequential algorithms can make up the insufficiency of currently used snapshot methods.

## Introduction

Inside a *Receiver Autonomous Integrity Monitoring* (RAIM), the problem of the availability of the *Fault Detection* function ($FD$) and the *Fault Detection and Exclusion* function ($FDE$) is strongly related to the geometry of the visible satellites constellation. Due to a poor geometry some faults with a small Signal-to-Noise Ratio (SNR) in residuals can produce a significant impact on the position precision without being detected by conventional snapshot RAIM. Therefore, using algorithms adapted to GPS measurements and typical GPS faults, a sequential RAIM can detect such faults in one of the measured pseudoranges or pseudorange rates.

The aim of this paper is to provide the theoretical basis of the sequential RAIM and to show that the performances in terms of availability that can be achieved by using such algorithms will be better than any snapshot method (under Differential GPS condition or with the Selective Availability turned off).

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# 1 BACKGROUND

## 1.1 System performance requirements

For civil aviation applications, major problems of the existing systems consist of their lack of continuity and integrity. The Radio Technical Commission for Aeronautics (RTCA) has defined in [RTCA98] the Minimum Operational Performance Standards (MOPS) for airborn equipment using the GPS augmented by a Satellite Based Augmentation Systems (SBAS). In this paper we follow these definitions and requirements in order to compare the proposed FD and FDE algorithms with the well-known snapshot approaches.

## 1.2 Integrity Monitoring

As stated in [RTCA98] SBAS *equipment shall have a FDE capability that utilizes redundant GPS and* SBAS *ranging measurements to provide independant integrity monitoring.* So, RAIM will be used whenever SBAS Integrity is not available but it also can be used *as a backup to detect some local anomalies that cannot be accounted for in range corrections supplied by* a SBAS during precision approach phases of flight (quoting [G97]).

For a given phase of flight three paramaters can be defined, namely : the ***Horizontal Alarm Limit*** (HAL), i.e. the radius of a horizontal region centered at the true position which contains the indicated horizontal position with the probability $1 - 10^{-7}$ per flight hour ; the ***Vertical Alarm Limit*** (VAL) which defines a vertical segment which contains the indicated vertical position with the probability $1 - 10^{-7}$ per flight hour (only for precision approach) ; the ***Time-to-Alert*** ($T_A$) which is the delay allowed for FD and FDE algorithms to detect or detect and exclude a faulty satellite. As an example, for a Non-Precision-Approach phase of flight (NPA), we have $HAL = 0.3NM$ and $T_A = 10s$ (There is no VAL for NPA).

Here is RTCA's definitions of the availability of the detection and the exclusion functions :

*The detection function is defined to be available when the constellation of satellites provides a geometry for which the missed alert and false alert requirements can be met on all satellites being used for the applicable alert limit and time-to-alert. (...)*

*The exclusion function is defined to be available when the constellation of satellites provides a geometry for wich the FDE algorithm can meet the failed exclusion requirement (...). Therefore, exclusion must occur before the duration of a positioning failure exceeds the time-to-alert, and the detection function as defined above must be available after exclusion* (quoting [RTCA98]).

Also, in order to qualify an FD or an FDE algorithms, [RTCA98] specifies the false alarm rate ($p_{fa}$) to $10^{-5}/h$, the missed alarm rate ($p_{ma}$) to $10^{-3}$ and the failed exclusion rate ($p_{fe}$) to $10^{-3}$.

## 1.3 Notions of Protection Levels

The ***Horizontal Protection Level***$_{\textit{Fault Detection}}$ *($HPL_{FD}$)* is the radius of a circle in the horizontal plane, with its center being at the true position, which describes the horizontal region of protection for which the missed alert and false alert requirements are met for the chosen set of satellites when autonomous fault detection is used.

The ***Vertical Protection Level***$_{\textit{Fault Detection}}$ *($VPL_{FD}$)* is half the length of a segment on the vertical axis, with its center being at the true position, which describes the vertical region of protection for which the missed alert and false alert requirements are met for the chosen set of satellites when autonomous fault detection is used.

In other words, given a false alarm rate $p_{fa}$ and a missed detection rate $p_{md}$, a $FD$ algorithm should be able to detect within the time-to-alert any failure that will cause an error above the $HPL_{FD}$ or the $VPL_{FD}$.

Following the same idea, given a false alert rate $p_{fa}$, a false exclusion rate $p_{fe}$, and a missed alert rate $p_{ma}$, a $FDE$ algorithm should be able to detect and exclude within the time-to-alert any failure that will cause an error above the $HPL_{FDE}$ or the $VPL_{FDE}$.

We can also define protection levels that will ensure that the $FD$ algorithm remains able to detect a failure within the time-to-alert after the exclusion of a single satellite ; we call this function the $FD^*$ function. These more stringent corresponding protection levels are noted $HPL^*_{FD}$ and $VPL^*_{FD}$.

For a given phase of flight, the availability of the $FD$ or the $FDE$ fonctions will be determined by comparing the computed Protection Level(s) with the specified Alarm Limit(s) (c.f. 1.1). For non-

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precision approach phases of flight, the different availabilities are defined as :

- $FD$ function available if $HPL_{FD} < HAL$
- $FDE$ function available if $HPL_{FDE} < HAL$
- $FD^*$ function available if $HPL_{FD}^* < HAL$

# 2 GNSS Navigation and Failure impact

## 2.1 Position Solution

### 2.1.1 Notations and definitions

$n$ Number of Satellites

$t_j$ Discretized time

$PR$ Pseudorange measurement $n \times 1$ vector

$\widehat{PR}$ Pseudorange estimation $n \times 1$ vector

$Y$ Linearized measurement $n \times 1$ vector

$b \sim N(0, W)$ Measurement noise $n \times 1$ vector

$W = diag(\sigma_1^2, ..., \sigma_n^2)$ Measurement noise covariance $n \times n$ matrix

$P_{WGS}$ True receiver position

$P_{Sat}$ Satellites postions

$\widehat{P}_{WGS}$ Estimated receiver position

$X$ Linearized state $4 \times 1$ vector

$\hat{X}$ Least square estimation of the state $4 \times 1$ vector

$w$ Least square residuals $n \times 1$ vector

$H$ Direction cosine $n \times 4$ matrix

$G = (H^T W^{-1} H)^{-1} . H^T W^{-1}$ Projection $4 \times n$ matrix

$P = HG$ Idempotent $n \times n$ matrix

$Q = I_{n \times n} - P$ Residuals projection $n \times n$ matrix

### 2.1.2 System linearization

$$Y = PR(t_j) - \widehat{PR}(t_j) \quad (1)$$
$$X = P_{WGS}(t_j) - \widehat{P}_{WGS}(t_j) \quad (2)$$
$$Y = HX + b \quad (3)$$

### 2.1.3 Least square solution

The retained solution $\widehat{X}$ minimizes the impact of the measurement noise $b$ :

$$\widehat{X} = G.Y = G.H.X + G.b \quad (4)$$
$$w = Q.Y \quad (5)$$
$$\widehat{P}_{WGS}(t_j) = \widehat{P}_{WGS}(t_{j-1}) + \widehat{X} \quad (6)$$
$$\widehat{PR}(t_j) = \left\| \overrightarrow{\widehat{P}_{WGS}(t_j) P_{Sat}(t_j)} \right\| \quad (7)$$

It results from these equations that the position error $(G.b)$ is a four-dimensional centered Gaussian variable. It follows by projection of this error on the horizontal plane and on the vertical axis that the horizontal and vertical position errors are centered Gaussian variables which standard deviations are respectively :

$$\sigma_H = \sqrt{\sum_{k=1}^{n} (G_{1k}^2 + G_{2k}^2) . \sigma_k^2} \quad (8)$$
$$\sigma_V = \sqrt{\sum_{k=1}^{n} G_{3k}^2 . \sigma_k^2} \quad (9)$$

## 2.2 Failure Impact

### 2.2.1 Notations and definitions

Suppose that the current GPS constellation counts $n$ visible satellites at time $t$ and that a failure occurs at time $T_F$ on satellite number $k$ $(1 \leq k \leq n)$. This failure will be represented as an additional bias $B$ in pseudorange $PR_k$. We assume that this bias may be positive or negative : we will name *type-$2k$ failure* a positive failure on satellite $k$ and *type-$2k+1$ failure* a negative failure on satellite $k$. This convention is not usefull at all for **the snaphot RAIM** (cf. §3.1) but it is required for the **sequential RAIM** as we will see in §3.2. Here are the resulting notations :

hypothesis 0 No failure

hypothesis $l$ Type-$l$ failure

$l = 2k$ Positive failure on satellite $k$

$l = 2k-1$ Negative failure on satellite $k$

$(-1)^l B$ Magnitude of type-$l$ failure

$\Gamma_l = (0 \ldots 0\,(-1)^l B\, 0 \ldots 0)^T$ Resulting measurement error vector

$EH(l)$ Resulting horizontal position error

$EV(l)$ Resulting vertical position error

$X_0, Y_0, w_0$ Variables under normal conditions (no failure)

#### 2.2.2 Least Squares Solution in case of a type-$l$ failure

System modelling :

$$\begin{aligned} Y &= HX + b + \Gamma_l & (10)\\ &= Y_0 + \Gamma_l & (11)\end{aligned}$$

Least square solution :

$$\begin{aligned} w &= Q.Y & (12)\\ &= w_0 + Q.\Gamma_l & (13)\end{aligned}$$

Resulting position error :

$$\begin{aligned} \hat{X} &= G.Y & (14)\\ &= \hat{X}_0 + G.\Gamma_l & (15)\end{aligned}$$

The position error $G.\Gamma_l + Gb$ is a four-dimensional Gaussian variable with $G.\Gamma_l$ mean. By projection on the horizontal plane and on the vertical axis, we can deduce the following :

$$EH(l) \sim N\left(\sqrt{G_{1k}^2 + G_{2k}^2}.B\,,\, \sigma_H\right) \qquad (16)$$

$$EV(l) \sim N\left(|G_{3k}|\,B\,,\, \sigma_V\right) \qquad (17)$$

where $\sigma_H$ and $\sigma_V$ have been defined in §2.1.3. As these errors do not depend on the sign of the failure, we will note them $EH_k$ and $EV_k$ where $k$ is the faulty satellite in a type-$l$ failure ($l = 2k$ or $l = 2k-1$).

# 3 Integrity Monitoring

## 3.1 snapshot RAIM

#### 3.1.1 *FD* fonction

The decision criterion of the detection function is the weighted norm of the least squares residuals vector $w$ :

$$r_{FD} = \frac{\|w\|}{\sqrt{n-4}} \qquad (18)$$

Under normal conditions (i.e. no failure), $r_{FD}^2$ will follow a central $\chi^2$-distribution with $(n-4)$ degrees of freedom as it is a quadratic sum of centered gaussian variables. The detection threshold $h_{FD}$ is set so that $P(r_{FD} > h_{FD}) = pfd_{FD}$ where $pfd_{FD}$ is the required false detection probability of this algorithm ($pfd_{FD}$ only depends on the required false alert rate $p_{fa}$).

For a given type-$l$ failure of magnitude $B$ (affecting satellite $k$), the horizontal position error is a gaussian variable : $EH_k \sim N(\overline{EH_k}, \sigma_H)$ (c.f. §2.2.2) ; for a given detection threshold, the missed detection rate for a type-$l$ failure is $pmd_{FD} = P(r_{FD} < h_{FD})$ where $r_{FD}^2$ follows a non central $\chi^2$-distribution with $(n-4)$ degrees of freedom and centrality parameter $\gamma = \|Q\Gamma_l\|$. If the measurement noise vector is equal to zero, the value for $EH_k$ which would give $r_{FD} = h_{FD}$ is approximated by $ARP_k$ which is computed as :

$$ARP_k = h_{FD} \times HSlope_k \qquad (19)$$

$$HSlope_k = \sqrt{\frac{(G_{1k}^2 + G_{2k}^2)(n-4)}{Q_{kk}}} \qquad (20)$$

where $HSlope_k$ relates the horizontal position error to the criterion $r_{FD}$.

So, for a type-$l$ failure and without any measurement noise, the $HPL_{FD}$ should be set to $ARP_k$ ; but the existing noise will spread the horizontal positionning impact of this failure around $ARP_k$. Then, in order to detect this failure with a given probability of detection $(1 - pmd_{FD})$, we shall set $HPL_{FD}$ to $ARP_k + \alpha(pmd_{FD}) \times \sigma_H$. Now, if we consider all the possible failure directions, the horizontal protection level for fault detection is defined by :

$$HPL_{FD} = \max_{1 \le k \le n} \{ARP_k\} + \alpha(pmd_{FD}) \times \sigma_H$$

$\alpha(p) = \sqrt{2}\mathrm{erf}^{-1}(1-2p)$ is the threshold for which the probability that a variable with a normal distribution will exceed it is equal to $p$.

Analogously, the vertical protection level for fault detection is defined by :

$$VPL_{FD} = h_{FD} \times VSlope_{Max} \quad (21)$$
$$+\alpha(pmd_{FD}) \times \sigma_V \quad (22)$$
$$VSlope_{Max} = \max_{1\leq k\leq n} \{VSlope(k)\} \quad (23)$$
$$VSlope(k) = \frac{|G_{3k}|}{\sqrt{Q_{kk}}} \quad (24)$$

**Requirement appliance :** If we assume that the noise is uncorrelated between measurements, the false detection probability should be set to the false alert rate per measurement and the missed detection probability should be set to the missed alert rate per measurement :

$$pfd_{FD} = p_{fa} \times \Delta t \quad (25)$$
$$pmd_{FD} = (p_{ma})^{\Delta t/T_A} \quad (26)$$

where $\Delta t$ is the measurement period.

#### 3.1.2 $FDE$ function

Within a *snapshot RAIM*, the exclusion algorithm is started only when a failure is detected. In classical architectures, the exclusion function consists in computing the mean square residuals for each subset of $n-1$ satellites. Then, the subset of satellites which presents the lowest residuals is the one that include no faulty satellite (c.f. [PA88]). But this exclusion function definition does not take into account the notion of probability of false exclusion. In order to satisfy the requirement on false exclusion rate, the fault detection algorithm should be applied to the $n$ subsets of $n-1$ satellites. Within the exclusion algorithm, the detection threshold should be set so that its false detection probability is equal to the specified $p_{fe}$ (c.f. §1.2). The latter has not been demonstrated and have no theoretical basis but simulations have shown that the resulting false exclusion rate is always under the specified $p_{fe}$. It results that the protection levels are defined as :

$$HPL_{FDE} = \max\left\{HPL^j_{FD}\right\} \quad (27)$$
$$VPL_{FDE} = \max\left\{VPL^j_{FD}\right\} \quad (28)$$

where $HPL^j_{FD}$ is the $HPL_{FD}$ of the subset excluding satellite number $j$ but which is computed with different parameters than in §3.1.1.

**Requirement appliance :** If we assume that the $FDE$ function must perform as soon as a failure has been detected, it should require only one set of measurements (as opposed to $T_A/\Delta t$ measurements within the $FD$ function) ; then the missed detection probability should be set to the missed alert rate. The false detection probability of the $FD$ algorithm applied to each subset of $n-1$ satellites will be set to the specified false exclusion rate :

$$pfd_{FDE} = p_{fe} \quad (29)$$
$$pmd_{FDE} = p_{ma} \quad (30)$$

#### 3.1.3 $FD^*$ function

The $FD^*$ function is available if all of the $FD$ functions for the $n$ subset of $n-1$ satellites are available. Then the parameter settings are the same as in §3.1.1. The resulting protection levels are definded as :

$$HPL_{FDE} = \max\left\{HPL^j_{FD}\right\} \quad (31)$$
$$VPL_{FDE} = \max\left\{VPL^j_{FD}\right\} \quad (32)$$

where $HPL^j_{FD}$ is the $HPL_{FD}$ of the subset excluding satellite number $j$ (see §3.1.1).

## 3.2 Sequential RAIM

### 3.2.1 Introduction

It is known that the sequential algorithms show high performances in detection of stochastic signals and system changes [BN93]. Based on the theory of hypotheses testing, these algorithms would make up the insufficiency of the existing snapshot methods used in GPS integrity monitoring. Instead of testing the least square residuals of the current GPS position and velocity resolution (what is done by commonly used snapshot RAIM), the sequential method accumulates these residuals and detects mean changes occurrence in these residuals. The sequential approach has two advantages over the snapshot approach : the capacity to detect faults with low signal-to-noise ratio in residuals (with a small detection delay) ; the essentially higher efficiency in the fault isolation step (namely, the lower probability of false isolation). This is a very powerful approach because a drift-type error could be detected before the failure occurs (according to civil aviation specifications). Moreover, these sequential algorithms do have clear theoretical results in terms of missed detection rate, false isolation rate and detection and isolation delays that are very helpful while designing an Integrity Module with respect to the Civil Aviation specifications.

### 3.2.2 The Log-likelihood ratio

Let us consider a time varying system based on observations $Y_0,...,Y_t$. If the probability density function of these observations is equal to $f_1$, then this system is said to satisfy hypothesis $H_1$, if it is equal to $f_2$, then the system is said to satisfy hypothesis $H_2$. The log-likelihood ratio between $H_1$ and $H_2$is a function of time $t$ and is defined as $S_t(H_1, H_2) = \log \frac{f_1(Y_0,...,Y_t)}{f_2(Y_0,...,Y_t)}$. As a consequence, if $H_1$ is more likely than $H_2$ ($f_1(Y_0,...,Y_t) > f_2(Y_0,...,Y_t)$), the log-likelyhood ratio $S(H_1, H_2)$ is positive and will increase with time.

### 3.2.3 Detection and exclusion rules

In the case of a single satellite failure, for a known failure magnitude without a priori assumption on the direction of the failure, the detection and exclusion algorithm will test each possible direction of the failure (c.f. [N95]). If there are $n$ visible satellites, there will be $2n$ possible directions because the change could be either positive or negative for each satellite. Hence, there are a total of $2n+1$ hypotheses, including hypothesis 0 (c.f. §2.2.1). The sequential theory allows the recursive calculation of the log-likelihood ratio between different hypotheses.

Log-likelihood ratio at time $t$ between hypotheses $l$ and 0 ($l = 2k$ or $l = 2k-1$) given $S_0(l, 0) = 0$ :

$$S_t(l,0) = \left\{ S_{t-1}(l,0) + \frac{(-1)^l \widetilde{v} Y(k)}{\sigma_k^2 \sqrt{Q_{kk}}} - \frac{\widetilde{v}^2}{2\sigma_k^2} \right\}^+$$

where $\{x\}^+ = \max\{x, 0\}$ and $\widetilde{v}$ is a tuning parameter (Idealy $\widetilde{v} = B\sqrt{Q_{kk}}$ as stated in §3.2.4).

Log-likelihood ratio between hypotheses $l$ et $q$ at time $t$ :

$$S_t(l,q) = S_t(l,0) - S_t(q,0) \tag{33}$$

where $l$ and $q$ are elements of $1, ..., 2n$.

Intuitively, one might say that at least one log-likelihood ratio $S_t(l, 0)$ is positive if there is a failure and that hypothesis $l$ is correct if all of the log-likelihood ratios $S_t(l, q)$ between hypotheses $l$ and $q$ are positive (c.f. §3.2.2). If the hypothesis $l$ remains true, all $S_t(l, q)$ will increase with time because this hypothesis will be more likely than the others. If $h_D$ and $h_E$ are the detection and exclusion thresholds, the decision rules for detection and detection-exclusion are :

$$r_{FD}(t) = \max_{1\le l\le 2n} [S_t(l,0) - h_D] \tag{34}$$

$$r_{FDE}^l(t) = \min_{0\le q\ne l\le 2n} [S_t(l,q) - h_q] \tag{35}$$

where $h_q = h_D$ if $q = 0$ and $h_q = h_E$ if $q \neq 0$.

It results that the stopping times for detection and for detection and exclusion are defined as :

$$T_{FD} = \min\{t \ge 1 : r_{FD}(t) > 0\} \tag{36}$$

$$T_{FDE} = \min\left\{t \ge 1 : \max_{1\le l\le 2n} \left[r_{FDE}^l(t)\right] > 0\right\} \tag{37}$$

In the case of exclusion, the excluded satellite is the one for which the exclusion threshold is reached first. In other words, satellite $k$ is excluded if $r_{FDE}^{2k}(t) > 0$ or $r_{FDE}^{2k-1}(t) > 0$.

### 3.2.4 Parameter Settings

**The tuning parameter $\widetilde{v}$ :** For an a priori known failure magnitude $B$ in the pseudorange of satellite number $k$, this algorithm is optimal only if parameter $\widetilde{v}$ is equal to $B\sqrt{Q_{kk}}$. Because of the regression model, the optimal value for $\widetilde{v}$ will vary from satellite to satellite and should be set in accordance. But in general case, the magnitude failure $B$ is also unknown. To solve this problem, we propose to apply several CUSUM tests in parallel in order to cover a large range of magnitudes $[\widetilde{v}_{min}, \widetilde{v}_{max}]$. So, the fault detection algorithm will be composed of $L$ parallel CUSUMs with parameters $\widetilde{v}_1, \widetilde{v}_2, ...\widetilde{v}_L$ ranging from $\widetilde{v}_{min}$ to $\widetilde{v}_{max}$. As was shown in [N98], the choice of $L$ and parameters $\widetilde{v}_i$ can be made so as to minimize the asymptotic theoretical value of the detection delay, L is the smallest integer that verify :

$$L \ge \ln \frac{v_{max}}{v_{min}} \left\{ \ln \frac{e+1}{e-1} \right\}^{-1} \tag{38}$$

$$v_i = v_{min} \frac{(e+1)^i}{e(e-1)^{i-1}} \tag{39}$$

where $e$ is defined as the asymptotic efficiency of the CUSUM (the more $e$ is close to 1, the greater $L$ will be). It shall be noticed that these parameters are not designed to minimize the asymptotic theoretical value of the exclusion delay ; but monte-carlo simulations have shown that the resulting simulated mean isolation delay follows its theoretical value (c.f. [YNB97]).

**The thresholds :** According to sequential theory (c.f. [W47]), the detection threshold in a CUSUM should be set to $\sim \ln(n/pfd_{FD})$ as $pfd_{FD} \to 0$ so as to get a given false detection probability $pfd_{FD}$. If we use $L$ parallel CUSUMs, then the detection thresholds become :

$$h_D \sim \ln\left(\frac{n \times L}{pfd_{FD}}\right) \quad (40)$$

Unfortunately, there is no theoretical result concerning the threshold for exclusion but Monte-Carlo simulations results show that, if we set $h_{FDE} \times \ln(L/pfe_{FDE})$, where $pfe_{FDE}$ is the required false exclusion probability, the resulting false exclusion probability is well under the required one ; but we have to emphasize that this value have no theoretical basis and might lead to wrong results.

### 3.2.5 Distribution law of the delay of detection $\tau d_D$

It was shown by Wald that the detection delay for a type-$l$ failure $\tau_D^l$ follows (asymptotically) a normal distribution $N(\overline{\tau}_D^l, \sigma_D^l)$ (c.f. [W47]) ; after simplifications we get :

$$\overline{\tau}_D^l = \frac{h_D}{K_D^l} \;,\; \sigma_D^l = \frac{\sqrt{2h_D}}{K_D^l} \quad (41)$$

where $K_D^l = \frac{B^2}{2\sigma_k^2} Q_{kk}$ is the Kullback information relative to hypothesis $l$.

### 3.2.6 Distribution law of the delay of detection-exclusion $\tau_E$

Monte-Carlo simulations show that the detection and exclusion delay of a type-$l$ failure ($\tau_E^l$) follows a normal distribution $N(\overline{\tau}_E^l, \sigma_E^l)$. The only clear theoretical result concerning exclusion is on $\overline{\tau}_E^l$ (c.f. [W47]). The expression below for $\sigma_E^l$ has been obtained by a generalisation of the detection problem to the detection-exclusion problem. But simulations have shown that it is a good estimation of the standard deviation of variable $\tau_E^l$ :

$$\overline{\tau}_E^l = \max(\overline{\tau}_D^l, \frac{h_E}{K_E^l}) \;,\; \sigma_E^l = \max(\sigma_D^l, \frac{\sqrt{2h_E}}{Z_E^l}) \quad (42)$$

where :

$$K_E^l = B^2 Q_{kk} \min_{j \neq k} \left\{ \frac{\beta_{kj}}{\sigma_{kj}^2} \right\} \quad (43)$$

$$\sigma_{kj} = \sqrt{2}\left(\frac{1}{\sigma_k^2} + \frac{1}{\sigma_j^2}\right)^{-\frac{1}{2}} \quad (44)$$

$$\beta_{kj} = \min\left[1 \pm \frac{Q_{kj}}{\sqrt{Q_{kk} + Q_{jj}}}\right] \quad (45)$$

### 3.2.7 *FD* function

We have to ensure that, for a given type-$l$ failure, the detection delay $\tau_D^l$ will be lower than the specified time-to-alert $T_A$ (c.f. §1.2) with a suitable probability $1 - pmd$, where $pmd$ is the required missed detection probability. This consideration brings the following condition :

$$\overline{\tau}_E^l + \alpha(pmd) \times \sigma_E^l \leq T_A \quad (46)$$

Combining these equations we get the expression of the minimum detectable magnitude of a type-$l$ failure (i.e. $B \geq B_k^{md}$) :

$$B_k^{md} = \sigma_k \sqrt{\frac{2}{T_A Q_{kk}}\left(h_D + \alpha(pmd).\sqrt{2h_D}\right)} \quad (47)$$

The horizontal and vertical components of this failure magnitude leads to the expressions of the protection levels for detection :

$$HPL_{FD} = \max_{k=1..n}\left\{B_k^{md}\sqrt{G_{1k}^2 + G_{2k}^2}\right\} \quad (48)$$

$$VPL_{FD} = \max_{k=1..n}\left\{B_k^{md}|G_{3k}|\right\} \quad (49)$$

### 3.2.8 *FDE* function

We have to ensure that, for a given type-$l$ failure, the exclusion delay $\tau_D^l$ will be lower than the specified time-to-alert $T_A$ (c.f. §1.2) with a suitable probability $1 - pmd$, where $pmd$ is the missed detection probability. This consideration brings the following condition :

$$\overline{\tau}_I^l + \alpha(pmd) \times \sigma_I^l \leq T_A \quad (50)$$

Combining these equations the minimum excludable magnitude of a type-$l$ failure (i.e. $B \geq B_k^{me}$) is given by :

$$(B_k^{me})^2 = \frac{1}{T_A Q_{kk}} \max\left[2\sigma_k^2 h_D \,,\, \max_{j\neq k}\left(\frac{\sigma_{kj}^2}{\beta_{kj}}\right).h_E\right] + \frac{\alpha(pmd)}{T_A Q_{kk}} \max\left[2\sigma_k^2\sqrt{2h_D} \,,\, \max_{j\neq k}\left(\frac{\sigma_{kj}^2}{\beta_{kj}}\right).\sqrt{2h_E}\right]$$

The horizontal and vertical components of this failure magnitude leads to the expressions of the horizontal and vertical protection levels for detection and exclusion :

$$HPL_{FE} = \max_{l=1..2n+1} \left\{ B_k^{me} \sqrt{G_{1k}^2 + G_{2k}^2} \right\} \quad (51)$$

$$VPL_{FE} = \max_{l=1..2n+1} \{ B_k^{me} G_{3k} \} \quad (52)$$

### 3.2.9 $FD^*$ function

As in §3.1.3, the $FD^*$ function is available if all of the $FD$ functions for the $n$ subset of $n-1$ satellites are available. Then the parameter settings are the same as in §3.2.7. The resulting protection levels are definded as :

$$HPL_{FDE} = \max \left\{ HPL_{FD}^j \right\} \quad (53)$$

$$VPL_{FDE} = \max \left\{ VPL_{FD}^j \right\} \quad (54)$$

where $HPL_{FD}^j$ is the $HPL_{FD}$ of the subset excluding satellite number $j$ (see §3.2.7).

### 3.2.10 Requirement appliance

The specified false alert rate and false exclusion rate must be transposed on a per measurements basis, but the missed alert rate has to be considered on a time-to-alert basis :

$$pfd_{FD} = p_{fa} \times \Delta t \quad (55)$$

$$pfe_{FDE} = p_{fe} \times \frac{\Delta t}{T_A} \quad (56)$$

$$pmd = p_{ma} \quad (57)$$

# 4 snapshot RAIM versus sequential RAIM

## 4.1 Hypotheses for simulation

**GPS** As it has been annonced that the *Selective Availability* (S.A.) will be turned off in a few years, the chosen error model is based on Standard Positionning System (SPS) without S.A. characteristics. The applied pseudorange measurement noise is then a normally distributed noise with a $12.5m$ standard deviation. The GPS pseudoranges used in the simulations are generated by a GPS constellation simulator based on an almanac file (GPS week 973). At this time, 27 satellites were available. The degraded constellations are obtained by disabling the first satellite(s).

**Geostationary Satellites (Geo)** A GNSS system may include one or more additional geostationary satellites ; the INMARSAT satellites that will be used in future SBAS systems are named AOR-E (Atlantic ocean region east), AOR-W (Atlantic ocean region west), POR (Pacific ocean region) and IOR (Indian ocean region).

## 4.2 Criteria for comparison

We will use three criteria for comparison of the two RAIM methods, that is to say the $HPL_{FD}$, the $HPL_{FDE}$ and the $HPL_{FD}^*$. The considered phase of flight is the Non-Precision approach (NPA) which specifies $HAL = 555m$ and $T_A = 10s$ (c.f. §1.1).

## 4.3 Simulation results

The plots in figures 1, 2 and 3 represent the evolution of the $HPL_{FD}$, the $HPL_{FDE}$ and the $HPL_{FD}^*$ as fonctions of time for both sequential (dotted) and snapshot (solid) algorithms. Time ranges from 1998-09-05 0h00 to 1998-09-06 0h00 with a 2 minutes step. GNSS receiver position is located in Toulouse (France).

In table 1, the $FD$, $FDE$ and $FD^*$ functions availabilities are given for Snapshot RAIM (left value) and Sequential RAIM (right values) in regards to NPA requirements. These values are the percentage of time when $HPL \leq HAL$.

# Conclusion

The key problem concerning GPS is the RAIM function availability. $HPL_{FD}$ ploting on figure 3.1.1 shows that there is an improvement in fault detection availability by using Sequential RAIM rather than Snapshot RAIM, and the improvement is very substantial with regards to fault detection and exclusion (see fig. 2). This is even more substantial when the $FD$ has to remain available after a satellite exclusion (see fig. 3). These results show that the constellation geometry affects much less the Sequential RAIM than the snapshot RAIM and that the Sequential exclusion function is much more powerful. These improvements, as summarized in table 1, lead to the conclusion that a SBAS system should use of a Sequential RAIM in order to satisfy NPA requirements for example. As for precision approach where RAIM function needs to be used as a backup, Sequential RAIM will surely outperform Snapshot RAIM in detection of local small errors.

| **Snap. / Seq.** | $FD$ (%) | $FDE$ (%) | $FD^*$ (%) |
|---|---|---|---|
| 27GPS+3Geo | 100/100 | 100/100 | 100/100 |
| 27GPS+1Geo | 100/100 | 100/100 | 95.42/100 |
| 27GPS | 99.86/100 | 98.06/100 | 72.82/98.20 |
| 26GPS | 99.86/100 | 96.95/100 | 64.91/97.64 |
| 25GPS | 99.86/100 | 92.09/100 | 51.04/93.20 |

Table 1: NPA RAIM availability (snapshot/sequential)

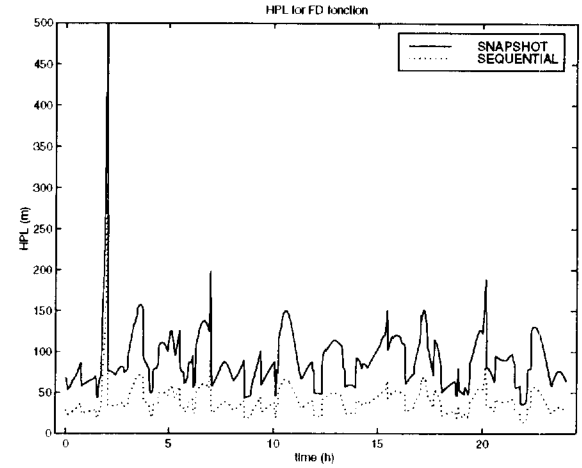


Figure 1: $HPL_{FD}$ for 27 GPS satellites

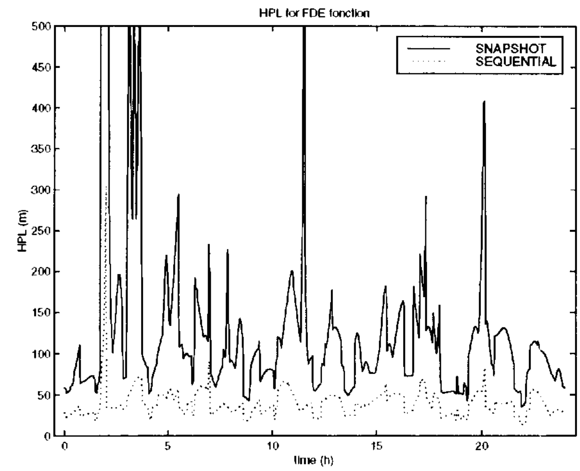


Figure 2: $HPL_{FDE}$ for 27 GPS satellites

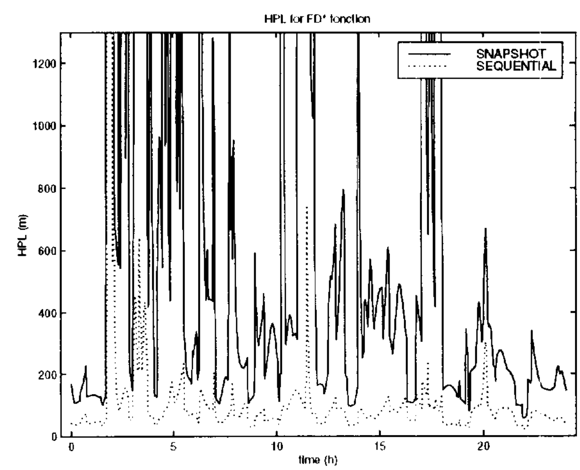


Figure 3: $HPL^*_{FD}$ for 27 GPS satellites

# References


[RTCA98] SC-159 ; *Minimum Operational Performance Standards for Global Positioning System/Wide Area Augmentation System Airborne Equipment* ; RTCA, Inc ; May 15, 1998.

[W47] Wald, A. ; *Sequential analysis* ; New York : John Wiley ; 1947.

[BN93] Basseville, M. and Nikiforov, I.V. ; *Detection of abrupt changes. Theory and applications* ; Prentice Hall, Information and System Sciences Series ; 1993.

[RTCA96] SC-159 ; *Minimum Operational Performance Standards for Global Positioning System/Wide Area Augmentation System airborne equipment* ; RTCA DOCUMENT NO. RTCA/DO-229 ; January 16, 1996.

[ICAO97] *ICAO All Weather Operations Panel. Draft Manual on RNP for Approach, Landing and Departure Operations* ; AWOP.

[N98] Nikiforov, I.V. *On a class of quadratic tests for detection of abrupt changes in signals and systems* ; American Control Conference, Philadelphia, Pennsylvania USA, pp. 751-755, v.1,1998.

[YNB97] Younes, A., Nikiforov, I.V. , Benhallam, A. 1997; *Improvements in GPS Integrity monitoring for non-precision sole means of navigation using hybrid GPS-INS* ; Proceedings of ION GPS-97, Kansas City ; September 15-18, 1997.

[N97] Nikiforov, I.V. ; *Two strategies in the problem of change detection and isolation, IEEE Transactions on Information Theory* ; vol.43, no.2, 770-776 ; March 1997.

[N96] Nikiforov, I.V. ; *New optimal approach to Global Positioning System/Differential Global Positioning System integrity monitoring* ; Journal of Guidance, Control, and Dynamics, 19, 5, pp. 1023-1033 ; Sep-Oct 1996.

[G97] Grover Brown, R. ; *Solution of the Two-failure GPS RAIM Problem Under Worst-Case Bias Conditions : Parity Space Approach* ; Navigation, Vol. 44, No. 4, Winter 1997-1998.

[S97] Strachan, V.F. ; *The Global Positioning System As A Sole Means of Navigation ; Civil Avionics '97 Conference* ; January 16-17, 1997.

[PA88] Parkinson, B.W. and Axelrad, P. ; *Autonomous GPS Integrity Monitoring using the pseudorange residual* ; Navigation, 35, no.2, pp 255-274 ; 1988.

[Ashtech89] Ashjaee, J., Lorentz, R., Sutherland, R., Dutilloy, J., Minagio, J., Abtahi, R., Eichner, J., Kosmalska, J., and Helkey, R. ; *New Developments and Ashtech M-XII* ; Proceedings of ION GPS-89, Colorado Springs, CO, 1989, pp. 195-98 ; September 27-29, 1989.

[N80] Nikiforov, I.V. ; *Modification and analysis of the cumulative sum procedure* ; Avtomatika i Telemekhanika (Automation and Remote Control) vol.41, no.9, part 1, p.1247-1252 ; 1980.

[N95] Nikiforov, I.V. ; *A generalized change detection problem, IEEE Transactions on Information Theory* ; IT-41, 1, 171-187 ; January 1, 1995.