A global model for long term planning of multimodal freight transportation systems

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A GLOBAL MODEL FOR LONG TERM PLANNING

OF MULTIMODAL FREIGHT TRANSPORTATION SYSTEMS

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ABSTRACT

In this communication is considered the problem of long term forecasting of the traffic growth in a large transportation network. This problem is crucial when planning the necessary investments in terminals, fleets and traffic control equipments.
The proposed approach makes use of two different optimization models: One model is devoted to freight demand forecasting, the other one defines the transport supply on a multimodal basis. A general framework is proposed and a solution scheme composed of an iterative process between the current solutions of the demand and the supply optimization problems, is introduced. Convergence conditions are discussed for this iterative process between these two problems which can be seen as inverse of each other.

KEYWORDS: freight flows, transportation networks, bi-level programming

1. INTRODUCTION

In this communication is considered the problem of long term forecasting of freight traffic growth in a large transportation network. This problem is crucial when planning the necessary investments in terminals, transportation links and fleets. One of the main difficulty of this task is related with the estimation of future demand over the network which has direct influence on the operational conditions and costs. The proposed approach makes use of two different optimization models: One model is devoted to demand forecasting, the other one defines the global transport supply according with a profit maximization behavior for the whole transport sector operating in this area. The freight demand forecasting process is based a new entropy maximization approach to determine the distribution of origin-destination matrices. The supply optimization model considers simultaneously two classes of flows: vehicle flows providing transportation capacity and freight flows generating revenues to the transportation operators. Hence, no classical flows in networks optimization technique is available to solve this problem while two level solution techniques considering vehicle flows at the first level and passengers/freight flows at the second level can be considered. A global optimization problem is associated to a full scenario with respect to demand and operations costs. Each of the optimization problems, taken separately is convex, however, the whole problem, through the global definition of the revenue and of a cost constraint is non convex. A proposed solution schemes is composed of an iterative process between the current solutions of the demand and the supply optimization problems: the entropy maximizing/minimum freight distribution cost problems provides the freight origin-destination matrices given a fare structure, while the supply optimization problem provides the fare structure given passengers/freight origin-destination matrices. To enforce convergence while maintaining convexity of the two problems, the effective-potential demand level constraints of the supply optimization problem are modified.

2.DISTRIBUTION OF FREIGHT DEMAND

To perform the prevision of freight demand, it is supposed that a priori trip distribution is available:

$$O_i \ , i \in \{1, \ldots, N\} \text{ and } D_j \ , j \in \{1, \ldots, N\} \tag{1}$$

We consider that the demand for a given origin destination i-j and for a given modality *m* is such that there exists a positive constant λ_{ij}^m with :

$$T_{ij}^m(\bar{\pi}_{ij}) = T_{ij} \ (e^{-\lambda_{ij}^m \bar{\pi}_{ij}^m} / \sum_{n \in M} e^{-\lambda_{ij}^n \bar{\pi}_{ij}^n}) \tag{2}$$

where $\bar{\pi}_{ij}^m$ is the mean freight fare between *i* and *j* with modality *m*. Considering that the potential origin and destination levels are $O_i \ , i \in \{1, \ldots, N\}$ and $D_j, j \in \{1, \ldots, N\}$, we have the following constraints:

$$\sum_{m \in M} \sum_{j=1, j \neq i}^{N} T_{ij} e^{-\lambda_{ij}^m \bar{\pi}_{ij}^m} = O_i (\sum_{n \in M} e^{-\lambda_{ij}^n \bar{\pi}_{ij}^n}) = \tilde{O}_i \quad i = 1 \, to \, N \tag{3-1}$$

$$\sum_{m \in M} \sum_{i=1, i \neq j}^{N} T_{ij} e^{-\lambda_{ij}^m \bar{\pi}_{ij}^m} = D_j (\sum_{n \in M} e^{-\lambda_{ij}^n \bar{\pi}_{ij}^n}) = \tilde{D}_j \quad j = 1 \, to \, N \tag{3-2}$$

The demand distribution estimation problem is taken here as a constrained entropy maximization problem. Then to a choice of an instance $I=\{[\hat{T}_{ij}^m], \ O_i \ , i \in \{1, \ldots, N\}, \ D_j \ , j \in \{1, \ldots, N\}\}$ is associated the following maximization problem, Problem *D*:

$$\max \quad - \sum_{i=1}^{N} \sum_{j=1, j \neq i}^{N} T_{ij} \ Ln\left(T_{ij} / \hat{T}_{ij}\right) \tag{4}$$

under constraints (3-1), (3-2) and

$$T_{ij} \geq 0 \, , i = 1 \, to \, N, \ j = 1 \, to \, N, i \neq j \tag{5}$$

The adopted optimization criteria, a conditional entropy function, is representative of the global distortion between the a priori and the predicted demand distributions. The above optimization problem being convex it is useful to introduce the Lagrangian associated to this problem :

$$L = - \sum_{i=1}^{N} \sum_{j=1, j \neq i}^{N} T_{ij} \ Ln\left(T_{ij_{ij}} / \hat{T}_{ij}\right) + \sum_{i=1}^{N} \alpha_i \ (\sum_{m \in M} \sum_{j=1, j \neq i}^{N} T_{ij} e^{-\lambda_{ij}^m \bar{\pi}_{ij}^m} - \tilde{O}_i) + \sum_{j=1}^{N} \beta_j \ (\sum_{m \in M} \sum_{i=1, i \neq j}^{N} T_{ij} e^{-\lambda_{ij}^m \bar{\pi}_{ij}^m} - \tilde{D}_j) \tag{6}$$

The first order optimality conditions are such as :

$$\partial L / \partial T_{ij} = 0 \quad i = 1 \text{ to } N,\ j = 1 \text{ to } N, i \neq j \tag{7}$$

or :

$$-1 - Ln(T_{ij}/\hat{T}_{ij}) + \sum_{m \in M} (\alpha_i \, e^{\lambda_{ij}^m \bar{\pi}_{ij}^m} + \beta_j \, e^{\lambda_{ij}^m \bar{\pi}_{ij}^m}) = 0 \qquad i = 1 \text{ to } N,\ j = 1 \text{ to } N, i \neq j \tag{8}$$

Then the solution is such as :

$$T_{ij}^* = \hat{T}_{ij} \, e^{-(1+(\alpha_i+\beta_j)\sum_{m \in M} e^{\lambda_{ij}^m \bar{\pi}_{ij}^m}} \qquad i = 1 \text{ to } N,\ j = 1 \text{ to } N, i \neq j \tag{9}$$

The dual variables are such as:

$$\sum_{j=1, j \neq i}^{N} \rho_{ij} \ p_{ij}^{(\alpha_i + \beta_j)} = O_i \quad i = 1 \text{ to } N \tag{10}$$

$$\sum_{i=1, i \neq j}^{N} \rho_{ij} \ p_{ij}^{(\alpha_i + \beta_j)} = D_j \quad j = 1 \text{ to } N \tag{11}$$

where

$$\rho_{ij} = \hat{T}_{ij} e^{(\sum_{m \in M} \lambda_{ij}^m \bar{\pi}_{ij}^m - 1)} \quad \text{and} \quad p_{ij} = e^{e^{\lambda_{ij} \bar{\pi}_{ij}^n}} \tag{12}$$

The numerical solution of an instance of *Problem D* can be obtained using an extension of the Furness algorithm. Linear convex algorithms such as the Simplex-Convex or the Frank-Wolfe algorithms (Assad, 1978) appears to be very cumbersome in computational terms. Another interesting direction for numerical resolution is through the solution of the geometric primal associated to this problem (Mora-Camino, 1978).

3. ASSIGNMENT OF FREIGHT FLOWS

The different vehicle flows along each network provide the physical support for the freight flows. Then considering that $[\phi_{ij}^m]$ are the freight flows over the network corresponding to transportation mode *m*, they obey to the following capacity constraints :

$$0 \leq \phi_{ij}^m \leq \sigma_\phi f_{ij}^m \quad (i,j) \in G_m, \quad m \in M \tag{13}$$

where $[f_{ij}^m]$ is the flow of vehicles of the m^{th} modality over $\boldsymbol{G_m}$, σ_ϕ is a scaling factor. For each origin-destination pair a set of concurrent paths composed of a succession of links is retained according to directness criteria for freight:

$$Ch_{ij} = \left\{ Ch_{ij}^n, n = 1 \text{ to } n_{\max} \right\} \quad (i,j) \in \bigcup_{m \in M} G_m \tag{14}$$

Here it is considered that demand for a given O-D pair, is assigned to these possible paths according with the available capacities. This approach is acceptable when available capacity is close to demand levels.

Let [β(i,j,n,k,l,m)] be the incidence matrix between path $Ch^n{}_{ij}$ and arc (k,l) of $\boldsymbol{G_m}$: β($i,j,n,k,,l,m$) = 1 if (k,l)∈ $Ch^n{}_{ij}$ $\beta(i,j,n,k,l,m)$ = 0 else. Then the flow of freight between vertices i and j is given by :

$$\phi_{ij} = \sum_{m \in M} \sum_{(k,l) \in G_m}^{N} \left(\sum_{n=1}^{N_{ij}} \beta(k,l,n,i,j,m) \theta_{k\ln} \right) \qquad (i,j) \in \bigcup_{m \in M} G_m \tag{15}$$

where θ_{ijn} is the flow of freight between origin i and destination j using the n[th] path between them.

The mean freight fare for the i-j origin-destination pair is then given by :

$$\bar{\pi}_{ij} = \left(\sum_{n=1}^{N_{ij}} \pi_{ij}^{n}\, \theta_{ijn} \right) / \left(\sum_{n=1}^{N_{ij}} \theta_{ijn} \right) \qquad (i,j) \in \bigcup_{m \in M} G_m \tag{16}$$

4. TRANSPORTATION SUPPLY MODEL

The proposed model takes into account m+1 types of flows: m vehicle flows according to the different transport modalities and freight flows using the resulting transport capacity. The fleets of vehicles and their operation generate fixed and variable costs, while freight flows are the main source of revenue for the freight transportation sector. To each transportation modality is associated a network linking the N transportation centers such as:

$$\boldsymbol{R_m} = [\boldsymbol{G_m}, [\, f_{ij}^{m} \,]] \tag{17}$$

where $\boldsymbol{G_m}$ is the graph associated to the m^{th} modality, [f_{ij}^{m}] is the flow of vehicles of the m^{th} modality over $\boldsymbol{G_m}$. The different networks some common vertices and links. The flow of vehicles associated to each transportation mode satisfies to conservation and positivity constraints:

$$\sum_{(j,i) \in G_m} f_{ji}^{m} = \sum_{(i,j) \in G_m}^{N_m} f_{ik}^{m} \qquad i \in G_m, \quad m \in M \tag{18}$$

$$f_{ij}^{m} \geq 0, \quad (i,j) \in \boldsymbol{G_m} \quad m \in M \tag{19}$$

where N_m is the number of vertices of graph $\boldsymbol{G_m}$. Here flows integrity constraints are not taken into account since no scheduling or routing problem will be formulated according to these flows which should only provide a global view of the future development of the multimodal transportation system. However for sake of realism, a fleet capacity constraint can be introduced:

$$\sum_{(i,j) \in G_m}^{N_m} f_{ij}^{m}\, d_{ij}^{m} \leq D_m\, F_m \qquad m \in M \tag{20}$$

where d_{ij}^{m} is the block time for travel, including departure and arrival activities, between vertices i and j , D_m is the average time availability of a vehicle for modality m.
It can be also of interest to introduce node capacity constraints related with intermodal stations and terminals:

$$\sum_{m,n \in M} \sum_{(i,j) \in G_m \cup G_n} S_{mn}(i,j,t)\, f_{ij}^{m} \leq Q^{t} \quad t \in T \tag{21}$$

where Q_m^t is the traffic capacity of terminal t, $S_{mn}(i, j, t)$ are incidence matrices between vehicle flows of different transportation modes.

5. SUPPLY OPTIMIZATION MODEL

Given the distribution of potential demands $[T_{ij}^{*}]$ associated to an instance of $[\hat{T}_{ij}]$, O_i , $i \in \{1, ..., N\}$ and D_j , $j \in \{1, ..., N\}$, as well as to mean transportation fares $[\bar{\pi}_{ij}^{m}]$, the optimization of transportation supply (capacities and fares) can be considered: This problem (*Problem S*) is concerned with the optimization of the global economic performance over a period of time of the multimodal freight transportation network. Here the decision variables are the vehicle flows (f_{ij}^{m}) between the different centers and the fares (π_{ij}^{mn}) applied to each selected path between the centers. To solve this problem it is also necessary to introduce the effective flows of goods (θ_{ijn}) associated to each selected path between the connected centers. The optimization criterion of *Problem S* is given by:

$$\max_{f_{ij}, \pi_{ij}^{n}, \theta_{ij}} \sum_{m \in M} \Big\{ \sum_{(i,j) \in G_m} \sum_{n=1}^{N_{ij}^{m}} (\pi_{ij}^{mn}\ \theta_{ijn})/(1+\tau_r) - \left((c_{cf}^{m} + c_{cv}^{m} F^{m} + \sum_{(i,j) \in G_m}^{N} c_{ij}^{mcv} f_{ij}^{m}) \right) \Big\} \tag{22}$$

where τ is a return rate and the c_{uv}^{m} are coefficients related with fixed and variable fleets and flows costs. *Problem S* must satisfy the following constraints:

$$\sum_{(j,i) \in G_m} f_{ji}^{m} = \sum_{(i,j) \in G_m}^{N_m} f_{ik}^{m} \qquad i \in G_m, \quad m \in M \tag{23}$$

$$\sum_{(i,j) \in G_m}^{N_m} f_{ij}^{m}\ d_{ij}^{m} \leq D_m\ F_m \qquad m \in M \tag{24}$$

$$\sum_{(k,l) \in G_m} (\sum_{n=1}^{N_{ij}} \beta(k,l,n,i,j,m)\theta_{k\ln}) \leq \sigma_{\phi}^{m} f_{ij}^{m} \qquad (i,j) \in G_m, \quad m \in M \tag{25}$$

$$\sum_{m,\,n\in M}\ \sum_{(i,j)\in G_m \cup G} S_m(i,j,t)\, f_{ij}^m \le Q^t \quad t\in T \tag{26}$$

$$\sum_{n=1}^{N_{ij}^m} \theta_{ijn} \le T_{ij}^*\left[\bar{\pi}_{ij}\right] \quad (i,j)\in G_m,\, m\in M \tag{27}$$

with the positivity conditions:

$$\theta_{k\ln} \ge 0, \qquad n\in\left\{1,..,N_{ij}^m\right\},(k,l)\in G_m, m\in M \tag{28}$$

$$f_{ij}^m \ge 0 \qquad (i,j)\in G_m, \quad m\in M \tag{29}$$

$$\pi_{ij}^{\phi mn} \ge 0 \qquad (i,j)\in G_m,\ n\in N_{ij}^m, \quad m\in M \tag{30}$$

6. GLOBAL SOLUTION SCHEME

It appears that *Problems D* on one side and *Problem S* on the other side, are strongly interdependent: while *Problem D* provides to *Problem S* potential levels of demand $[T_{ij}^*]$ (constraints (27)), *Problem S* provides mean fare values $[\bar{\pi}_{ij}]$ to *Problems D*.

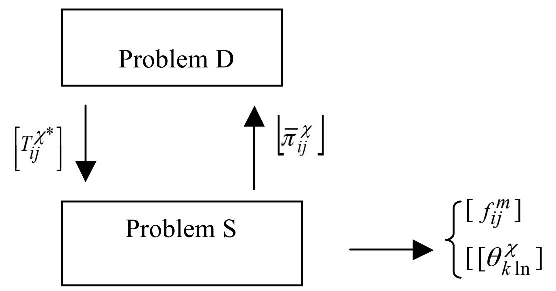

Figure 1: Interaction between problems

Each of the optimization problems, taken separately is a convex programming problem, however, the whole problem when integrating *Problem D* through constraint (27) is non convex. Then it seems interesting to solve numerically these problems separately and design an interactive process towards equilibrium. In this case, some questions are of interest:

- the easiness to solve numerically each of the optimization problems,
- the guarantee of convergence of the iterative process towards global solution,
- the speed of convergence,
- the quality of the limit solution.

These three linked problems constitute a non standard bi-level programming problem (Dempe, 2000) where Problem S is the leader's problem while Problem D are the followers' problems. This results from the adoption of a deregulated point of view in which transport

firms operate as a whole the transport network according to their direct economic interest without considering any social surplus.
Constraint (27) plays a central role in the articulation of the two optimization levels and the convergence of their solutions towards a common global solution. Since these constraints transmit to the revenue optimization problem the reaction of demand with respect to changes in mean fares between the different origins and destinations, it is useful to makes apparent this effect so that limited fare values will be provided by the solutions of Problem S. However, the relation between origin-destinations flows $([T^*_{ij}])$ and fare levels $[\bar{\pi}_{ij}])$ is quite complex and at least non linear. So, to maintain the convexity of *Problem S*, these constraints can be replaced by their first order approximation where the reference values are the solutions of the two problems at the previous iteration:

$$\sum_{n=1}^{N_{ij}} \theta_{ijn} \leq T_{ij}^{*(k-1)} + \sum_{l=1}^{N} \sum_{p=1, p\neq l}^{N} \frac{\partial T^*_{ij}}{\partial \bar{\pi}_{lp}}\Big|_{k-1} \left(\sum_{n=1}^{N_{lm}} \frac{\partial \bar{\pi}_{lp}}{\partial \pi^n_{lp}}\Big|_{k-1} (\pi^{vn}_{lp} - \bar{\pi}^{n\,(k-1)}_{lp}) \right) \tag{31}$$

$$(i,j)\in G_m, \quad m\in M$$

Since the following inequalities are likely:

$$\left.\begin{array}{l} \dfrac{\partial T^{v*}_{ij}}{\partial \bar{\pi}_{ij}\big|_{k-1}} \leq 0 \quad \dfrac{\partial T^*_{ij}}{\partial \bar{\pi}_{lp}}\Big|_{k-1} \geq 0 \\[2em] \left| \dfrac{\partial T^{v*}_{ij}}{\partial \bar{\pi}^v_{lp|k-1}} \right| << \left| \dfrac{\partial T^{v*}_{ij}}{\partial \bar{\pi}^v_{ij|k-1}} \right| \end{array}\right\} \tag{32}$$

$$(i,j)\in G_m, \quad l,p\in A_m, l\neq p, \quad m\in M$$

it appears that starting from low fares, an increase of fares on trips linking origin *i* and destination *j* at solution of *Problem S* at iteration *k*-1 will imply a decrease of potential demand on the same origin-destination pair at the same iteration of the corresponding *Problem D* and then a negative effect on the revenue of transport firms. Fares will then be increased by this process until no more improvement of revenue is obtained. Starting from high level fares, the inverse process will be obtained. his iterative process can be sketched with a very simple example:

$$\left.\begin{array}{l} \max\limits_{\pi,\theta} \pi\,\theta - c\,\theta \\[1em] \text{under} \quad \theta \leq \theta_0\, e^{-\lambda\,\pi} \end{array}\right\} \tag{33}$$

where θ_0 and λ are positive constants. Its exact solution is given by: $\pi^* = (1+c\,\lambda)/\lambda$ and $\theta^* = \theta_0\, e^{-(1+\lambda c)}$. Similarly to (41), its feasible set can be approximated around (π_{k-1} , θ_{k-1}) by:

$$\theta \leq \theta_{k-1}(1+\lambda\,\pi_{k-1}) - \lambda\,\theta_{k-1}\,\pi_{k-1} \tag{34}$$

and the solution of problem $\{\max\limits_{\pi,\theta} \pi\,\theta - c\,\theta$ under (44)} is :

$$\begin{cases} \pi_k = (1 + \lambda\ \pi_{k-1} + \lambda\ c)/(2\ \lambda) \\ \theta_k = \theta_0\ \exp(-\lambda\ \pi_k) \end{cases} \tag{45}$$

It is clear that the limit when k $\rightarrow +\infty$ of (π_k, θ_k) is ((1+c λ)/λ, θ_0 $e^{-(1+c\ \lambda)}$). Here, the convergence rate is such as:

$$\pi_k - \pi_{k-1} = (1/2)\ (\pi_{k-1} - \pi_{k-2}) \tag{46}$$

Other bi-level schemes have been considered in (Alou et al, 2006) and (Brotcome et al, 2000) for different transportation problems while numerical convergence conditions have been discussed in (Scheel et al, 2000).

7. CONCLUSION

This communication has considered the problem of long term forecasting of freight transportation in a large network. The proposed approach has introduced two different optimization models: A model devoted to demand forecasting and another one describing a profit maximization supply behavior by transportation firms. The distribution demand forecasting model introduces in a new way elasticity of demand with respect to fares. The supply optimization model considers simultaneously two flows: vehicle flows providing transportation capacity and freight flows generating revenues to the transport operators. Then a global optimization problem is associated to each scenario with respect to demand and operations costs. A solution scheme composed of an iterative process between the current solutions of the demand and the supply optimization problems is proposed: the entropy maximizing problem provides the freight origin-destination matrices given the fare structure, while the supply optimization problem provides the fare structure given freight origin-destination matrices. Convergence conditions are discussed for this iterative process between the two levels, which can be seen as inverse of each other.

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