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Abstract – To overcome the traffic growth predicted by current ATM research programs in Europe and the US, we propose a new model to avoid conflicts based on small speed regulations, as depicted in the ERASMUS project [6], with interval conflict constraints as in [2].

After a first conflict detection phase, a centralized solver computes new RTAs to dynamically adjust the flight plans during the flight, taking operational costs for airlines and for ATC into account. The resolution would be iteratively performed over a rolling horizon to handle the uncertainties inherent to trajectory prediction.

The described model is currently being implemented using Constraint Programming and Local Search as optimization techniques. Simulations will be carried out with Europe-wide traffic data.

Key words – Air Traffic Management, Speed Regulation, Conflict Avoidance

I. INTRODUCTION

For more than forty years, global air traffic has never ceased to increase. The current traffic control systems are reaching their structural limits, so that the traffic growth might reduce the safety level of the airspace. Thus, new methods and concepts are to be set up in order to adapt to future traffic, as advocated by research programmes such as SESAR[1] [5] in Europe or *NextGen* in the US. Therefore, in this paper, we propose a model for speed regulation in order to avoid a maximum number of conflicts. This work is built upon the conflict model presented in [2] and uses small speed regulations like in [4] to avoid conflicts.

A standard day of traffic within the european airspace is made of approximately 30,000 flights. Each flight follows a flight plan described as a sequence of waypoints that the aircraft will fly over. The aim of this project is to avoid the air space conflicts in advance by the use of slight modifications of flight speeds, or equivalently by constraining fly over times on waypoints (RTA[2]), in order to decrease the controllers' workload. Our model assumes that a conflict between two flights that follow intersecting trajectories can only happen on a common waypoint of their routes – catching-up flights are handled by considering multiple waypoints. After a first potential conflict detection step, resulting in a combinatorial optimization problem that generalizes the one presented in [2], the RTAs are computed by a centralized system so as to avoid as many conflicts as possible and

to optimize operating costs. Then they are transmitted to aircraft during the flight to modify their flight plans. To take uncertainties into account, the problem would be iteratively solved over a rolling horizon (typically 20–30 min) as presented in [4].

This combinatorial optimization problem could be solved by state-of-the-art techniques such as Constraint Programming as in [2] or Local Search as in [1].

II. SEPARATION CONSTRAINTS

As in [4], this model is designed to take into account the functionalities of future Flight Management Systems (FMS) in the SESAR context, which will be able to dynamically accommodate several RTAs on the waypoints of their trajectories with an accuracy of a few seconds. Its output is therefore a set of RTAs for each flight involved in a potential conflict within the time window of the resolution, trying to minimize the number of actual conflicts and their durations.

In this context, the trajectory of a flight i is represented by a sequence of 3D points and associated times corresponding to the waypoints of its route indicated by the flight plan:

$$\left\{ \left(\omega_i^k, \theta_i^k \right), \ k \in [1, n_i] \right\}$$

where the $\left(\theta_i^k \right)_{i,k}$ are the decision variables and n_i the number of waypoints of flight i. Furthermore, we note:

- σ_i^k, the curvilinear abscissa (or oriented length along the trajectory) of each waypoint k of flight i;

- v_i^k (for all $k \in [1, n_i - 1]$), the speed, considered constant, between waypoints ω_i^k and ω_i^{k+1}.

DISCRETIZATION OF TRAJECTORIES

Following the conflict model presented in [2], potential conflicts between two flights i and j are detected by pairwise checks on the points of a discretization of each trajectory, with a grain fine enough to ensure that even the shortest potential conflicts will be taken into account, as described in [3] for example. We note:

$$\left\{ (p_i^{k'}, t_i^{k'}), \ k' \in [1, m_i] \right\}$$

the discretization of the trajectory of flight i consisting of a sequence of m_i 3D points $p_i^{k'}$ and associated times $t_i^{k'}$, as illustrated in 1.

Similarly to the waypoints, we note $s_i^{k'}$ the curvilinear abscissa of each point $p_i^{k'}$. These abscissæ can be easily
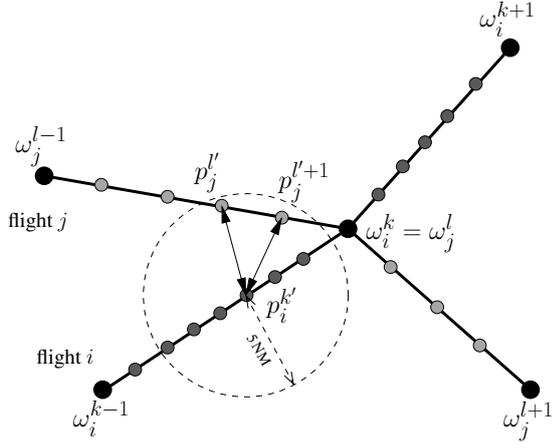

Figure 1: Projection in the horizontal plane of the trajectories of flights i and j, which are in *potential conflict* at their common waypoint $\omega_i^k = \omega_j^l$. Point $p_i^{k'}$ is in potential conflict with points $p_j^{l'}$ and $p_j^{l'+1}$.

computed by adding the distances between the previous points of the trajectory:

$$s_i^{k'} = \sum_{l'=1}^{k'-1} dist(p_i^{l'}, \ p_i^{l'+1})$$

providing that the turning points of the trajectory, i.e. the waypoints ω_i^k, are included in the discretization points $p_i^{k'}$. A similar relation would be true for the abscissæ σ_i^k of the waypoints (as they indeed are the turning points), but only during cruise, as the waypoints are too distant from each other to approximate the vertical profile of the descent or climb phase precisely enough.

CONFLICT DETECTION

Two points $p_i^{k'}$ and $p_j^{l'}$ of flight i and j are in *potential conflict* if their horizontal *and* vertical distances both violates the separation norm (usually $5\,\mathrm{NM}$ horizontally and $1000\,\mathrm{ft}$ vertically), as depicted in figure 1 for the horizontal plane. For two such points, there would be an actual *conflict* between flights i and j if they are located at these points at the same time. To avoid a conflict, it is therefore necessary that:

$$\forall k' \in [1, m_i], \forall l' \in [1, m_j],$$
$$\mathrm{dist_H}(p_i^{k'}, p_j^{l'}) < 5\,NM \ \wedge \ \mathrm{dist_V}(p_i^{k'}, p_j^{l'}) < 1000\,ft$$
$$\implies t_i^{k'} \neq t_j^{l'}$$

(1)

where $\mathrm{dist_H}$ is the distance in the horizontal plane and $\mathrm{dist_V}$ in the vertical plane.

For intersecting trajectories, we assume, as in [4], that successive potentially conflicting points are all located in the vicinity of the same waypoint. As the speed is considered constant between two consecutive waypoints, it is possible to translate these conflict inequations 1 to their closest waypoint, such that the conflict constraints can be expressed as a temporal separation at this waypoint.

The resolution will therefore consist in regulating the speed of these flights (through the issuing of RTAs at the waypoints only) so as to avoid conflicts, i.e. ensure that the corresponding inequation $\theta_i^k \neq \theta_j^l$ holds for all the neighbouring pairs of potentially conflicting points $p_i^{k'}$ and $p_j^{l'}$.

However, catch-up conflicts along the same route portion (or if the angle of intersecting trajectories is very low) cannot be considered local to a single waypoint. Thus, all potentially conflicting points located between waypoints ω_i^{k-1} and ω_i^{k+1} are reported to a potential conflict associated with ω_i^k. Several such conflicts will then be defined at each successive waypoints as long as the flights follow the same route.

We can now define a generalized (intersecting or catching-up) potential conflict between two flights i and j as the set of conflicting pairs of trajectory points around a single waypoint:

$$C_{ij}^{kl} = \ \{(k', l') \in [1, m_i] \times [1, m_j], \text{ s.t.}$$
$$p_i^{k'} \text{ and } p_j^{l'} \text{ are in potential conflict near } \omega_i^k = \omega_j^l,$$
$$\text{and } \sigma_i^{k-1} < s_i^{k'} < \sigma_i^{k+1}, \ \sigma_j^{l-1} < s_j^{l'} < \sigma_j^{l+1}\}$$

To compute the resulting contraint between θ_i^k and θ_j^l, we first need to express $t_i^{k'}$ as a function of θ_i^k and the speed of the aircraft, which is v_i^{k-1} before waypoint ω_i^k and v_i^k afterwards. Therefore:

$$\forall k' \in [1, m_i] \ , \exists k \in [1, n_i] \ ,$$
$$t_i^{k'} = \begin{cases} \theta_i^k + \frac{s_i^{k'} - \sigma_i^k}{v_i^k} \text{ si } s_i^{k'} \geq \sigma_i^k \\ \theta_i^k + \frac{s_i^{k'} - \sigma_i^k}{v_i^{k-1}} \text{ si } s_i^{k'} < \sigma_i^k \end{cases}$$

(2)

Inequation 1 and equation 2 can then be combined, with four different cases depending on the locations of points $p_i^{k'}$ and $p_j^{l'}$ with respect to waypoint $\omega_i^k = \omega_j^l$. Note that if flights i and j have several distinct (non continuous) potential conflicts, there will be as many (non empty) sets C_{ij}^{kl} of conflicting points:

$$\forall (k, l) \in [1, n_i] \times [1, n_j] \text{ s.t. } C_{ij}^{kl} \neq \emptyset, \forall (k', l') \in C_{ij}^{kl},$$
$$t_i^{k'} \neq t_j^{l'} \ \Leftrightarrow$$
$$\theta_i^k - \theta_j^l \neq \begin{cases} \frac{s_j^{l'} - \sigma_j^l}{v_j^l} - \frac{s_i^{k'} - \sigma_i^k}{v_i^k} \text{ if } s_j^{l'} \geq \sigma_j^l \text{ and } s_i^{k'} \geq \sigma_i^k \\ \frac{s_j^{l'} - \sigma_j^l}{v_j^l} - \frac{s_i^{k'} - \sigma_i^k}{v_i^{k-1}} \text{ if } s_j^{l'} \geq \sigma_j^l \text{ and } s_i^{k'} < \sigma_i^k \\ \frac{s_j^{l'} - \sigma_j^l}{v_j^{l-1}} - \frac{s_i^{k'} - \sigma_i^k}{v_i^k} \text{ if } s_j^{l'} < \sigma_j^l \text{ and } s_i^{k'} \geq \sigma_i^k \\ \frac{s_j^{l'} - \sigma_j^l}{v_j^{l-1}} - \frac{s_i^{k'} - \sigma_i^k}{v_i^{k-1}} \text{ if } s_j^{l'} < \sigma_j^l \text{ and } s_i^{k'} < \sigma_i^k \end{cases}$$

(3)

However, these expressions depend on the (unknown) variable speeds of aircraft, whereas the conflict model presented in [2] uses static bounds for the time difference between flights i and j at waypoint $\omega_i^k = \omega_j^l$. In the next section, we explain how the speed variations are tightly bounded in our operational context, which allows us to approximate the values of equation 3 by small intervals.

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III. SMALL SPEED ADJUSTMENT AND CONFLICT APPROXIMATION

Following [4], we consider the same kind of small speed variation as in the ERASMUS project [6]. Two ratio parameters $\underline{\alpha} \leq 1$ and $\overline{\alpha} \geq 1$ are introduced to bound the possible speed adjustment, such that if we note v_i^0 the reference speed of flight i, all other speed variables are restricted to take values in a small interval $\left[\underline{\alpha}\, v_i^0, \overline{\alpha}\, v_i^0\right]$:

$$\forall i \in [1, n], \forall k \in [1, n_i - 1],\ v_i^k \in \left[\underline{\alpha}\, v_i^0, \overline{\alpha}\, v_i^0\right] \quad (4)$$

where n is the number of flights in the instance.

These ratio parameters are typically chosen in the range $\underline{\alpha} = 0.97$ and $\overline{\alpha} = 1.06$ to limit the cost of regulation on the fuel consumption of aircraft and to have the smallest impact possible on standard Air Traffic Control (ATC) practices.

As the speed variations considered are small and bounded, we can bound the values of $\theta_i^k - \theta_j^l$ described in equation 3 by constants $\underline{r_{ij}^{k'l'}}$ (lower bound) and $\overline{r_{ij}^{k'l'}}$ (upper bound) defined as follows:

$$\forall (k,l) \in [1, n_i] \times [1, n_j] \text{ s.t. } C_{ij}^{kl} \neq \emptyset,\ \forall (k', l') \in C_{ij}^{kl},$$

$$\underline{r_{ij}^{k'l'}} = \begin{cases} \frac{s_j^{l'} - \sigma_j^l}{\overline{\alpha}\, v_j^0} - \frac{s_i^{k'} - \sigma_i^k}{\underline{\alpha}\, v_i^0} & \text{if } s_j^{l'} \geq \sigma_j^l \text{ and } s_i^{k'} \geq \sigma_i^k \\[6pt] \frac{s_j^{l'} - \sigma_j^l}{\overline{\alpha}\, v_j^0} + \frac{\sigma_i^k - s_i^{k'}}{\overline{\alpha}\, v_i^0} & \text{if } s_j^{l'} \geq \sigma_j^l \text{ and } s_i^{k'} < \sigma_i^k \\[6pt] -\frac{\sigma_j^l - s_j^{l'}}{\underline{\alpha}\, v_j^0} - \frac{s_i^{k'} - \sigma_i^k}{\underline{\alpha}\, v_i^0} & \text{if } s_j^{l'} < \sigma_j^l \text{ and } s_i^{k'} \geq \sigma_i^k \\[6pt] -\frac{\sigma_j^l - s_j^{l'}}{\underline{\alpha}\, v_j^0} + \frac{\sigma_i^k - s_i^{k'}}{\overline{\alpha}\, v_i^0} & \text{if } s_j^{l'} < \sigma_j^l \text{ and } s_i^{k'} < \sigma_i^k \end{cases}$$

$$\overline{r_{ij}^{k'l'}} = \begin{cases} \frac{s_j^{l'} - \sigma_j^l}{\underline{\alpha}\, v_j^0} - \frac{s_i^{k'} - \sigma_i^k}{\overline{\alpha}\, v_i^0} & \text{if } s_j^{l'} \geq \sigma_j^l \text{ and } s_i^{k'} \geq \sigma_i^k \\[6pt] \frac{s_j^{l'} - \sigma_j^l}{\underline{\alpha}\, v_j^0} + \frac{\sigma_i^k - s_i^{k'}}{\underline{\alpha}\, v_i^0} & \text{if } s_j^{l'} \geq \sigma_j^l \text{ and } s_i^{k'} < \sigma_i^k \\[6pt] -\frac{\sigma_j^l - s_j^{l'}}{\overline{\alpha}\, v_j^0} - \frac{s_i^{k'} - \sigma_i^k}{\overline{\alpha}\, v_i^0} & \text{if } s_j^{l'} < \sigma_j^l \text{ and } s_i^{k'} \geq \sigma_i^k \\[6pt] -\frac{\sigma_j^l - s_j^{l'}}{\overline{\alpha}\, v_j^0} + \frac{\sigma_i^k - s_i^{k'}}{\underline{\alpha}\, v_i^0} & \text{if } s_j^{l'} < \sigma_j^l \text{ and } s_i^{k'} < \sigma_i^k \end{cases}$$

$$(5)$$

For one conflict set C_{ij}^{kl}, bounds of a forbidden interval for the difference of times $\theta_i^k - \theta_j^l$ at the waypoint can then be computed by cumulating the inequations over the pairs of conflicting points $(k', l') \in C_{ij}^{kl}$:

$$\underline{r_{ij}^{kl}} = \min_{(k', l') \in C_{ij}^{kl}} \underline{r_{ij}^{k'l'}}$$

$$\overline{r_{ij}^{kl}} = \max_{(k', l') \in C_{ij}^{kl}} \overline{r_{ij}^{k'l'}}$$

Moreover, the bounds on the speed of each aircraft further constrain the θ_i^k variables for two consecutive waypoints ω_i^k and ω_i^{k+1}:

$$\forall k \in [1, n_i - 1],\ \theta_i^{k+1} - \theta_i^k \in \left[\frac{d_i^k}{\overline{\alpha}\, v_i^0}, \frac{d_i^k}{\underline{\alpha}\, v_i^0}\right] \quad (6)$$

with $d_i^k = \sigma_i^{k+1} - \sigma_i^k$, the distance between ω_i^k and ω_i^{k+1}. In the following, these bounds on the travel time of flight i between two consecutive waypoints are respectively noted $\underline{T_i^k} = \frac{d_i^k}{\overline{\alpha} v_i^0}$ and $\overline{T_i^k} = \frac{d_i^k}{\underline{\alpha} v_i^0}$.

IV. MODEL

The conflict detection processing of the previous sections can now be sum up with a more standard (and concise) combinatorial decision problem formulation, with variables θ_i^k representing the RTA of flight i on waypoint ω_i^k, for a set of n potentially conflicting flights over the time window considered:

$$
\begin{aligned}
\text{Find:}\quad & \forall i \in [1, n],\ (\theta_i^k)_{k \in [1, n_i]} \\
\text{s.t.:}\quad & \forall (i, j) \in [1, n]^2, i < j, \\
& \forall (k, l) \in [1, n_i] \times [1, n_j] \text{ s.t. } C_{ij}^{kl} \neq \emptyset, \\
& \quad \theta_i^k - \theta_j^l \notin \left[\underline{r_{ij}^{kl}}, \overline{r_{ij}^{kl}}\right] \\
& \forall i,\ \forall k \in [1, m_i - 1], \\
& \quad \theta_i^{k+1} - \theta_i^k \in \left[\underline{T_i^k}, \overline{T_i^k}\right]
\end{aligned}
$$

The first set of constraints are derived from the conflicts between two potentially conflicting flights i and j, $\underline{r_{ij}^{kl}}$ and $\overline{r_{ij}^{kl}}$ being the bounds of the forbidden values for the difference of fly times over the conflicting waypoint $\omega_i^k = \omega_j^l$. The second one characterizes the speed limitation for each aircraft as stated in equation 6.

OPTIMIZATION

Among the admissible solutions of this decision problem, the ones that minimize airlines costs should be preferred. To optimize their operating costs, airlines generally tune the Cost Index (CI) parameter (used by the FMS to optimize the flight parameters along its trajectory) for each flight, which represents the relative importance of the cost of fuel with respect to the cost of flight time, i.e.:

$$\text{CI} = \frac{\text{cost}_{\text{time}}}{\text{cost}_{\text{fuel}}}$$

where $\text{cost}_{\text{time}}$ is in \$ per time unit, and $\text{cost}_{\text{fuel}}$ in \$ per mass unit.

For a given flight i, we can therefore consider that the airline cost is the sum of the extra cost of time and of the extra cost of fuel multiplied by CI_i (the Cost Index of flight i), compared to the reference trajectory at the reference speed:

$$\text{cost}_i = \text{CI}_i \times \text{cost}_i^{\text{fuel}} + T_i^0 - \left(\theta_i^{n_i} - \theta_i^1\right)$$

where $T_i^0 = \sum_{k=1}^{n_i - 1} \frac{d^k}{v_i^0}$ is the total flight time of flight i at its reference speed v_i^0.

If we assume that v_i^0 is the optimal speed of the flight and that a discrepancy from v_i^0 leads to a proportional increase of fuel consumption during the time flown at this speed, then:

$$
\begin{aligned}
\text{cost}_i^{\text{fuel}} &= \sum_{k=1}^{n_i - 1} \frac{|v_i^k - v_i^0|}{v_i^0} \left(\theta_i^{k+1} - \theta_i^k\right) \\
&= \sum_{k=1}^{n_i - 1} \left|\frac{d_i^k}{v_i^0} - \left(\theta_i^{k+1} - \theta_i^k\right)\right|
\end{aligned}
$$

So the cost for one flight can be expressed as:

$$\text{cost}_i = \text{CI}_i \sum_{k=1}^{n_i-1} \left| \frac{d_i^k}{v_i^0} - \left(\theta_i^{k+1} - \theta_i^k\right) \right| + T_i^0 - \left(\theta_i^{n_i} - \theta_i^1\right)$$

and the total cost of one solution simply is the sum of the costs of all flights:

$$\text{cost} = \sum_{i=1}^{n} \text{cost}_i \qquad (7)$$

Note that other parameters could be taken into account, like the number of speed changes, or could use a more realistic function to estimate the effect of the speed discrepancy from the reference speed, provided we could gather enough data from airlines and manufacturers.

INFEASIBILITY

If some conflict constraints cannot be satisfied, it may still be of interest to relax these constraints and search for solutions with as few violated constraints as possible. Furthermore, when a conflict constraint is violated, the longer the conflict lasts, the more hazardous is the operational situation, so it is interesting as well to search for solutions that minimize the duration of all actual conflicts, as proposed in [4].

As the conflict constraints of our model enforce that $\theta_i^k - \theta_j^l$ does not belong to $\left[\underline{r_{ij}^{kl}}, \overline{r_{ij}^{kl}} \right]$, we have to compute the "distance" of the time difference from the center of the interval $M_{ij}^{kl} = \frac{1}{2} \left(\underline{r_{ij}^{kl}} + \overline{r_{ij}^{kl}} \right)$:

$$\text{dist}_{ij}^{kl} = \left| \left(\theta_i^k - \theta_j^l\right) - M_{ij}^{kl} \right|$$

The longer the conflict lasts, the closer $\theta_i^k - \theta_j^l$ is to M_{ij}^{kl}, and the smaller is dist_{ij}^{kl}. So to obtain a measure of the duration of the conflict, the distance is subtracted from the half of the length of the interval $\left[\underline{r_{ij}^{kl}}, \overline{r_{ij}^{kl}} \right]$, i.e. from $L_{ij}^{kl} = \overline{r_{ij}^{kl}} - \underline{r_{ij}^{kl}}$. Moreover, the cost should be 0 outside the conflict interval:

$$\max\left(0, \frac{1}{2} L_{ij}^{kl} - \left|\left(\theta_i^k - \theta_j^l\right) - M_{ij}^{kl}\right|\right)$$

The total cost over all remaining conflicts is the sum of all the actual conflicts durations, for all pairs of flights in potential conflict (i.e. $\forall i < j$ s.t. $\exists(k,l), C_{ij}^{kl} \neq \emptyset$):

$$\text{cost}_{\text{conflict}} = \sum_{i<j} \sum_{\substack{k,l \text{ s.t.} \\ C_{ij}^{kl} \neq \emptyset}} \max\left(0, \frac{1}{2} L_{ij}^{kl} - \left|\left(\theta_i^k - \theta_j^l\right) - M_{ij}^{kl}\right|\right)$$

This cost could be combined with the cost defined in equation 7 with suitable weighting to balance the cost of remaining conflicts with the operational one, so as to take both criteria into account for overconstrained instances.

V. CONCLUSIONS AND PERSPECTIVES

To overcome the traffic growth predicted by current ATM research programs in Europe, we propose a novel deconfliction model based on small speed regulations, following the mixed integer program presented in [4] with a conflict model that generalizes the work presented in [2]. The associated centralized solver would output new RTAs to dynamically adjust the flight plans during the flight, taking operational costs for airlines *and* for ATC into account. The resolution would be iteratively performed over a rolling horizon (20–30 min) to handle the uncertainties inherent to trajectory prediction.

The described model is currently being implemented using Constraint Programming and Local Search as optimization techniques. Simulations will be carried out with Europe-wide traffic data.

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