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# Mixed Conflict Model for Air Traffic Control

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**Abstract.** Airspace congestion is today the most critical issue European Air Traffic Management (ATM) has to face. Current real-time Air Traffic Control (ATC) is achieved by human controllers. One of their main tasks is to keep separation between aircraft, asking to the pilots to do basic avoidance manoeuvres. We propose here two mixed CSP models of this separation issue, combining discrete and continuous variables. An implementation of these models allows to produce optimal solutions for problems where numerous aircraft are conflicting.

## 1 Introduction

The capacities of European en-route Air Traffic Control (ATC) centres are far exceeded by a constant growth in air traffic demand, resulting in ever increasing flight delays.

To overcome this issue, novel ATM schemes are designed while keeping the hard constraint of a minimal 5 NM (9260 m) horizontal separation between every pair of aircraft to ensure safety. Nowadays, solutions to avoid conflicts are empirical, and human controllers rely on standard routes and traffic organization to devise them. However, the complexity of conflicts could tremendously grow within such future ATM systems, should the aircraft fly on direct routes, from taking-off airport to destination. Then human controllers would no longer be able to solve them efficiently by their own, thus requesting automated solvers.

Former approaches like [DA95] use local search (namely genetic algorithms) to solve the conflict problem. These kind of meta-heuristics

are well suited to solve large scale and difficult problems when no other relevant techniques are known, but stochastic search inherently lacks existence and optimality proofs.

We propose here to model the problem using CSP in order to get the optimal solution for a given criteria (e.g. the real cost of the manoeuvres, in time and/or fuel consumption). The difficulty to handle the required constraints is related to the fact that the separation must be kept at any time. One simple solution would be to discretize the time and set the constraints for each time step. However we choose here to express the separation between straight pieces of trajectory, using continuous constraints.

After presenting the models and some optimization strategies, we give some indication about their implementation, and eventually provide preliminary results validating the approach.

## 2 Mixed Models

An aircraft is characterized by its initial position, speed and heading, which are represented by floating point numbers. In our model we are only taking into account horizontal manoeuvres between aircraft at the same altitude.

Two aircraft are in conflict at a given time if the distance between them is smaller than the safety separation. The considered manoeuvres for maintaining separation involve deviations of the aircraft headings. The starting times of the manoeuvres and the deviation angles are discrete variables since it is more representative of the orders given to a pilot. More precise orders would indeed be irrelevant according to the pilots and aircraft equipment performances.

### 2.1 Horizontal TCAS Model

This horizontal TCAS<sup>1</sup> model is the most simple for horizontal deviations: at the initial time, deviation angles are imposed to the aircraft headings in order to avoid conflicts, so there is only one discrete decision variable  $\alpha_i$  for the aircraft  $i$ . This simple model is for emergency situations and could be used for a real-time TCAS-like system. This model is not suitable for common traffic control since it does not take into account returning back to the main trajectory.

<sup>1</sup> Traffic Control Avoidance System: airborne device issuing emergency vertical manoeuvres to help short-term avoidance for a single pair of aircraft.

Let  $\vec{v}_{ij}$  be the relative speed and  $\vec{p}_{ij}(t)$  the relative position between two aircraft  $i$  and  $j$ , and  $d$  the safety distance. We have  $\vec{p}_{ij}(t) = \vec{p}_{ij}(t_0) + \vec{v}_{ij}(t - t_0)$ . There is no conflict between them at a given time  $t$  if  $\vec{p}_{ij}(t)^2 - d^2 \geq 0$ , meaning that the distance between them is greater than  $d$ . Two aircraft will not be in conflict if the discriminant of this polynomial is negative, which lead to an inequality constraint per couple of aircraft, stated as follow:

$$(\vec{p}_{ij}(t_0) \cdot \vec{v}_{ij})^2 - (\vec{p}_{ij}(t_0)^2 - d^2)v_{ij}^2 \leq 0 \quad (1)$$

with

$$\vec{p}_{ij}(t_0) = \begin{pmatrix} x_i(t_0) - x_j(t_0) \\ y_i(t_0) - y_j(t_0) \end{pmatrix}$$

$$\vec{v}_{ij} = \begin{pmatrix} v_i \cos(\theta_i + \alpha_i) - v_j \cos(\theta_j + \alpha_j) \\ v_i \sin(\theta_i + \alpha_i) - v_j \sin(\theta_j + \alpha_j) \end{pmatrix}$$

where  $\theta_i$  and  $\theta_j$  (initial headings),  $v_i$  and  $v_j$  (speeds) and  $x_i(t_0)$ ,  $x_j(t_0)$ ,  $y_i(t_0)$ , and  $y_j(t_0)$  (initial positions) characterize the problem, whereas  $\alpha_i$  and  $\alpha_j$  are the decision variables.

## 2.2 Horizontal Human Controller Model

Now we consider that the aircraft is initially heading toward a waypoint. The autorised manoeuvres still are deviations from the original heading but they can be delayed: a decision variable  $t_1$  for each manoeuvre starting time is added (see figure 1). Moreover each aircraft heads back to its original waypoint after some time  $\delta t$ , so another decision variable is added.

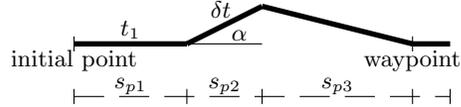
The path of an aircraft  $p$  is then composed of three segments  $s_{p1}$ ,  $s_{p2}$ ,  $s_{p3}$ : during the first one, the aircraft is heading toward its waypoint (initial route); during the second one, the aircraft is deviated by an angle  $\alpha$ ; and during the last one, it is heading back toward its waypoint. Each segment of each aircraft trajectory is potentially in conflict with the segments of the other aircraft, resulting in 9 constraints per pair of aircraft.

Considering the polynomial similar to the one studied in (1), there is a conflict between two segments  $s_{ai}$  and  $s_{bj}$  if the three following conditions are verified:

- there is a common time when the aircraft  $a$  is on the segment  $s_{ai}$  and the aircraft  $b$  is on the segment  $s_{bj}$ ;
- the discriminant of the corresponding polynomial is positive;

- at least one root of the polynomial is in the common time of both segments.

The negation of these statements leads us to the disjunction of reified constraints *no\_common\_time*  $\vee$  *is\_le(discriminant)*  $\vee$  *root\_not\_in\_common\_time*.



**Fig. 1.** Decision variables for the Horizontal Human Controller Model

## 2.3 Optimization and Search Strategy

Different optimization strategies were studied, starting with the simple “minimize maximal deviation angle” (in absolute value).

However, two solutions with the same maximal deviation angle can be very different from a human point of view, so we considered more precise optimization criteria. To improve the quality of solutions, an efficient strategy was to perform a multi-stage hierarchical optimization on the angles, corresponding to the following lexicographic minimization. Let

$$c(\eta) = \text{card}(\{|\alpha_i| = \eta\})$$

be the number of (absolute values of) deviations equal to a given angle  $\eta$ , and  $\{\eta_1, \dots, \eta_n\}$  their possible values sorted in decreasing order, the objective is then to lexicographically minimize the tuple:

$$(c(\eta_1), \dots, c(\eta_n)) = (c(30), c(20), c(10), c(0))$$

as in our application, deviations can range over  $\{-30, -20, -10, 0, 10, 20, 30\}$ .

Another optimization strategy is to maximize the number of aircraft not deviated. It is a relevant criterion for a human controller but the computation times are more costly. A third one, more focused at airlines concern, would be to minimize the total lengthening of the paths; however, this objective is yet too hard to achieve, at least for our complete CP system.

### 3 Implementation

To handle these problems the support for reals variables and constraints was added to FaCiLe [BB01]. Programs may mix real and integer variables, as it is the case with our models.

#### 3.1 Interval Arithmetic

Since it is difficult to make assumptions on the definition range of functions or the domains of variables, the arithmetic considered is an extended interval arithmetic. No exceptions are raised during operations over intervals but the domains of interval functions are consistently extended to infinite and empty values. The interval arithmetic library used to implement constraints is Filib++ [MGJ01,Zil05].

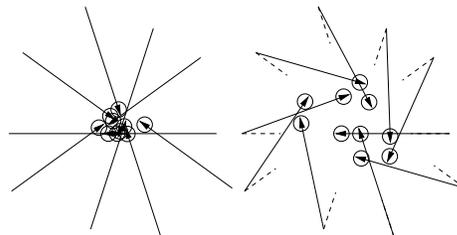
#### 3.2 Propagation

The consistency algorithm implemented is based upon BC4 proposed by [BGGP99] but enforces a weakening of the  $\text{box}_\varphi$ -consistency [GGB99]. The latter only considers intervals of a single width  $\varphi$  when looking for the leftmost and rightmost quasi-zeros, which can be costly when narrowing intervals of very different sizes.

While searching for the quasi-zeros, our FaCiLe implementation handles intervals of width  $\alpha l + \varphi$ ,  $l$  being the width of the narrowed interval and  $\alpha \in [0, 1]$  an additional tuning parameter. The key idea behind this scheme is to work with a floating precision of the narrowing to efficiently handle domains of different sizes. Hence, costly computations of precise bounds are avoided for large intervals.

#### 3.3 Integration within FaCiLe

FaCiLe is a Fonctionnal Constraint Library written in Ocaml. The module system of Ocaml allows to define functors, equivalent to functions at the module level. A functor builds a new module from a module taken as argument. The addition of variables over the reals to FaCiLe simply amounts to the application of the variables functor to a new domain module. This module provides a new type of domains and the necessary functions to handle them. Goals and constraints over these new variables can easily be written using the provided generic mechanisms.

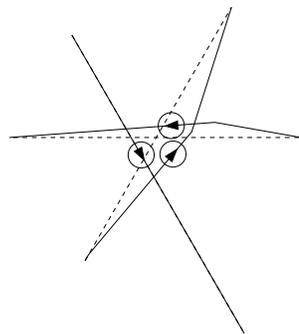


**Fig. 2.** Routes, at a given time, without and with conflicts solving for a 10 aircraft problem solved with the TCAS model.

## 4 Results

A common conflicting situation in ATC involves aircraft converging to the same waypoint. This has first been simulated with perfectly converging aircraft. Aircraft are disposed regularly on a circle, converging to the centre. The addition of noise to the positions, speeds and headings of aircraft breaks the symmetry and simulates a more realistic ATC event.

Problems up to 15 aircraft have been solved in a few seconds using the TCAS model and the hierarchical optimization procedure (see figure 2), which is a reasonable computation time for emergency avoidance situations. For this problem, the deviation angles may range over  $\{-30, -20, -10, 0, 10, 20, 30\}$ , which are realistic ATC orders. Problems up to 30 aircraft have been solved as well in a few minutes.



**Fig. 3.** Conflict solving with return on main trajectory.

The horizontal human controller model have been used to optimally solve problems of two and three aircraft (see figure 3), which represent almost 90% of the total number of conflicts

detected by the CATS<sup>2</sup> simulator [DA97] during a typical day of traffic. The same deviation angles were used for this model with a time step of 10 seconds over 10 minutes time periods.

## 5 Conclusion and Further Works

We have presented two mixed CSP models to optimally solve air traffic conflicts with horizontal manoeuvres: a simple TCAS-like model for emergency avoidance and a more complete human controller model to automate ATC. These models have been implemented with FaCiLe by adding real constraints and variables to the library.

Computational results for the first model compare well with former local search approaches [DA95] like genetic algorithms, additionally providing existence and optimality proofs. This model features a major improvement over the current TCAS, being able to solve conflicts involving more than 2 aircraft and issuing less drastic manoeuvres than changes of altitudes.

The second one is much more complex, and only conflicts involving 2 or 3 aircraft could be solved with the search strategies we have tried.

These results also demonstrates the versatility of FaCiLe and its ability to simply integrate new solvers. However, some work still has to be done to overcome the limitations of the "human" model (smarter strategies, more powerful propagations, relaxations of the model...), and the algorithms have to be tested on conflicts situations from real traffic data to be validated.

## References

- [BB01] Nicolas Barnier and Pascal Brisset. FaCiLe: a Functional Constraint Library. In *CICLOPS'01*, 2001.
- [BGGP99] Frédéric Benhamou, Frédéric Goualard, Laurent Granvilliers, and Jean-François Puget. Revising hull and box consistency. In *Proceedings of the 16th International Conference on Logic Programming (ICLP'99)*, pages 230–244. MIT Press, 1999.
- [DA95] Nicolas Durand and Jean-Marc Alliot. Automatic aircraft conflict resolution using Genetic Algorithms. In *11th annual ACM conference on applied computing (ACM/SAC 96, philadelphia)*, 1995.
- [DA97] Nicolas Durand and Jean-Marc Alliot. Optimal Resolution of En Route conflicts. *Séminaire Europe/USA, Saclay*, June 1997.
- [GGB99] Laurent Granvilliers, Frédéric Goualard, and Frédéric Benhamou. Box consistency through weak box consistency. In *ICTAI*, pages 373–380, 1999.
- [MGJ01] Michael Lerch, German Tischler, and Jürgen Wolff von Gudenberg. *Filib++ - Interval Library, Specification and Reference Manual*, August 2001.
- [Zil05] Julius Zilinskas. Comparison of packages for interval arithmetic. *Informatica, Lith. Acad. Sci.*, 16(1):145–154, 2005.

<sup>2</sup> Complete Air Traffic Simulator