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Reduced RLT constraints for polynomial programming

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Abstract. An extension of the reduced Reformulation-Linearization Technique constraints from quadratic to general polynomial programming problems with linear equality constraints is presented and a strategy to improve the associated convex relaxation is proposed.

1. Introduction

Reduced RLT constraints (rRLT) are a special class of Reformulation-Linearization Technique (RLT) constraints, that apply to nonconvex (both continuous and mixed-integer) quadratic programming problems subject to linear equality constraints [2, 4, 3]. rRLT are obtained by replacing some of the quadratic terms with suitable linear constraints. These turn out to be a subset of the RLT constraints for quadratic programming [7].

We present an extension of the rRLT theory to the case of general polynomial programs. Then, we show a strategy to choose the basis of a matrix involved in the rRLT constraints generation so as to tighten the bound of the associated convex relaxation. This allows to improve the performance of a spatial Branch-and-Bound algorithm applied to nonconvex NLP and MINLP problems where such convex relaxation is computed at each node.

2. Extending rRLT to polynomial programs

Let \( n \) be the number of variables, \( q \) the degree of the polynomials in the targeted problem and \( N = \{1, \ldots, n\}, Q = \{2, \ldots, q\} \). For each monomial \( x_{j_1} \cdots x_{j_p} \), \( p \in Q \), appearing in the problem, we define a finite sequence \( J = (j_1, \ldots, j_p) \) and consider defining constraints of the following form:

\[
\forall p \in Q, J' \in \bar{N}^{p-1} \quad A w_{J'} = b w_{J'}
\]

(2.1)

(for \( |J| = 1 \), i.e. \( J = (j) \), we also define \( w_J = x_j \)). For all \( p \in Q \), \( J \in \bar{N}^p \) and any permutation \( \pi \) in the symmetric group \( S_p \) we have that \( w_J = w_{\pi J} \) by commutativity. We therefore define an equivalence relation \( \sim \) on \( \bar{N}^p \) stating that for \( J, K \in \bar{N}^p \), \( J \sim K \) only if \( \exists \pi \in S_p \) such that \( J = \pi K \).

We then consider the index tuple set \( \bar{N}^p = N^p/\sim \) to quantify over when indexing variables \( w_{J'} \).

We multiply the original linear constraints \( Ax = b \) by all monomials \( \prod_{\ell \leq p-1} x_{j_{\ell}} \) and replace them by the corresponding added variables \( w_{(J',j)} \), where \( J' \in \bar{N}^{p-1} \). This yields the following rRLTS:

\[
\forall p \in Q, J' \in \bar{N}^{p-1} \quad A w_{J'} = b w_{J'}
\]

(2.2)

where \( w_{J'} = (w_{(J',1)}, \ldots, w_{(J',n)}) \). We then consider the companion system:

\[
\forall p \in Q, J' \in \bar{N}^{p-1} \quad A z_{J'} = 0.
\]

(2.3)

Since (2.3) is a linear homogeneous system, there is a matrix \( M \) such that the companion system is equivalent to \( M z = 0 \), the columns of which are indexed by sequences in \( \bar{N}^p \). We let \( B \subseteq \bar{N}^p \).
and $N \subseteq \mathcal{N}_p$ be index sets for basic and nonbasic columns of $M$. We define the following sets:

$$C = \{ (x, w) | Ax = b \land \forall p \in Q, J \in \mathcal{N}_p (w_j = \prod_{\ell \leq |J|} x_{\ell_j}) \}$$

$$R_N = \{ (x, w) | Ax = b \land \forall p \in Q, J' \in \mathcal{N}_p^{-1} (A w_{J'} = bw_{J'}) \land \forall J \in N (w_J = \prod_{\ell \leq |J|} x_{\ell_J}) \}.$$

**Theorem 1.** For each partition $B, N$ into basic and nonbasic column indices for the companion system $Mz = 0$, we have $C = R_N$.

3. Tightening the convex relaxation

Replacing $C$ with $R_N$ for some nonbasis $N$ effectively replaces some monomial terms with linear constraints, and therefore contributes to simplify the problem. A convex relaxation for the reformulated problem is readily obtained by applying monomial convexification methods in the literature [5, 6, 1]. We observe that for any given linear system there is in general more than one reformulated problem is readily obtained by applying monomial convexification methods in the ear constraints, and therefore contributes to simplify the problem. A convex relaxation for the $N$:

For each partition $B, N$ the volume

$$V = \sum_{J \in S} \prod_{\ell \leq |J|} x_{\ell_J}$$

of (3.1) is equivalent to a single-objective problem: any solution $B$ of (3.1) maximizing $V_B$ also maximizes $V_{B,p}$ for all $p \in Q$. In this way, we have derived a technique to choose a good basis for the companion system so as to improve the chances of tightening the lower bound of the convex relaxation associated to rRLT.

Preliminary computational experiments carried out on a set of randomly generated instances of the convex Quadratic Knapsack Problem (cQKP) show that the proposed strategy is promising in improving performances of a spatial Branch-and-Bound algorithm.

**Bibliography**


