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Improvements in GPS Integrity monitoring for non-precision sole means of navigation using hybrid GPS-INS.

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Biography

Abdelrazak Younes was born in 1971 in Toulouse, France. He graduated in 1995 as an electronics engineer at the *Ecole Nationale de l'Aviation Civile (ENAC)* in Toulouse. He is specialized in signal processing and in radionavigation electronics. He became a Ph.D. candidate at the *Laboratoire de Traitement du Signal et des Télécommunications (LTST)* of the ENAC in November 1995. He is working on the application of the GPS integrity monitoring techniques to aeronautics.

Igor Nikiforov received the M.S. degree in automatic control from the *Moscow Physical - Technical Institute* in 1974, and the Ph.D. in automatic control from the *Institute of Control Sciences, Moscow*, in 1981. He joined the *University of Technology of Troyes* in 1995, where he is currently Professor. His scientific interest is statistical decision theory. I. Nikiforov has authored and co-authored three books and a collective monograph. The last book (co-authored with M. Basseville) is *Detection of abrupt changes - Theory and applications* (New Jersey: Prentice Hall Information and System Sciences Series, 1993).

Abdelahad Benhallam obtained his Ph.D. in communications from the *Institut National Polytechnique de Toulouse* in 1988. His areas of research include satellite communications, radionavigation and non-stationary signal processing. He is currently responsible of the LTST activities, at the ENAC.

Abstract

GPS by itself is unsatisfactory as a sole means of navigation for civil aviation users. 'Receiver Autonomous Integrity Monitoring' (RAIM) has been proposed whereby a receiver makes use of redundant satellite information to check the integrity of the navigation solution. Two types of algorithms can provide RAIM function: the currently used snapshot methods only process the current measurements and the sequential ones process all past and current measurements. The principal limitation of snapshot RAIM is its availability. Indeed, for a Non-Precision Approach (NPA) phase of flight, there are periods when the five satellites (with sufficiently good geometry) required for fault detection are not available; these periods sometimes last more than 10 minutes. As well, the fault detection and exclusion function

can be unavailable for more than an hour. Use of sequential algorithm will with no doubt improve this situation but will probably not be sufficient to allow GPS to meet Civil Aviation requirements. As a solution, an attempt could be made to hybridize GPS with an Inertial Navigation System (INS). Two solutions for hybridization are considered. The simplest solution is to use the INS as a primary system of navigation and to update it periodically with the GPS solution. The GPS position must then be carefully monitored by a sequential algorithm that tests the least squares residuals of the GPS solution. The integrity of the INS must then be monitored by another algorithm. The other solution is to hybridize more finely the two systems by using a bank of Kalman filters that take into account all the measurements from GPS satellites and INS. Then a sequential algorithm will try to detect and isolate any faulty GPS channel or INS sensor.

I. System performance requirements

For the civil aviation application, major problems of the existing systems consist in their lack of accuracy and integrity and their vulnerability in presence of multipath or jamming. The Required Navigation Performance (RNP) concept has been defined by the International Civil Aviation Organization (ICAO) to specify performances to be respected by a universal civil navigation system. The proposed requirements on continuity and integrity for different phases of flight are given for the total system in table (1). For RNP 0.5 to 0.3/125 the outer containment limit is twice the 95 % accuracy value and assumes a probability of 10^{-5} per hour that the aircraft will exceed the containment limit.

RNP Type TSE 95% Lat/vert	Containment limit Lat/vert	Continuity	Integrity
0.5 NM	1 NM	$1 \cdot 10^{-4} / h$	$1 \cdot 10^{-5} / h$
0.3 NM	0.6 NM	$1 \cdot 10^{-4} / h$	$1 \cdot 10^{-5} / h$
0.3 NM / 125 ft	0.6 NM / 250 ft	$1 \cdot 10^{-4} / h$	$1 \cdot 10^{-5} / h$

Table (1): Total system performance requirements for approach, landing and departure operations (see [ICAO97]).

II. Loosely Coupled GPS / INS

An INS that has not been calibrated in flight by GPS has a specification of 2NM/h (2dRMS) [ARINC 704-6]. It would therefore not be capable to meet the NPA accuracy requirement during the integrity outage periods. On the other hand, calibration of INS with GPS dramatically increases its performance; but the problem is that a soft GPS failure can contaminate the integrated GPS/INS solution before the failure is detected. Because of this, if the GPS becomes unavailable due to a failure that could not be isolated, both the INS and GPS would become unavailable. By using sequential algorithms such as CUSUM (Cumulative SUM, see [1,2,3,4,5]) rather than conventional snapshot methods to check the integrity of GPS, a hybrid GPS/INS receiver can detect a soft bias range error of one of the satellites. If the faulty GPS channel is isolated then it should be eliminated from the navigation solution, if the channel isolation step is failed, then the integrity algorithm disable the INS updating with GPS.

A. hybridization scheme

We define the INS periodically calibrated by GPS as the primary navigation system. Three tuning parameters can be introduced :

- ΔT_c : period between calibrations of the INS under normal conditions
- $\tau_{Calibration}$: delay to apply to the GPS solution before it can be used for INS calibration
- τ_{INS} : delay since the last calibration before INS would become unusable for the current phase of flight. (when the position given by INS is outside the Horizontal Protection Level or HPL)

It should be noted that ΔT_c will essentially depend on computing facilities. Ideally, this parameter would be equal to zero but INS calibration needs a lot of calculation. Furthermore, the GPS solution must be delayed by $\tau_{Calibration}$ before being used to calibrate INS. Indeed we have to be sure that GPS based position and velocity are not infected by a failure before we can use them. Figure (1) shows how GPS and INS information will be mixed in our hybridization scheme. Several instants and delays involved in this scheme under the hypotheses of a failure are defined below.

- $T_{Failure}$: instant of the failure
- $T_{Detection}$: instant of the detection of the failure
- $\tau_{Detection}$: delay for detection
- $T_{Isolation}$: instant of the isolation of the faulty satellite
- $\tau_{Isolation}$: delay for isolation
- $T_{Calibration}^k$: instant of the calibration of INS with delayed GPS position and velocity before the failure
- $T_{Calibration}^{k+1}$: instant of the calibration of INS with delayed GPS position and velocity after the failure
- T_{INS} : instant when INS solution errors would become too large for being used for the current phase of flight.

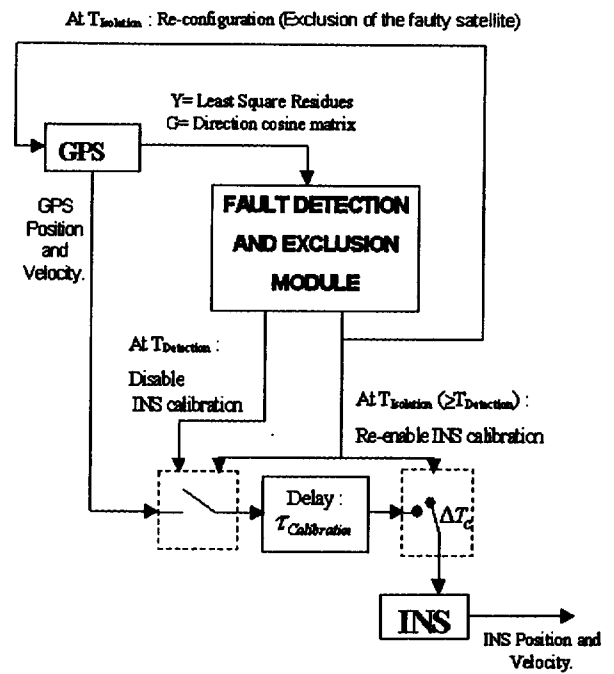


figure (1): hybridization scheme

Let us suppose that a failure occurs at time $T_{Failure}$ and is detected at time $T_{Detection}$. If the failure is also isolated i.e. $T_{Detection} = T_{Isolation}$ there is no need to disable the calibration since GPS has already been reconfigured. If not, INS calibration will be disabled at time $T_{Detection}$ until the faulty satellite is isolated at time $T_{Isolation}$. Then, in order to be sure that a new failure will not appear, the next calibration can only be made after the necessary delay $\tau_{Calibration}$.

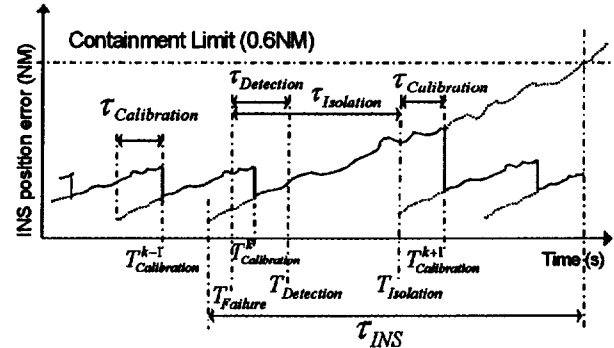


figure (2): INS error position in case of a failure

1. Temporal considerations

1. In order to avoid contamination of the INS solution, the GPS delayed solution used at time $T_{Calibration}^k$ to calibrate the INS must be taken before the instant failure. This gives the condition:

$$T_{Calibration}^k - \tau_{Calibration} < T_{Failure} \quad (1)$$

2. At the instant of detection $T_{Detection}$, INS calibration must be disabled until the faulty satellite is isolated. Then, after the necessary delay $\tau_{Calibration}$, a new calibration can take place:

$$T_{Detection} - \Delta T_c \leq T_{Calibration}^k < T_{Detection} \quad (2)$$

$$T_{Calibration}^{k+1} = T_{Isolation} + \tau_{Calibration} \quad (3)$$

3. INS solution errors increase with time, so we shall ensure that a new calibration happens before INS would become unusable for the current phase of flight. This gives the condition: $T_{Calibration}^{k+1} < T_{INS}$ (4)

4. There are some obvious relations between the different instants and delays:

$$\begin{aligned} T_{Detection} &= T_{Failure} + \tau_{Detection} \\ T_{Isolation} &= T_{Failure} + \tau_{Isolation} \\ T_{Calibration}^{k+1} &= T_{Isolation} + \tau_{Calibration} \\ &= T_{Failure} + \tau_{Isolation} + \tau_{Calibration} \\ T_{INS} &= T_{Calibration}^k - \tau_{Calibration} + \tau_{INS} \end{aligned} \quad (5)$$

Using equations (2), (3) and (5), we can show that, to satisfy inequalities (1) and (4), it is sufficient to have the following inequalities verified:

$$\tau_{Calibration} > \tau_{Detection} \quad (6)$$

$$2\tau_{Calibration} + \Delta T_C + \tau_{Isolation} - \tau_{Detection} < \tau_{INS} \quad (7)$$

As $\tau_{Detection}$ is necessarily positive, we can bound it by zero. Hence, to satisfy (7) it is sufficient to have:

$$2\tau_{Calibration} + \Delta T_C + \tau_{Isolation} < \tau_{INS} \quad (8)$$

2. Statistical consideration

Let us now suppose that $\tau_{Detection}$ and $\tau_{Isolation}$ are gaussian variables with means and variances:

$$\begin{cases} E(\tau_{Detection}) = \bar{\tau}_{Detection} \\ Var(\tau_{Detection}) = \sigma_{Detection}^2 \end{cases} \quad \begin{cases} E(\tau_{Isolation}) = \bar{\tau}_{Isolation} \\ Var(\tau_{Isolation}) = \sigma_{Isolation}^2 \end{cases} \quad (9)$$

Then, in accordance with the normal distribution, if we want to satisfy (8) with a given probability p_{11} , we shall choose:

$$\begin{aligned} \tau_{Calibration} &= \left\{ \tau : P(\tau_{Detection} \geq \tau) = 1 - p_{11} \right\} \\ &= \bar{\tau}_{Detection} + \alpha(p_{11}) \cdot \sigma_{Detection} \end{aligned} \quad (10)$$

where
$$\alpha(p) = \left\{ \alpha : \int_{\alpha}^{+\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx = 1 - p \right\} \quad (11)$$

$$= \sqrt{2} \times \text{erf}^{-1}(2p - 1)$$

For RNP 0.3, integrity must be equal to $1-10^{-5}$ /h or equivalently $1-2,78 \times 10^{-8}$ /s, so p_{11} should be fixed to this value. This gives the value $\alpha(p_{11}) = 5.43$.

Once $\tau_{Calibration}$ is fixed, inequality (13) still must be verified with a given probability p_{13} . Therefore, the performance of the INS must be good enough to have the following inequality verified:

$$\begin{aligned} \tau_{INS} &> 2\tau_{Calibration} + \Delta T_C + \left\{ \tau : P(\tau_{Isolation} \leq \tau) = p_{13} \right\} \\ &> 2\tau_{Calibration} + \Delta T_C + \bar{\tau}_{Isolation} + \alpha(p_{13}) \cdot \sigma_{Isolation} \end{aligned} \quad (12)$$

By replacing $\tau_{Calibration}$ with $\bar{\tau}_{Detection} + \alpha(p_{11}) \cdot \sigma_{Detection}$ we obtain the following requirement for τ_{INS} :

$$\begin{aligned} \tau_{INS} &> \Delta T_C + 2\bar{\tau}_{Detection} + 2\alpha(p_{11})\sigma_{Detection} \\ &\quad + \bar{\tau}_{Isolation} + \alpha(p_{13}) \cdot \sigma_{Isolation} \end{aligned} \quad (13)$$

So as to ensure integrity of the GPS solution, p_{13} should also be fixed to $1-10^{-5}$ /h.

3. Geometrical consideration:

Delays for detection and isolation strongly vary depending on the geometry of the GPS constellation and on which satellite is faulty. So, at a each instant, we must consider the satellite failure which gives the greater mean detection delays for detection and isolation. Let us introduce these values:

$\bar{\tau}_{Detection}^*$: worst case mean detection delay for a given geometry (at a given instant)

$\bar{\tau}_{Isolation}^*$: worst case mean isolation delay for a given geometry

$\sigma_{Detection}^*$: worst case standard deviation of the detection delay for a given geometry

$\sigma_{Isolation}^*$: worst case standard deviation of the isolation delay for a given geometry.

So, finally, we can define a minimum allowable value for τ_{INS} ($\tau_{INS} > \tau_{min}$):

$$\begin{aligned} \tau_{min} &= \Delta T_C + 2\bar{\tau}_{Detection}^* + 2\alpha(p_{11})\sigma_{Detection}^* \\ &\quad + \bar{\tau}_{Isolation}^* + \alpha(p_{13}) \cdot \sigma_{Isolation}^* \end{aligned} \quad (14)$$

It shall be noticed that the period of calibration ΔT_C is flexible and can be adjusted with the geometry.

4. Final requirement for INS performances

The aim of this paper is to define the minimal INS performances needed to always satisfy the inequality $\tau_{INS} > \tau_{min}$. Then, as GPS constellation has a 24 hours-periodicity, this hybridization scheme will function only if τ_{INS} is larger than $\tau_{min}^* = \sup_{24h} \{ \tau_{min}(t) \}$.

Of course, τ_{INS} will strongly depend on the accuracy and on the desired integrity of the GPS position and velocity that are used for calibration. This will result on a compromise :

- On one hand, the less the minimum detectable failure magnitude is, the more accurate will be the solution, and as a consequence the larger will be τ_{INS} . Indeed, the accuracy of the INS depends on its initial state, namely the GPS solution used for calibration.

- On the other hand, the less the minimum detectable magnitude failure is, the greater will be τ_{min}^* . Indeed, the detection and isolation delays are all the more larger that the magnitude of the failure is small.

B. Hypothesis for GPS

1. GPS regression model

The GPS navigation system follows the regression model with additive changes in case of failure :

$$\begin{aligned} w_i^p &= PR_{Measured} - PR_{Estimated} = G_i^p \cdot X_i + b_i^p + \Gamma_i^p \\ w_i^v &= PRR_{Measured} - PRR_{Estimated} = G_i^v \cdot V_i + b_i^v + \Gamma_i^v \end{aligned} \quad (15)$$

where if t is the current time and n the number of visible satellites, X_i^T is the user's position and clock bias, V_i^T is the user's velocity and clock drift, $PR_{Measured}$ and $PR_{Estimated}$ are the $n \times 1$ vectors of the pseudo-range measurements and estimates at time t , $PRR_{Measured}$ and $PRR_{Estimated}$ are the $n \times 1$ vector of the pseudo-range rate measurements and estimates at time t , G_i^p and G_i^v are $n \times 4$ direction-cosine matrices for position and velocity, b_i^p and b_i^v are signal perturbations and finally, Γ_i^p and Γ_i^v are additive changes corresponding to failure modes.

The least square residuals Y_i^p and Y_i^v are obtained by:

$$\begin{aligned} H^p &= I - G_i^p \cdot (G_i^{pT} G_i^p)^{-1} \cdot G_i^{pT} \\ Y_i^p &= H^p \cdot w_i^p \\ H^v &= I - G_i^v \cdot (G_i^{vT} G_i^v)^{-1} \cdot G_i^{vT} \\ Y_i^v &= H^v \cdot w_i^v \end{aligned} \quad (16)$$

The fault detection and the fault detection and exclusion algorithms will test Y_i^p to verify integrity of the GPS position and Y_i^v to verify integrity of the GPS velocity.

2. GPS error model

GPS without Selective Availability or Wide Area DGPS (WAAS or EGNOS):

In this model, the GPS range and range-rate measurements are normally distributed:

$$\begin{aligned} E(b_i^p) &= \theta = (\theta_1 \dots \theta_n)^T, & Var(b_i^p) &= \sigma^p I_n \\ E(b_i^v) &= 0, & Var(b_i^v) &= \sigma^v I_n \end{aligned}$$

with $\sigma^p = 4m$, θ_i ranging uniformly from $-1m$ to $1m$ [NAV88], and $\sigma^v = 0.01m/s$ [ION89].

As is described in [RTCA/DO-229], the following model is applied to simulate Selective Availability:

$$\begin{aligned} b_i^p &= gm_i^p + \theta \\ &= (gm_i^p(1) \dots gm_i^p(n))^T + (\theta_1 \dots \theta_n)^T \\ b_i^v &= gm_i^v \\ &= (gm_i^v(1) \dots gm_i^v(n))^T \end{aligned} \quad (17)$$

where $(gm_i^p(i) \ gm_i^v(i))$ is a second order Gauss-Markov process with an auto-correlation time of 118 seconds and

standard deviations of $\sigma^p = 23m$ for ranges and $\sigma^v = 0.28m/s$ for range-rates. θ is a random constant normally distributed with a mean of zero and a standard deviation of 23 m.

C. Hypotheses for INS

Following [ARINC 704-6], the INS have a specification of 2NM/h (2dRMS); but, in this scheme, we are more interested in the short term accuracy of the INS delivered position. Figure (3) shows the growth of the 95% and the 99.9% integrity limits for a generic inertial system. These curves are best case and only take account of velocity error. Furthermore, the curves start from zero whereas in practice, they will inherit the precision (and the integrity) of the last GPS calibration.

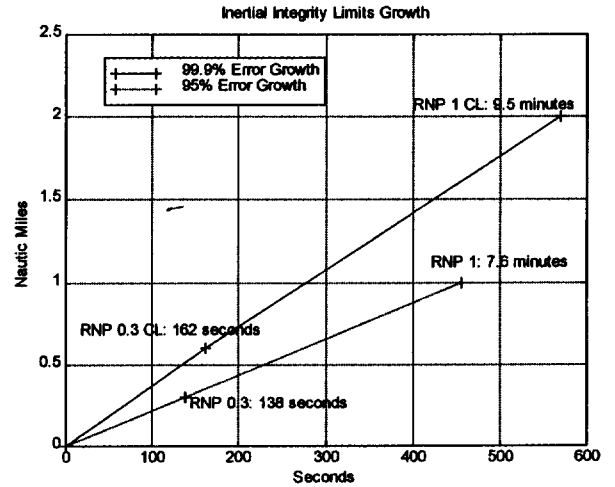


figure (3): Inertial Integrity Limits Growth

Actually, all we need to know in this hybridization scheme is τ_{INS} which is a function of the RNP. Let us consider the curve representing the 99.9 % error growth. Within the first 5 minutes, this curves is almost linear and have a rate of 6.8m/s. Obviously, the GPS should give relatively accurate position and velocity with very high integrity. For example, if we want to limit the influence of the errors of the GPS solution used for calibration to 2 % for position, this give a maximum allowable horizontal position error of 22m and a maximum allowable velocity error of 0.13m/s; this lead to a value of 155 seconds for τ_{INS} . Then the Fault Detection and Exclusion module will have to detect and isolate within 155 seconds any failure that will cause such position or velocity errors. In regards to the two GPS models (with and without S.A.) and to their variance, it is likely that GPS needs to be free of S.A. to allow the loosely coupled GPS/INS to function. We shall note also that, because of the error level affecting range measurements, a failure affecting a range measurement will be much harder to detect and isolate than one affecting a range-rate measurement. So, focus is on the detection and isolation of small bias affecting range measurements (called **range bias** in §II-E).

D. Fault Detection and Exclusion Module

1. CUSUM presentation

It is well known that sequential algorithms show high performances in detection of statistical characteristic changes of non-stationary [BN93]. Based on hypothesis test theory, these algorithms would make up the insufficiency of the existing snapshot methods used in GPS integrity monitoring. Indeed, the sequential approach has two advantages over the snapshot approach: the small detection delay for a given false alarm rate in the case of faults with a small magnitude-to-noise ratio and the essentially higher efficiency in the fault isolation step.

For a known failure magnitude without a priori assumption on the direction of the failure, the CuSum algorithm will test each possible direction of the failure [IEEE95, JGCD96]. If there are n visible satellites, there will be $2 \times n$ possible directions because the change could be either positive or negative for each satellite. Hence, there are a total of $2 \times n + 1$ hypotheses:

- hypothesis 0 : No failure
- hypothesis $2 \times i - 1$: Negative failure of satellite i
- hypothesis $2 \times i$: Positive failure of satellite i

For a given time of failure $T_{Failure}$ and a given failure magnitude v on satellite k , the log-likelihood ratio between hypotheses p and q is:

$$S_i(p, q) = S_i(p, 0) - S_i(q, 0)$$

$$S_i(p, 0) = \sum_{j=T_{Failure}}^i \left(\frac{(-1)^p \tilde{v} Y_j(i)}{\sigma^2 \sqrt{H_j(i, i)}} - \frac{\tilde{v}^2}{2\sigma^2} \right) \Bigg|_{p=2 \times i - 1 \text{ or } 2 \times i} \quad (18)$$

where p and q are elements of $\{1, \dots, 2 \times n + 1\}$, Y_i is either Y_i^p or Y_i^v , H_i is either H_i^p or H_i^v (depending on whether we test position or velocity) and $\tilde{v} = |v| \times \sqrt{H_i(k, k)}$ is the normalized magnitude.

Intuitively, one might say that at least one log-likelihood ratio $S_i(p, 0) \Big|_{p \in \{1, \dots, 2n\}}$ is positive if there is a failure and that hypothesis p is correct if all of the log-likelihood ratio $S_i(p, q) \Big|_{q \neq p}$ between hypotheses p and q are positive. In the case of a negative failure ($p = 2 \times k$), $S_i(2 \times k, q) \Big|_{q \neq 2 \times k}$ will obviously increase with time.

A recursive implementation of this notion is describe here:

$$S_0(p, 0) = 0 \Big|_{p=1, \dots, 2n}$$

$$S_i(p, 0) = \left\{ S_i(p, 0) + \frac{(-1)^p \tilde{v} Y_j(i)}{\sigma^2 \sqrt{H_j(i, i)}} - \frac{\tilde{v}^2}{2\sigma^2} \right\} \Bigg|_{\substack{p=2 \times i - 1 \text{ or } 2 \times i \\ p=1, \dots, 2n}}$$

$$S_i(p, q) = S_i(p, 0) - S_i(q, 0) \Big|_{p, q=1, \dots, 2n}$$

The stopping time for detection and isolation are defined as:

$$T_{Detection} = \min_{t>0} \left\{ t : \max_{p=1, \dots, 2n} [S_i(p, 0)] > h_{Detection} \right\} \quad (19)$$

$$T_{Isolation} = \min_{t>0} \left\{ t : \min_{0 \leq p \neq q \leq 2n} [S_i(p, q)] > h_{Detection} \right\}$$

where $\{x\}^+ = \max[x, 0]$ and \tilde{v} , $h_{Detection}$ and $h_{Isolation}$ are tuning parameters.

Theoretical results [IEEE95, JGCD96]:

- It shall be noticed here that γ have a theoretical lower bound: $\gamma < e^{h_{Detection}} \Leftrightarrow h_{Detection} > \ln(\gamma)$. (20)
- The mean delay for detection of a failure of magnitude v on satellite k has an asymptotic value:

$$\bar{\tau}_{Detection} \underset{h_{Detection} \rightarrow \infty}{\sim} \frac{h_{Detection}}{\rho_{fa}(k)} \quad (21)$$

$$\rho_{fa}(k) = \frac{v^2 H_i(k, k)}{2\sigma^2}$$

So the asymptotic worst case mean detection delay is:

$$\bar{\tau}_{Detection}^* \approx \frac{h_{Detection}}{\rho_{fa}^*} \quad (22)$$

$$\rho_{fa}^* = \min_{1 \leq k \leq n} (\rho_{fa}(k))$$

- The mean delay for isolation has an asymptotic value:

$$\bar{\tau}_{Isolation} \underset{h_{Isolation} \rightarrow \infty}{\sim} \frac{h_{Isolation}}{\rho(k)}$$

$$\rho(k) = \min(\rho_{fa}(k), \rho_{fi}(k)) \quad (23)$$

$$\rho_{fi}(k) = \frac{\tilde{v}^2}{\sigma^2} \min_{1 \leq i \neq k \leq n} \left(1 \pm \frac{H_i(i, k)}{\sqrt{H_i(i, i) \cdot H_i(k, k)}} \right)$$

So the asymptotic worst case mean isolation delay is:

$$\bar{\tau}_{Isolation}^* \approx \frac{h_{Isolation}}{\rho^*} \quad (24)$$

$$\rho^* = \min_{1 \leq k \leq n} (\rho(k))$$

Parameter settings:

The threshold for detection $h_{Detection}$ will be set to get a given level γ of false alarm. so $h_{Detection}$ should be set to $\ln(\gamma)$. In Civil Aviation application, a required level of false alarm is $0.002 / h = 5.10^{-7} / s$, then $h_{Detection}$ is set to 14.4.

Unfortunately, there is not simple theoretical result concerning the threshold for detection $h_{Isolation}$. This tuning parameter should be chosen in regards to practical results. We have made simulation with $h_{Isolation}$ set to 14.4, and we didn't notice any false isolation among 10000 trials.

For an a priori known failure magnitude v on satellite number k , this algorithm is optimal only if parameter \tilde{v} is equal to $|v| \times \sqrt{H_j(k, k)}$. So, because of the regression model, the optimal value for \tilde{v} will vary from satellite to satellite and should be fixed in consequence. But in general case, the magnitude failure v is also unknown. To solve this problem, an attempt could be made to use many CUSUM in parallel in order to cover a large range of magnitude $[\tilde{v}_{min}, \tilde{v}_{max}]$. So, the fault detection algorithm will be

composed of L parallel CUSUM with parameter $\tilde{v}_1, \tilde{v}_2, \dots, \tilde{v}_L$ ranging from \tilde{v}_{\min} to \tilde{v}_{\max} . The choice of L and parameters \tilde{v}_i can be made so as to minimize the asymptotic delay of detection:

$$L \geq \ln \frac{v_{\min}}{v_{\max}} \left\{ \ln \frac{c+1}{c-1} \right\}^{-1}$$

$$v_i = v_{\min} \frac{(c+1)^i}{c(c-1)^{i-1}} \quad (25)$$

$$c = (1-e)^{-\frac{1}{2}}$$

where e is defined as the asymptotic efficiency of the CUSUM (the more e is close to 1, the greater L will be).

Extensive 24h-simulations for failure magnitude v ranging from 3σ to 8σ have shown that optimal value for parameter \tilde{v} ranges from 0.8 to 16 in regard to the mean detection delay. For computational reason, we decided to limit the number of parallel CUSUM to five, the resulting parameters are: $v_{\min}=0.8, v_{\max}=16, e=0.9, L=5$,

$$\tilde{v}_i = 1.052; 2.028; 3.9, 5.712, 14.456$$

E. Simulations results

The GPS pseudo-ranges used in these simulations are generated by a GPS constellation simulator based on an almanac file (GPS week 856). At this time, 25 satellites were available. It shall be noticed that no reduced constellation have been considered but only the one given by the GPS simulator. So, results presented here are just for illustration purpose of the theory developed in this article.

1. GPS alone with S.A.

The CUSUM algorithm is only optimal for detection of change in gaussian variable with a priori known variance. So because of the time-correlated noise, the CUSUM algorithm is not well adapted to the model of GPS with S.A. because slowly varying noise can be interpreted as bias. But simulations shows that if we artificially presume a larger standard deviation than the actual one (50m rather than 33m), the CUSUM shows to be robust enough to detect failure that snapshot could not detect. As a consequence, by using sequential algorithm, the availability of the detection function can be brought to 100% (compared to 97% for snapshot RAIM). But because of the error level of this model and of the limitation of the INS, GPS must be free of S.A. before it can be used in this model (see II-C).

2. GPS alone without S.A.

In the following simulation results, the GPS model without SA is assumed. When a curve is plotted as a function of the time of failure $T_{Failure}$, this one ranges from 1997-06-29 0h00 to 1997-06-30 0h00 with a 2 minutes step. A Failure is modeled as a bias introduced on the generated pseudoranges. In many figures the term "trial" refer to a statistical consideration (a new noise b_k is used at each trial).

Distribution law of $\tau_{Detection}$:

Figure (4) shows the evolution of the worst case mean detection $\bar{\tau}_{Detection}^*$ and the asymptotic worst case mean detection $\bar{\tau}_{Detection}^{*A}$ as functions of $T_{Failure}$ for failure magnitudes $v = 12$ m.

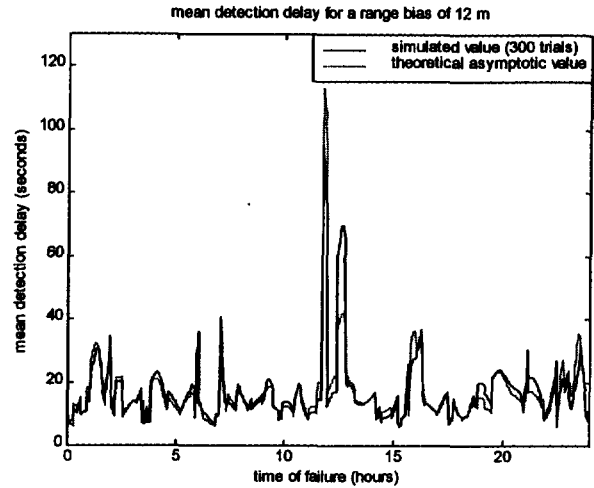


figure (4): Practical and asymptotical worst case mean detection delay for a range bias of 20 m

As one can see in this figure, the two curves are very close (same curves are obtained for $v = 16m, 20m, 24m, 28m$ and $32m$). We can conclude that the asymptotic worst case mean detection delay is a good criteria concerning the ability of the CUSUM algorithm to detect a failure with a given magnitude.

It shall be noted here that the pic-value at time 11h40 is common to all magnitude failure and corresponds to "constellation hole" as is shown in figure (5):

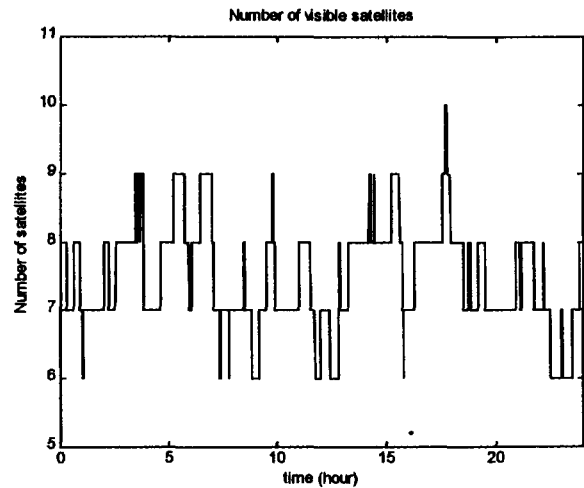


figure (5): Number of visible satellites upon 24h

In paragraph II-A-2, we made the assumption that the variables $\tau_{Detection}$ and $\tau_{Isolation}$ were normally distributed. In practice, this is "half true" because these variables are bounded by zeros. So, as we are more interested on the greater values of $\tau_{Detection}$, the calculation of $\sigma_{Detection}$ is based

on the upper part of the histogram ($\tau_{Detection} \geq \bar{\tau}_{Detection}$). Figure (6) shows an empirical Probability Density Function (PDF) of the worst case values of $\tau_{Detection}$ obtained at time $T_{Failure} = 11h40$ and for a failure magnitude $v=12m$. This curves is to be compared to the theoretical PDF of a normally distributed variable with same mean and variance ($\bar{\tau}_{Detection}, \sigma_{Detection}^2$). Despite the fact that the theoretical is a little bit shifted because of the asymmetric distribution of $\tau_{Detection}$, the two curves are quite close and confirm the assumption that $\tau_{Detection}$ is normally distributed.

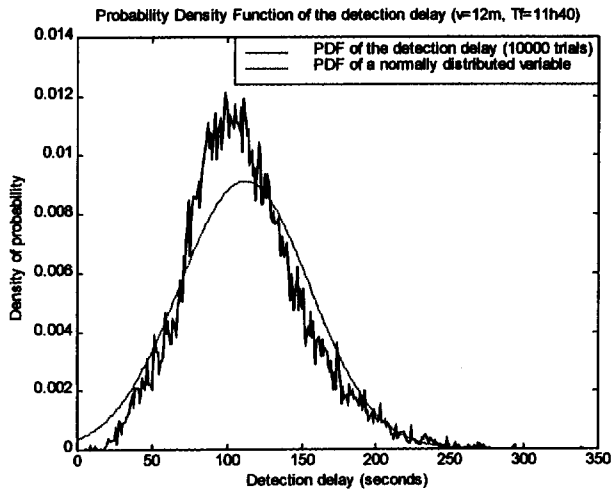


figure (6): Empirical and theoretical PDF of the worst case detection delay at time 11h40

Distribution law of $\tau_{Isolation}$:

Figure (7) shows the evolution of the worst case mean isolation $\bar{\tau}_{Isolation}^*$ and the asymptotic worst case mean isolation $\bar{\tau}_{Isolation}^{*A}$ as functions of $T_{Failure}$ for failure magnitudes $v = 32 m$.

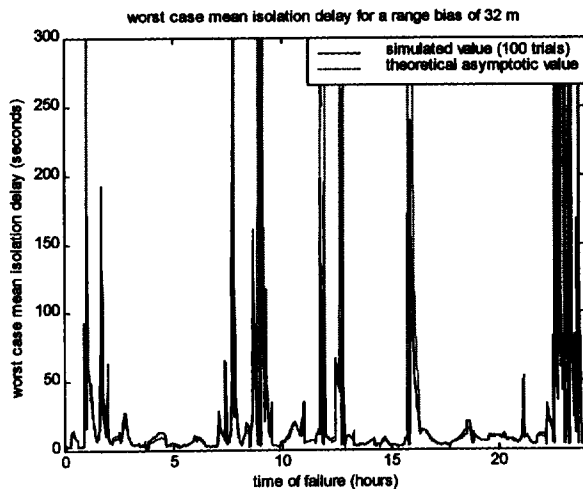


figure (7): Practical and asymptotical worst case mean isolation delay for a range bias of 32 m

Here again the empirical and the asymptotical curves are very close (delay larger than 300s are not plotted). So, the asymptotic worst case mean detection delay is a good

criteria concerning the ability of the CUSUM algorithm to isolate a failure with a given magnitude. Simulations show that $\tau_{Isolation}$ has, as for $\tau_{Detection}$, an asymmetric normal distribution.

An important conclusion of these curves is that, even if the worst case failure can always be detected, there are periods when it can not be isolated even for a failure magnitude as large as 32m. It shall be noted here that the asymptotic curve does reflect well this phenomena.

3. GPS with geostationary satellites

By using GPS alone the detection function is always available (more or less quickly), but there are periods when the isolation function is not available. To increase this availability we can use one or more geostationary satellites. Satellite Inmarsat AOR-E already deliver the ranging function for Euridis system (preliminary version of Egnos), and satellite Inmarsat IOR will come soon.

The following figures show that a minimum of two additional geostationary satellites is needed to obtain a 100% availability of the detection function:

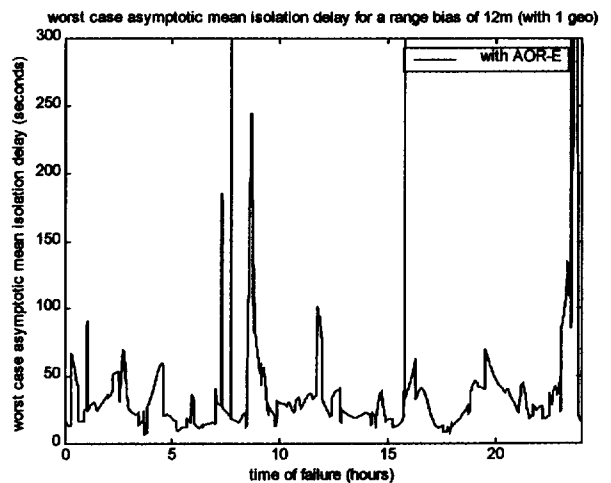


figure (8): Asymptotical worst case mean isolation delay for a range bias of 12 m and one geostationary satellite

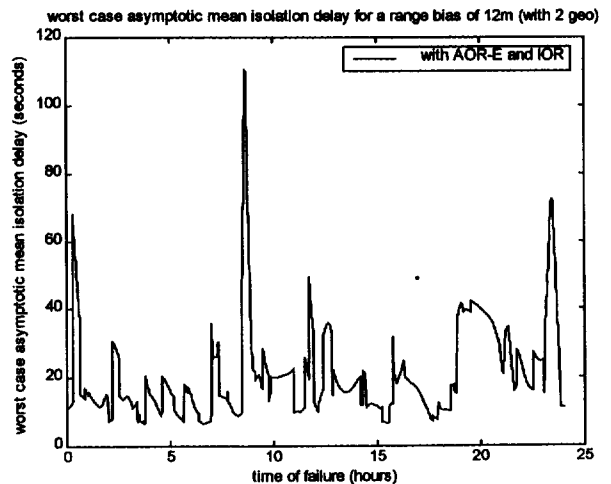


figure (9): Asymptotical worst case mean isolation delay for a range bias of 12 m and two geostationary satellites

4. Choice of the delay of calibration

It has been shown that the delay of calibration $\tau_{Calibration}$ should be greater than $\bar{\tau}_{Detection} + \alpha(p_{11}) \cdot \sigma_{Detection}$ (cf. §II-A-2) where $\alpha(p_{11}) = 5.43$ if $p_{11} = 1 - 10^{-5}/h$. The following figures give the minimum allowable value for $\tau_{Calibration}$ as a function of the time with and without additional geostationary satellites.

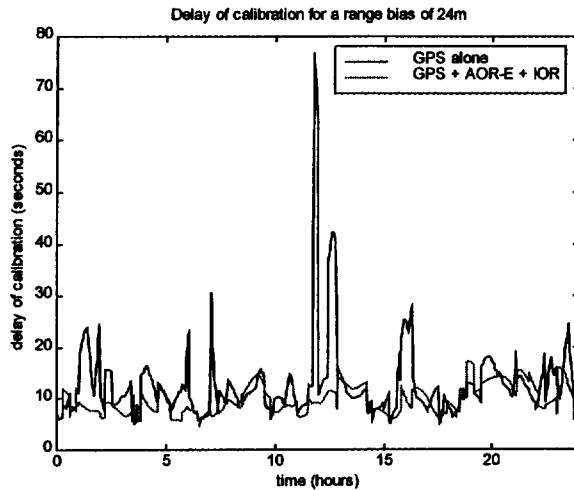


figure (10): 24 hours evolution of the delay of calibration.

5. Minimum allowable value for τ_{INS}

Following §II-A-4, the delay before INS become unusable must be larger than τ_{min}^* .

If we fix p_{11} and p_{13} to $1 - 10^{-5}/h$ and we suppose that ΔT_c is equal to zero (continuous calibration), we have:

Figure (8) and (9) in §II-E-2 show that a minimum of two additional geostationary satellites are needed in order to have reasonable value for the worst case isolation delay. Then, assuming that p_{11} and p_{13} correspond to $1 - 10^{-5}/h$ and that ΔT_c is equal to zero (continuous calibration), figure (11) gives the minimum allowable value for τ_{INS} for failure magnitudes ranging from 12m to 32m.

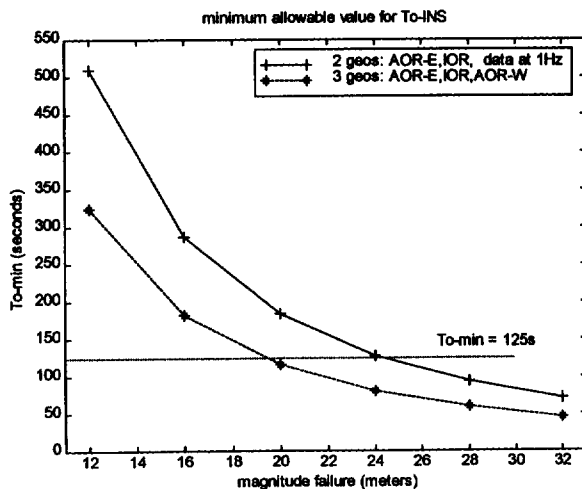


figure (11): minimum allowable value for τ_{INS}

In paragraph II-C, it has been shown that 155 seconds was a reasonable value for τ_{INS} assuming an horizontal position accuracy of 27 m. Then, if GPS solution is calculated every seconds (1Hz data rate) and if we fix ΔT_c to 30 seconds, the hybridization scheme will function with a minimum detectable failure magnitude of 24 meters if two additional geostationary satellites or with a minimum detectable failure magnitude of 20 meters if three additional geostationary satellites are available. In both case, as HDOP is always close to one, the resulting horizontal position error is always smaller than 27 meters.

F. Conclusion for Loosely Coupled GPS / INS

III. Tightly Coupled GPS / INS :

A. Hybridization Scheme

The second strategy of integrity monitoring is based on the tightly coupled GPS and INS. This means that a global Kalman filter is used to process all the measurements from GPS and INS together as what is done in AIME [AIME97] (see figure (12)). But unlike AIME, the measurements of GPS and INS are treated continuously together, hence, we can expect a very efficient integrity monitoring and the AIME is a very good application of this principle. But in this scheme, rather than Unfortunately, there are some difficulties in this case. First of all, we explain in brief these difficulties.

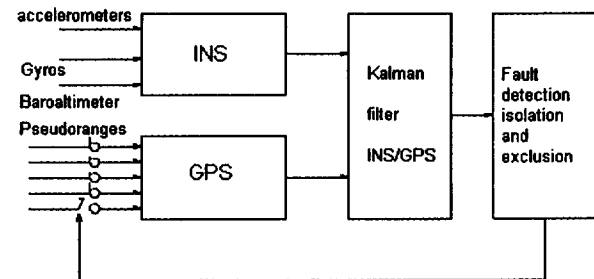


figure (12): Tightly coupled GPS/INS

B. State-space models

It is well known that the coupled GPS/INS navigation system can be reduced to the state-space model. We consider the state-space model with additive changes:

$$\begin{cases} X_{t+1} = \Phi(t+1, t)X_t + \xi_t \\ Y_t = H(t)X_t + v_t + \Gamma(t, t_0) \end{cases} \quad (26)$$

where ξ_t and v_t are two independent zero mean Gaussian white noises with covariance matrices $Q(t) \geq 0$ and $W(t) > 0$ respectively. The initial state X_0 is a Gaussian zero mean vector with a covariance matrix $P_0 > 0$. The matrices Φ , H , Q , W , P_0 are known. The faults are modeled as the additional pseudo-range biases

$$\Gamma(t, t_0) = \begin{cases} 0 & \text{if } t < t_0 \\ \Gamma(t - t_0) & \text{if } t \geq t_0 \end{cases} \quad (27)$$

in the measurement equation where t_0 is the time of failure. The likelihood function of this state-space model can be

computed by using the innovation sequence of the Kalman filter. Hence, first, we have to transform the initial data $(Y_i)_{i \geq 1}$ into the innovation sequence $(\varepsilon_i)_{i \geq 1}$ based upon the nominal (without fault) state-space model (26) :

$$\begin{cases} \varepsilon_i = Y_i - H(t)\hat{X}_{i/i-1} \\ \hat{X}_{i+1/i} = \Phi(t+1, t)\hat{X}_{i/i-1} + \Phi(t+1, t)K_i\varepsilon_i \end{cases} \quad (28)$$

where K_i is the Kalman gain, and, next, we have to detect/isolate a change in the innovation sequence $(\varepsilon_i)_{i \geq 1}$. It can be shown that the log-likelihood ratio between two hypotheses l and j may be expressed as below:

$$S'_k(l, j) = \ln \frac{p_l(Y_k, \dots, Y_l)}{p_j(Y_k, \dots, Y_l)} = \ln \frac{p_l(\varepsilon_k, \dots, \varepsilon_l)}{p_j(\varepsilon_k, \dots, \varepsilon_l)} \quad (29)$$

It shall be noted here that hypothesis l corresponds to a failure on satellite i ($l = 2 \times i$ or $l = 2 \times i + 1$, see §II-D-1 for details).

Let us assume now that the model (26) is time-invariant, that the Kalman filter corresponding to this model is stable and, moreover, that the steady-state has been reached:

$$\lim_{t \rightarrow \infty} R_t = R, \quad \lim_{t \rightarrow \infty} K_t = K \quad (30)$$

where R_t is a covariance matrix of the innovation of the Kalman filter. The innovation sequence can be modeled as a normal variable with different means before and after the failure:

$$\varepsilon_i \in \begin{cases} N(0, R_t) & \text{if } t < t_0 \\ N(\eta_l(t, t_0), R_t) & \text{if } t \geq t_0 \end{cases} \quad (31)$$

where $\eta_l(t, t_0)$ is the dynamic profile of the innovation sequence after fault number l . Since the innovation before and after a fault in model (3) is a Gaussian independent sequence, the theory developed in [3,4] can be applied in this case with some modifications: we have to compute all dynamic profiles for $1 \leq t_0 \leq t$ at every stage t . Unfortunately, this leads to a number of arithmetical operations at time t which grows to infinity with t . The second difficulty of this approach is the fact that the dynamic profiles $\eta_l(t, t_0)$ are functions of the unknown vectors $\Gamma_l(t, t_0)$. Unlike in the first hybridization scheme which uses a regression model for GPS (c.f. §II), the dynamic profiles $\eta_l(t, t_0)$ should here be known exactly. Therefore, it is too optimistic to recommend a direct implementation of the algorithms developed in [IEEE95, ACC95, JGCD96]. To solve the problem we propose the following heuristic solution. First, we split the measurement processing in several parallel Kalman filters (see Figure (11)). Hence, in every channel we process one pseudo-range of GPS (or DGPS) and the measurement of the INS together. If we have n visible satellites at the moment then the detection/isolation/exclusion scheme includes n parallel channels (see figure (13)).

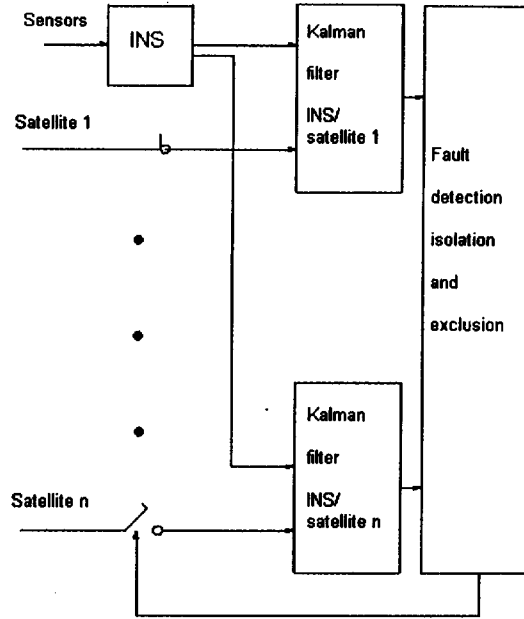


figure (13): Multi channel measurement GPS/INS integrity monitoring.

C. Simplified model

Let us consider the following simplified error model of the vertical channel of the GPS/INS navigation system. The vertical channel of the GPS/INS is based on the following sensors/channels : an accelerometer, a baro-altimeter, and n GPS channels. The plant equation in the continuous time is given by:

$$\frac{dz}{dt} = v_z, \quad \frac{dv_z}{dt} = b_z, \quad \frac{db_z}{dt} = \omega_a, \quad \frac{ds}{dt} = \omega_s$$

and the measurement equation is :

$$\begin{cases} h_{b-a} = z + \omega_b \\ R_{pr-a,1} = -U_{z,1}z + Cs + \omega_r + v_{r,1}(t, t_0) \\ \vdots \\ R_{pr-a,n} = -U_{z,n}z + Cs + \omega_r + v_{r,n}(t, t_0) \end{cases}$$

where $h_{b-a} = h(\text{baro}) - h(\text{acc})$, $R_{pr-a,i} = R(\text{GPS}) - R(\text{acc})$, R_i is the pseudorange from the i th satellite to the user ($i = 1, \dots, n$), s is a user clock bias, $C = 3.10^8$ m/s, $U_{z,i} = \sin(\phi_i)$, ϕ_i is the elevation angle of the i th satellite, $\omega_a, \omega_b, \omega_r, \omega_s$ are white noises such that $\sigma_b = 100\text{m}$, $\sigma_r = 4\text{m}$, $\sigma_a^2 = 2 \times (10^{-3})^2 / 3600 (m/s^2)^2 \times m/s$, $\sigma_s = 10^{-2}$ sec.

We assume that there are two types of fault for each of the n sensors/channel, namely:

$$\begin{aligned} \text{step: } & v_{r,j}(t, t_0) = \text{const} && \text{for } t \geq t_0 \\ \text{ramp: } & v_{r,j}(t, t_0) = \text{const} \cdot (t - t_0) && \text{for } t \geq t_0 \end{aligned}$$

D. Discrete time model

The discrete time error model of the vertical channel of the GPS/INS is represented by the following equations:

$$\begin{cases} X_{t+1} = \Phi \cdot X_t + V_t \\ Y_t = H \cdot X_t + W_t + \Gamma(t, t_0, l) \end{cases}$$

where $X_t = (z \ v_t \ b_x \ s)^T$, $\Phi \cong (I + \Delta t \cdot F)$,

$$Y_t = (h_{b-a} \ R_{pr-a,1} \ \dots \ R_{pr-a,n})^T,$$

$$F = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad H = \begin{pmatrix} 1 & 0 & 0 & C \\ -U_{z,1} & 0 & 0 & C \\ \vdots & \vdots & \vdots & \vdots \\ -U_{z,n} & 0 & 0 & C \end{pmatrix}$$

and V_t, W_t are Gaussian vector sequences:

$$\text{cov}(V) = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \Delta t \sigma_a^2 & 0 \\ 0 & 0 & 0 & \Delta t \sigma_a^2 \end{pmatrix}$$

$$\text{cov}(W) = \begin{pmatrix} \sigma_r^2 & 0 & 0 & 0 \\ 0 & \sigma_r^2 & 0 & 0 \\ 0 & 0 & \sigma_r^2 & 0 \\ 0 & 0 & 0 & \sigma_r^2 \end{pmatrix}$$

where $\Delta t = 1s$.

CONCLUSION

It has been shown that, at the location of the simulation (Toulouse, France) and for the current GPS constellation (25 satellites), this hybridization scheme could fulfill the RNP 0.3 with the help of two additional geostationary satellites (Inmarsat AOR-E and IOR). This is very promising because this approach does not need much hardware and could be quite easily adapted to existing equipment.

Of course, these simulations have been made on the assumption that a wide area differential GPS like EGNOS or WAAS is present. But, although these systems are planned in a near future to deliver the Integrity function, they will not be immunized against terrestrial jamming or spoofing. Furthermore, they will not be able to detect local degradation like strong multipath. The fact that INS is by nature immunized against external events and that sequential algorithm can detect and isolate small failure make this hybridization scheme worth developing.

Unlike the loosely coupled GPS-INS, the tightly coupled GPS/INS does not require a minimum of six satellites with good geometry to perform hence giving a much more higher availability. The promising performances of the hybridization scheme are still to be evaluated and this is the aim of a future study.

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