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# Dynamic Programming for Trajectory Optimization of Engine-out Transportation Aircraft

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**Abstract:** The purpose of this communication is to contribute to the development of a new trajectory management capability for an engine-out transportation aircraft. Engine-out is a dramatic situation for flight safety and this study focuses on the design of a management system for emergency trajectories at this special situation. First the gliding characteristics and flying qualities of a transport aircraft with total engine failure are analyzed while gliding range estimation is considered. Then a new representation of the flight dynamics of an engine-out aircraft is proposed where the space variable is chosen as independent parameter instead of the time variable. This allows to propose a new formulation of the corresponding trajectory optimization problem and to develop a reverse dynamic programming solution technique. Simulation results are displayed and new development perspectives are discussed.

**Key Words:** Flight Safety, Trajectory Optimization, Quasi Steady Glide, Reverse Dynamic Programming

## 1 INTRODUCTION

The failure of engines is a dramatic event for air flight safety and many incidents and accidents are resulting from engine failure. Here an undesirable and very special case, all engines out at a given point of the flight, is considered. This situation may lead to a crash unless a flyable descent trajectory towards a safe landing place is performed. There are many different reasons for engine-out while it appears that in this situation any wrong decision made by the pilots may lead to catastrophic consequences.

So it looks quite desirable to develop an emergency guidance mode for this situation. This new functionality could be integrated in a Flight Management System which should be able to select a proper landing site and propose a feasible trajectory towards this site.

To achieve this purpose there are major steps which should be performed: establish and analyze the flight dynamics of an air transportation aircraft with total engine failure (power off), study the gliding characteristics and flying qualities of a transportation aircraft, develop a method to establish safe reachable areas from a given situation and finally develop a method to optimize a gliding trajectory towards a possible safe landing place.

In this study, it is supposed that engine out occurs once the aircraft has already gained some speed and altitude after take-off. Only glide of engine-out airplane in the vertical plane is considered as a start.

## 2 ENGINE OUT FLIGHT DYNAMICS

To establish and analyze the flight dynamics of an air transportation aircraft with total engine failure (power off), the classical equations of flight should be slightly adapted to this particular case.

The aerodynamic forces (drag  $D$ , lift  $L$ , and side force  $Y$ ) are defined in terms of dynamic pressure, reference area and dimensionless aerodynamic coefficients [1]:

$$D = 1/2 \rho V^2 S \cdot C_D(\alpha, \delta_e, M_a) \quad (1-1)$$

$$L = 1/2 \rho V^2 S \cdot C_L(\alpha, \delta_e, M_a) \quad (1-2)$$

$$Y = 1/2 \rho V^2 S \cdot C_Y(\beta, \delta_r, p, r, M_a) \quad (1-3)$$

Here  $V$  is the airspeed,  $\rho$  is the air density ( $\text{kg/m}^3$ ),  $\alpha$  is the angle of attack,  $\delta_e$  is the elevator deflection,  $p$  is the aircraft roll rate,  $r$  is the yaw rate and  $M$  is the current Mach number,  $C_D$ ,  $C_L$  and  $C_Y$  are dimensionless aerodynamic coefficients.  $C_D$  and  $C_L$  are supposed related by the polar model  $C_D = C_{D0} + K C_L^2$  where  $K$  is a constant.

It is considered that some hydraulic power remains available to activate the elevators, ailerons and rudders aerodynamic surfaces, so that dynamic stability as well as attitude control can still be performed by the flight control system. Indeed, many transport aircraft are equipped with an deployable auxiliary turbine (RAM) which allows insuring in the control channels of the aircraft the availability of a residual hydraulic power. While the additional drag generated by the RAM remains minor [2], the extinction of the aircraft engines results in a noticeable increase of the drag, while lift and side forces remain quite the same. The drag coefficient is now given by:

$$C'_D = C_D(\alpha, \delta_e, M_a) + nC_{DE}(\alpha, M_a) \quad (2)$$

where  $C_{DE}$  is the additional drag of a shut down engine, and  $n$  is the number of engines of the aircraft.

The flight equations can be written as:

$$\dot{V} = -\frac{1}{m}(D(V, \rho, \theta - \gamma) + mg \sin \gamma) \quad (3-1)$$

$$\dot{\gamma} = \frac{1}{mV}(L(V, \rho, \theta - \gamma) - mg \cos \gamma) \quad (3-2)$$

$$\dot{x} = -V \cos \gamma \quad (3-3)$$

$$\dot{z} = V \sin \gamma \quad (3-4)$$

where  $\gamma$  is the path angle,  $\theta$  is the pitch angle. Once fuel dumping has been performed, the mass  $m$  of the aircraft is considered to remain constant. Here  $x$  and  $z$  are respectively the current longitudinal and vertical positions of the aircraft center of gravity. Then, its height above Earth is given by:

$$h = z - H(x) \quad (4)$$

where  $H(x)$  is ground level at position  $x$ .

### 3 ESTIMATION OF GLIDING RANGE

A first estimation of gliding range can be obtained by considering that the aircraft remains in a quasi steady gliding condition where air speed and path angle change steadily according to current air density during the whole descent.

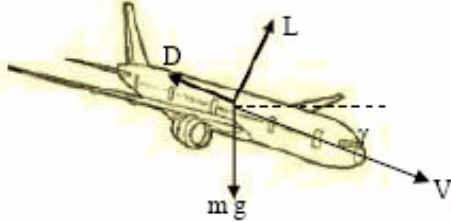


Fig 1. Aircraft forces for quasi steady descent

In this situation the path angle is such as [3]:

$$\gamma = -\arcsin((1/f) + (\dot{V}/g)) \quad (5)$$

or according to [4]:

$$\gamma = -\arcsin\left(\frac{mg}{L} + \frac{dE_T}{dz}\right) \quad \text{where} \quad E_T = mgz + \frac{1}{2}mV^2 \quad (6)$$

is the aircraft total energy. Here  $f=L/D$  is the lift to drag ratio. According to equation (5), a “minimum” glide angle,  $-\gamma_{\max}$ , is achieved with a maximum lift to drag ratio. Since  $L \approx mg$ , this corresponds to a minimum drag. Then it can be shown that:

$$-\gamma_{\max} = 1/(2\sqrt{C'_{D0} K}) \quad (7-1)$$

$$D_{\min} = \frac{1}{2}\rho(z)V^2(2C'_{D0}) = Q \cdot (2C'_{D0}) \quad (7-2)$$

A first approximation to the maximum range to sea level is then given by:

$$R_{a0} = 2z_0\sqrt{KC'_{D0}} \quad (8)$$

where  $z_0$  is the initial altitude. Now, considering the above expression of  $D_{\min}$ , we get:

$$V(z) = \sqrt{(2mg / (\rho(z) S)) \sqrt{K / C'_{D0}}} \quad (9)$$

Then the air speed decreases during the quasi static glide descent. For a wide body aircraft, from cruise level, about 8 m/s are lost for a quasi steady initial descent of 1000 m. A stall constraint can be considered to check the feasibility of the glide maneuver:

$$V(z) > V_{stall}(z) = \sqrt{2mg / (\rho(z) S C_{L\max})} \quad (10-1)$$

or

$$\sqrt{K / C'_{D0}} > 1 / C_{L\max} \quad (10-2)$$

This condition is a general aerodynamic condition for gliding feasibility of a given aircraft under a specific aerodynamic situation. Also, the expression of  $D_{\min}$  shows that dynamic pressure remains constant during the quasi static glide. Figure 2. displays the airspeed and stall speed during steady descent.

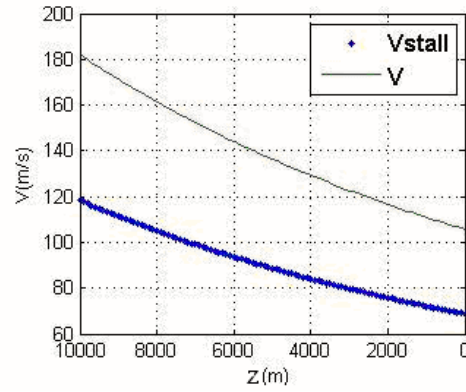


Fig 2. Airspeed during steady gliding

Then a relation between the quasi static glide path angle and the altitude can be introduced and the “minimum” glide path angle is given by:

$$-\gamma_{\max} \approx -\tan \gamma = \frac{2\sqrt{C'_{D0} K}}{1 + \frac{Q}{P_0((T_0 + a_0 z) / T_0)^{-g/(a_0 R)}}} \quad (11)$$

where air density in standard atmosphere can be expressed as  $\rho(z) = \rho_0 e^{-\frac{g}{RT}z}$  with  $T(z) = T_0 + a_0 z$ ,  $T_0 = 288.15^\circ K$ ,  $a_0 = -6.5 \times 10^{-3}^\circ K/m$ ,  $R = 287 m^2 / s^2 \cdot K$ ,  $\rho_0 = 1.2250 kg / m^3$ .

Then this angle increases while the altitude is decreasing during the quasi steady glide, shown in Figure 3. The maximum flight range  $R_a$  is then more accurately determined by:

$$R_a = \int_{z_0}^0 \frac{1}{\tan \gamma} dz = 2z_0\sqrt{KC'_{D0}} + \frac{mg}{2SC'_{D0}} \frac{1}{P_0} \frac{T_0 R}{g + a_0 R} \left( \left( 1 + \frac{a_0 z_0}{T_0} \right)^{\frac{g}{a_0 R} + 1} - 1 \right) \quad (12)$$

This is illustrated in Figure 4. If the aircraft loses engine power at a higher altitude, it can glide over an increased

range. In the case of the accident occurred on 24/08/2001, the A330 aircraft glided for 120 km.

With this information, the reachable landing site can be determined according to some flight planner [5], [6].

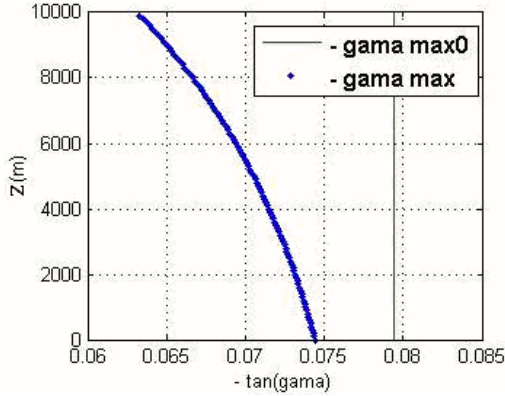


Fig 3. ‘- tan  $\gamma$ ’ during quasi steady gliding

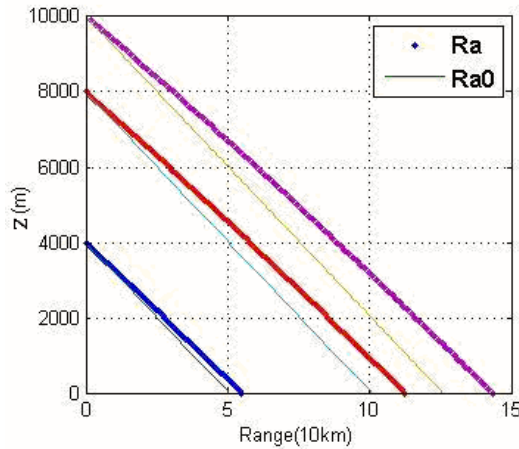


Fig. 4 Reachable range for quasi steady glide.

#### 4 GLIDE TRAJECTORY OPTIMIZATION FOR SAFETY

In this section the problem of managing the trajectory of a transportation aircraft gliding from a given initial flight situation is considered. Contrarily to the classical max range gliding problem, by the end of the gliding maneuver, the aircraft must be in conditions (speed and attitude) to perform a safe touch down at landing. In this case, the flight guidance equations written in the aircraft wind axis are given by equation (3).

Observe the equations that the only independent input parameter which is available here is the pitch angle,  $\theta$ , which can, even in an engine-out situation, be controlled by the pilot either through the hydraulic power provided by the

RAT or the auxiliary power unit-APU, or through the trim control channel.

Here the initial flight conditions are written as:

$$x(0) = x_0, h(0) = h_0, V(0) = V_0, \gamma(0) = \gamma_0 \quad (13)$$

while the final landing conditions are such as:

$$h(t_f) = h_G(x(t_f)), V(t_f) = V_1, \gamma(t_f) = \gamma_1 \quad (14)$$

where  $V_1$  and  $\gamma_1$  should allow a safe landing at altitude  $h_G(x(t_f))$  where function  $h_G$  is representative of the ground topography under the considered flight area.

Since final time is unknown and is only characterized by the satisfaction of the final conditions, the replacement of independent parameter  $t$  by the space variable  $x$  allows to diminish the complexity of the problem since now final  $x_f$  is known once the landing site has been chosen. Moreover, this approach should facilitate the consideration of ground separation constraints and could make easier the consideration of the effect of wind over the glide trajectory. From equations (3) with :

$$dt/dx = 1/(V \cos \gamma) \quad (15)$$

we get:

$$z' = tg \gamma \quad (16-1)$$

$$V' = -\frac{1}{mV \cos \gamma} (D(V, \rho, \theta - \gamma) + mg \sin \gamma) \quad (16-2)$$

$$\gamma' = \frac{1}{mV^2 \cos \gamma} (L(V, \rho, \theta - \gamma) - mg \cos \gamma) \quad (16-3)$$

where “ ’ ” represent the derivative with respect to the longitudinal position  $x$  of the aircraft. The additional instant constraints are:

$$V(x) \geq V_{\min}(z) \quad x \in [x_0, x_f], z \in [z_0, z_f] \quad (17-1)$$

$$\max\{\theta_{\min}, \alpha_{\min} + \gamma(x)\} \leq \theta(x) \leq \min\{\theta_{\max}, \alpha_{\max} + \gamma(x)\} \quad (17-2)$$

$$z(x) \geq h_G(x) \quad x \in [x_0, x_f] \quad (17-3)$$

Constraints (17-1) and (17-2) prevent from stalling and constraint (17-3) from some flight into terrain-FIT situation at an intermediary point of the glide.

Then, different formulations of an optimization problem [7] can be considered to design a safe glide trajectory. For example the following criterion could be minimized with respect to the successive values of  $\theta$  along the glide:

$$\min (h(x_f) - h_G(x_f))^2 \quad (18)$$

under final constraints

$$V_1(1 - v_{\min}) \leq V(x_f) \leq V_1(1 + v_{\max}) \quad (19-1)$$

$$\gamma_1(1 - g_{\min}) \leq \gamma(x_f) \leq \gamma_1(1 + g_{\max}) \quad (19-2)$$

where  $v_{\min}, v_{\max}, g_{\min}$  and  $g_{\max}$  are positive margins and with state equations (16), flight constraints (17) and initial conditions (13).

The solution of this non linear, strongly constrained trajectory optimization problem is difficult from the numerical point of view and a direct on line computation of its solution does not appear to be feasible. For instance, an approach based on the minimum principle [8] should result in a very difficult two point boundary problem since the resulting Hamiltonian has not an affine structure with

respect to the input parameter. Many other complex techniques have been developed for trajectory generation [9], [10], [11] while Dynamic Programming [12] appears to provide some good perspectives [13],[14]. To apply effectively a Dynamic Programming solution strategy, a discretization of this problem appears necessary and the choice of the space variable  $x$  as independent variables for the flight equations appears most convenient.

## 5 THE PROPOSED SOLUTION STRATEGY

Here dynamic programming is used to generate a feasible glide trajectory towards a safe landing place. To insure the satisfaction of the final landing configuration given by the quality constraints (14), which is a more critical condition, a reverse approach is adopted. Then the gliding trajectory is computed backward from these final conditions through the feasible glide set defined by constraints (17) and the space discretized state equations (16). With the objective of getting a smooth flyable trajectory which avoids wasting unnecessarily the remaining hydraulic energy used to control the aerodynamic actuators (elevator, THS, flaps and aero brakes) along the engine-out glide trajectory, a new optimization criterion is adopted here. This surrogate criteria allows penalizing large variations on pitch attitude angle, descent path angle, speed and flight level, so that its evaluation along a feasible path  $P_k^i$  leading to state  $i$  at stage  $k$  is given by a formula such as:

$$C_k^i = \sum_{s \in P_k^i} (\lambda_\theta^s |\Delta\theta_s| + \lambda_\gamma^s |\Delta\gamma_s| + \lambda_{E_T}^s |\Delta E_{T_s}|) \quad (20)$$

Here  $\lambda_\theta^s$ ,  $\lambda_\gamma^s$  and  $\lambda_{E_T}^s$  are positive weights whose values change with the distance to the landing site.

Dynamic programming, either direct or reverse, considers at each stage different feasible states and selects for each of them the best path leading to them from the initial state at the first stage of the search process. Under a given value of input parameter  $\theta_s^i$  at stage  $s$ , backwards integration is used to assess the additional cost involved in going from state  $(s,i)$  to a new feasible state at the next stage of the search process.

However, whatever the size of the discrete steps adopted to perform this reverse search process, from one stage to another, a large number of new states should be generated to guarantee the accuracy of the resulting solution. This leads to an explosive number of solutions to be considered when the stage order increases. So the explosion of the points must be avoided to insure the computer processes the problem. After each backward integrating, many points should be cut by using the dynamic programming principle. Here, to alleviate this foreseeable computational burden, a heuristic melting procedure is developed where closer states to a central state of the current stage in the search process are deleted while this central state is maintained.

The distance  $\Delta_s^{ij}$  between two states  $i$  and  $j$  of stage  $s$  which

has been adopted to generate these clusters within one stage is given by:

$$\Delta_s^{ij} = \sqrt{\lambda_V \frac{(V_s^i - V_s^j)^2}{V_{\max}^2} + \lambda_z \frac{(z_s^i - z_s^j)^2}{z_{\max}^2} + \lambda_\gamma \frac{(\gamma_s^i - \gamma_s^j)^2}{\gamma_{\max}^2}} \quad (21)$$

Here a two level weighting has been adopted:  $V_{\max}$ ,  $z_{\max}$  and  $\gamma_{\max}$  are scaling parameters and  $\lambda_V$ ,  $\lambda_z$  and  $\lambda_\gamma$  with  $\lambda_V + \lambda_z + \lambda_\gamma = 1$  are positive relative weightings.

The above approach which has been developed is basically an open loop approach and requires a very large computational effort which is unlikely to be performed on board an aircraft which is already in a critical engine-out situation. Our proposal here, which should be developed in the near future is to take profit of the amount of data generated by the reverse dynamic programming search process, considering different situations and parameters such as aircraft initial flight level, altitude and mass, to train a neural network devise designed to generate pitch angle directives at each point along the descent so that the glide trajectory leads safely to the landing situation. Here the computational burden associated with reverse dynamic programming is taken into profit to generate the training data base for the neural network [15].

The generated pitch angle directives can be either sent to the autopilot when it is still operating or to a flight director. In that last case this will allow this maneuver to be performed efficiently in manual mode by the pilot. Observe that along the glide trajectory, each new solicitation of the neural network will generate new piloting directives in accordance with the current situation of the aircraft which is also the result of external perturbations such as wind.

## 6 SIMULATION RESULTS

A simulation study has been performed using the RCAM wide body transportation aircraft model [16]. Then considering the case in which an engine failure occurs 150km away from a possible landing site, different glide trajectories obtained by reverse dynamic programming are displayed on Figures 5. and on Figure 6 according to different initial situations.

It appears that if the aircraft has a large initial total energy, which means high speed and/or high altitude, the resulting glide trajectory is not be very smooth: the speed and altitude are subject to large and rapid changes so that the aircraft loses energy in excess sufficiently quickly to arrive to the landing site with acceptable flying parameters. When initial total energy is not too much excessive, the resulting glide trajectories result to be smoother.

For example, for initial conditions with an Figure 7. and Figure 8. display an optimized glide trajectory in the case in which initial altitude is 10km (FL330) and initial airspeed is 200m/s (about 400 knot).

Figure 9. and Figure 10. display the landing range which can be reached safely by an aircraft whose initial glide conditions are an altitude of 10km and an airspeed of

180m/s (about 360 knot). The largest obtained glide range is about 137 km while the shortest obtained glide range is 116 km. Then in that case, the gliding aircraft can reach safely landing sites located between 116km and 137km away. Observe on these figures that, the shorter the range, the rougher is the trajectory. Comparing figures 7. and Figure 9. , it appears also that with a higher initial airspeed the gliding aircraft range is also higher.

These numerical results indicate that reverse dynamic programming can be used to solve the glide trajectory generation problem and contribute to the design of a glide trajectory generator either off line or on line.

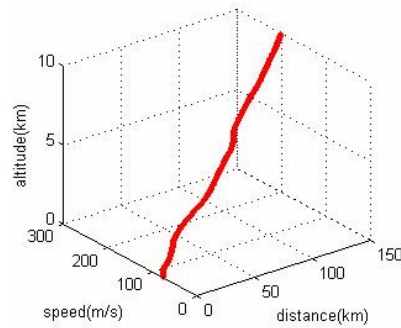


Fig 7. A smooth optimal glide trajectory in 3-D

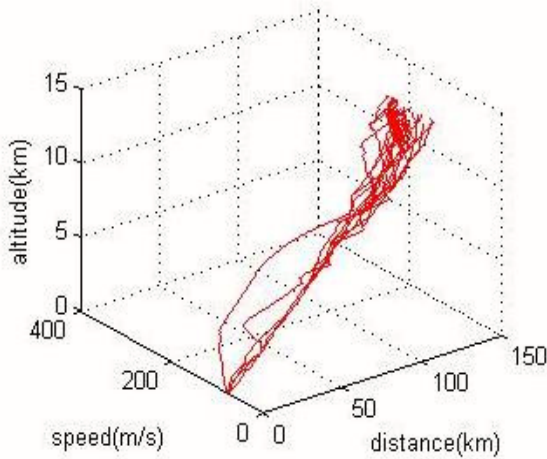


Fig 5. Optimal glide trajectories with different initial speeds 3-dimensional representation

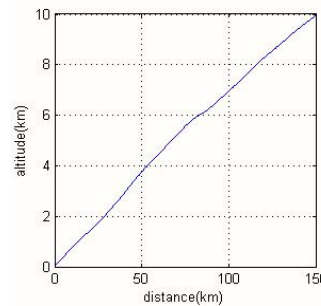


Fig 8. A smooth optimal glide trajectory in vertical plane

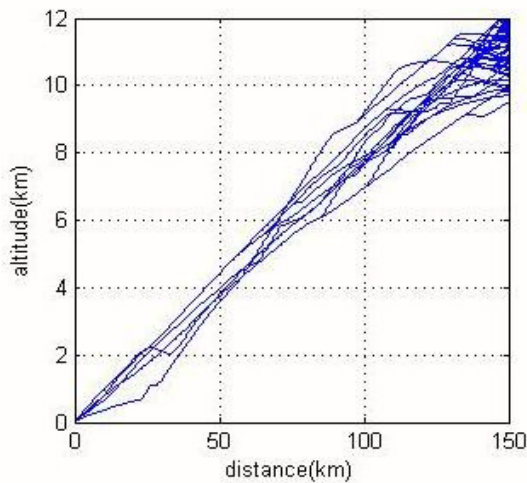


Fig. 6 Optimal glide trajectories in vertical plane with different initial altitudes

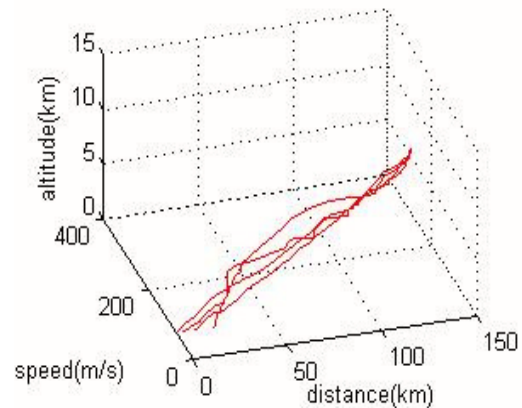


Fig 9. Trajectories of aircraft in vertical plane with different landing ranges, 3D view

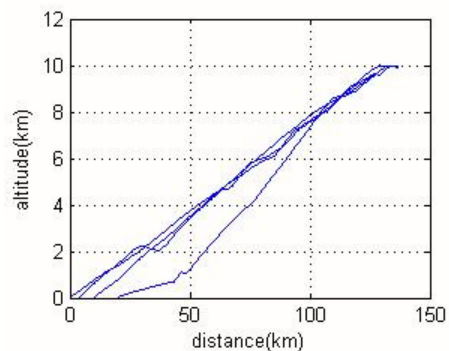


Fig 10. Trajectories of aircraft in vertical plane with different landing ranges

## 7 CONCLUSION

The purpose of this communication has been to present the first results of a study turned towards the design of an emergency management system able to cope with an engine-out situation for a transportation aircraft. The main contributions of this communication are:

- a review of the quasi steady glide range for a transport aircraft;
- the proposal of a new representation of the flight dynamics of a gliding aircraft with the introduction of a spatial dimension as independent variable;
- the development of a solution strategy based on backwards integration and reverse dynamic programming whose feasibility is supported by the displayed simulation results.

This work should be completed by the integration of lateral maneuvers and the consideration of the effect of wind over the glide trajectory. This last point could be tackled by the development of an adaptive approach based on the online estimation of wind and the use of a neural machine to generate control directives on a reactive basis. This remains for further studies.

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