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# Differential Flat Control for Rotorcraft Trajectory Tracking

Nan Zhang, Geanina Andrei, Antoine Drouin, and Félix Mora-Camino

**Abstract**—The purpose of this communication is to investigate the usefulness of the differential flatness control approach to solve the trajectory tracking problem for a four rotor aircraft. After introducing simplifying assumptions, the flight dynamics equations for the four rotor aircraft are considered. A trajectory tracking control structure based on a two layer non linear approach is then proposed. A supervision level is introduced to take into account the actuators limitations.

**Index Terms**—differential flatness, guidance supervision, rotorcraft flight mechanics, trajectory tracking.

## I. INTRODUCTION

In the last years a large interest has risen for the four rotor concept since it appears to present simultaneously hovering, orientation and trajectory tracking capabilities of interest in many practical applications [1]. The flight mechanics of rotorcraft are highly non linear and different control approaches (integral LQR techniques, integral sliding mode control [2], reinforcement learning [3]) have been considered with little success to achieve not only autonomous hovering and orientation, but also trajectory tracking. In this paper, after introducing some simplifying assumptions, the flight dynamics equations for a four rotor aircraft with fixed pitch blades are considered.

The purpose of this study is to investigate the usefulness of the differential flatness control approach to solve the trajectory tracking problem for this class of rotorcraft. This approach has been already considered in the case of aircraft trajectory tracking by different authors [8, 9]. It appears that the flight dynamics of the considered rotorcraft present a two level affine structure which is made apparent by the definition of a new set of equivalent inputs. It can be shown then that the rotorcraft flight dynamics are composed of a differentially flat structure followed by a non linear invertible structure. This allows to introduce a new non linear control structure devoted to orientation and trajectory tracking.

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## II. ROTORCRAFT FLIGHT DYNAMICS

The considered system is shown in figure 1 where rotors one and three are clockwise while rotors two and four are counter clockwise. Annex 1 describes the rotor dynamics.

The main simplifying assumptions adopted with respect to flight dynamics in this study are a rigid cross structure, no wind, negligible aerodynamic contributions resulting from translational speed, no ground effect as well as negligible air density effects and very small rotor response times. It is then possible to write simplified rotorcraft flight equations [1].

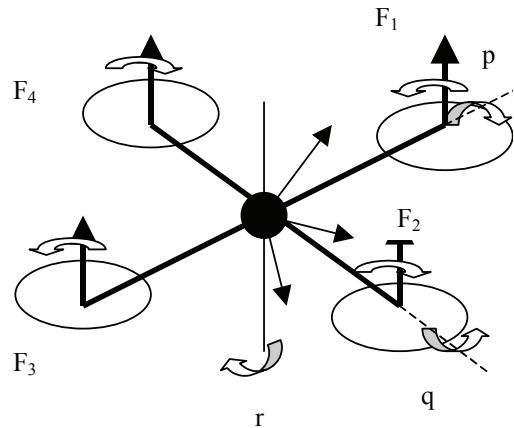


Fig 1. Four engine rotorcraft

The moment equations can be written as:

$$\begin{cases} \dot{p} = (a(F_4 - F_2) + k_2 q r - k_p p) / I_{xx} \\ \dot{q} = (a(F_1 - F_3) + k_4 p r - k_q q) / I_{yy} \\ \dot{r} = (k(F_2 - F_1 + F_4 - F_3) - k_r r) / I_{zz} \end{cases} \quad (1)$$

where  $p$ ,  $q$ ,  $r$  are the components of the body angular velocity, with  $k_2 = (I_{zz} - I_{yy})$  and  $k_4 = (I_{xx} - I_{zz})$ ,  $I_{xx}$ ,  $I_{yy}$  and  $I_{zz}$  being the moments of inertia in body-axis and  $m$  the total mass of the rotorcraft. Here the terms  $k_p p$ ,  $k_q q$ ,  $k_r r$  are related with the drag of the whole structure.

The Euler equations are given by:

$$\begin{cases} \dot{\phi} = p + \tan(\theta) \sin(\phi) q + \tan(\theta) \cos(\phi) r \\ \dot{\theta} = \cos(\phi) q - \sin(\phi) r \\ \dot{\psi} = ((\sin(\phi) / \cos(\theta)) q + (\cos(\phi) / \cos(\theta)) r \end{cases} \quad (2)$$

where  $\theta$ ,  $\phi$ , and  $\psi$  are respectively the pitch, bank and heading angles.

The acceleration equations written directly in the local Earth reference system are such as:

$$\begin{cases} \ddot{x} = (1/m)((\cos(\psi) \sin(\theta) \cos(\phi) + \sin(\psi) \sin(\phi)) F - k_x \dot{x}) \\ \ddot{y} = (1/m)((\sin(\psi) \sin(\theta) \cos(\phi) - \cos(\psi) \sin(\phi)) F - k_y \dot{y}) \\ \ddot{z} = -g + (1/m)(\cos(\theta) \cos(\phi) F - k_z \dot{z}) \end{cases} \quad (3)$$

where  $x, y$  and  $z$  are the centre of gravity coordinates and where :

$$F = F_1 + F_2 + F_3 + F_4 \quad (4)$$

The terms  $k_x \dot{x}, k_y \dot{y}, k_z \dot{z}$  are also related with the drag of the whole structure.

The rotor forces satisfy the constraints:

$$0 \leq F_{i_i} \leq F_{\max} \quad i \in \{1,2,3,4\} \quad (5)$$

### III. FLAT CONTROL LAW FOR ROTORCRAFT TRAJECTORY TRACKING

#### A. Differential Flatness of Smooth Systems

A general nonlinear system given by:

$$\dot{\underline{X}} = f(\underline{X}, \underline{U}), \underline{X} \in \mathbf{R}^n, \underline{U} \in \mathbf{R}^m \quad (6)$$

where  $f$  is a smooth mapping, is said explicitly flat with respect to the output vector  $\underline{Z}$ , if  $\underline{Z}$  is an  $m^{\text{th}}$  order vector which can be expressed analytically as a function of the current state, the current input and its derivatives, while the state and the input vectors can be expressed analytically as a function of  $\underline{Z}$  and a finite number of its derivatives. Then there exists smooth mappings  $G_X, G_U$ , and  $G_Z$  such as:

$$\begin{cases} \underline{Z} = G_Z(\underline{X}, \underline{U}, \dots, \underline{U}^{(n_z)}) & (7-1) \\ \underline{X} = G_X(\underline{Z}, \underline{\dot{Z}}, \dots, \underline{Z}^{(n_x)}) & (7-2) \\ \underline{U} = G_U(\underline{Z}, \underline{\dot{Z}}, \dots, \underline{Z}^{(n_x+1)}) & (7-3) \end{cases}$$

where  $n_z$  and  $n_x$  are integer numbers. Vector  $\underline{Z}$  is called a *flat output* for the nonlinear system given by equation (6). Although today there is no systematical way to determine flat outputs and eventually to prove its uniqueness, the flat outputs usually possess some physical meaning.

The explicit flatness property is of particular interest for the solution of control problems when physically meaningful flat outputs can be related with their objectives. In many situations, the control problem can be formulated as a flat output trajectory following problem. In general, for these cases, the flat output of equation (7-1) can be reduced, through state transformation, to a function of a single argument, the new system state itself:

$$\underline{Z} = G_Z(\underline{X}) \quad (8)$$

Then a possible control law providing to the flat outputs linear decoupled dynamics towards reference values is given by:

$$\underline{U} = G_U\left(\underline{Z}, \underline{\dot{Z}}, \dots, -\sum_{i=1}^{n_z} A_i \underline{Z}^{(i)} - A_0(\underline{Z} - \underline{Z}_c)\right) \quad (9)$$

where the  $A_i$  matrices are diagonal matrices such as the  $m$  polynomials :

$$s^{n_x+1} + \sum_{i=1}^{n_z} a_i(j, j) p^i + a_0(j, j) \quad j = 1, \dots, m \quad (10)$$

#### B. Differential Flatness of Rotorcraft Dynamics

The equations of motion of the rotorcraft can be written in non linear state form as:

$$\dot{\underline{x}} = f(\underline{x}, \underline{u}) \quad (11-1)$$

$$\text{where } \underline{x} = (p, q, r, \phi, \theta, \psi, \dot{x}, \dot{y}, \dot{z}, x, y, z)' \quad (11-2)$$

$$\text{and } \underline{u} = (F_1 - F_3, F_2 - F_4, F_1 + F_3, F_2 + F_4)' \quad (11-3)$$

Considering the output vector  $\underline{Z} = (\phi, \theta, \psi, z)'$ , the inversion of the Euler equations (2) provides expressions such as:

$$\begin{cases} p = p(\theta, \dot{\phi}, \dot{\psi}) \\ q = q(\phi, \theta, \dot{\psi}) \\ r = r(\phi, \theta, \dot{\psi}) \end{cases} \quad (12)$$

or more specifically:

$$\begin{cases} p = \dot{\phi} - \sin \theta \dot{\psi} & (12-1) \\ q = \cos \phi \dot{\theta} + \sin \phi \cos \theta \dot{\psi} & (12-2) \\ r = -\sin \phi \dot{\theta} + \cos \phi \cos \theta \dot{\psi} & (12-3) \end{cases}$$

while  $\underline{u}$  can be expressed as:

$$\underline{u} = \underline{u}(\phi, \theta, \dot{\phi}, \dot{\theta}, \dot{\psi}, \ddot{\phi}, \ddot{\theta}, \ddot{\psi}, \dot{z}, \ddot{z}) \quad (13)$$

by inversion of the set of equations (1-1), (1-2), (1-3) and (3-3), or more specifically:

$$\begin{cases} u_1 = (I_{yy} \dot{q} - k_4 p r + k_q q) / a & (14-1) \\ u_2 = (-I_{xx} \dot{p} + k_2 q r - k_p p) / a & (14-2) \\ u_3 = (((\ddot{z} + g)m + k_z \dot{z}) / (\cos \theta \cos \phi) - (I_{zz} \dot{r} + k_r r) / k) / 2 & (14-3) \\ u_4 = (((\ddot{z} + g)m + k_z \dot{z}) / (\cos \theta \cos \phi) + (I_{zz} \dot{r} + k_r r) / k) / 2 & (14-4) \end{cases}$$

Then, it can be concluded that the attitude and heading dynamics as well as the vertical dynamics of the rotorcraft are differentially flat when considering the input-output relation between  $\underline{u}$  and  $\underline{Z}$ .

#### C. Design of Flat Flight Control Laws

We adopt for the flat outputs second order dynamics such as:

$$\begin{cases} \ddot{\phi} = -2 \zeta_\phi \omega_\phi \dot{\phi} - \omega_\phi^2 (\phi - \phi_c) & (15-1) \\ \ddot{\theta} = -2 \zeta_\theta \omega_\theta \dot{\theta} - \omega_\theta^2 (\theta - \theta_c) & (15-2) \\ \ddot{\psi} = -2 \zeta_\psi \omega_\psi \dot{\psi} - \omega_\psi^2 (\psi - \psi_c) & (15-3) \\ \ddot{z} = -2 \zeta_z \omega_z \dot{z} - \omega_z^2 (z - z_c) & (15-4) \end{cases} \quad (8)$$

The control expressions of the control inputs in relations (14-1), (14-2), (14-3) and (14-4) are fed by  $p, q, r$  given by (12-1), (12-2) and (12-3) and by  $\dot{p}, \dot{q}, \dot{r}$  given by:

$$\begin{aligned} \dot{p} &= \ddot{\phi} - \cos \theta \dot{\theta} \dot{\psi} - \sin \theta \ddot{\psi} & (16-1) \\ \dot{q} &= \cos \phi \ddot{\theta} + \sin \phi \cos \theta \ddot{\psi} - \sin \phi (1 + \sin \theta) \dot{\theta} \dot{\psi} & (16-2) \\ &\quad + \cos \phi \cos \theta \dot{\phi} \dot{\psi} \\ \dot{r} &= -\sin \phi \ddot{\theta} + \cos \phi \cos \theta \ddot{\psi} - \cos \phi \dot{\phi} \dot{\theta} & (16-3) \\ &\quad - \sin \phi \cos \theta \dot{\phi} \dot{\psi} - \cos \phi \sin \theta \dot{\theta} \dot{\psi} \end{aligned}$$

where  $\ddot{\phi}, \ddot{\theta}, \ddot{\psi}$  are given by (15-1), (15-2) and (15-3).

Following the non linear inverse control approach (NLI), to insure that  $x$  and  $y$  follows second order dynamics such as:

$$\begin{cases} \ddot{x} + 2 \zeta_x \omega_x \dot{x} + \omega_x^2 (x - x_c) = 0 & (17-1) \\ \ddot{y} + 2 \zeta_y \omega_y \dot{y} + \omega_y^2 (y - y_c) = 0 & (17-2) \end{cases}$$

$\phi_c$  and  $\theta_c$  are chosen such as:

$$\begin{aligned} (1/m)((\cos(\psi) \sin(\theta_c) \cos(\phi_c) + \sin(\psi) \sin(\phi_c)) F - k_x \dot{x}) & \\ + 2 \zeta_x \omega_x \dot{x} + \omega_x^2 (x - x_c) = 0 & (18-1) \end{aligned}$$

$$\begin{aligned} (1/m)((\sin(\psi) \sin(\theta) \cos(\phi_c) - \cos(\psi) \sin(\phi_c)) F - k_y \dot{y}) & \\ + 2 \zeta_y \omega_y \dot{y} + \omega_y^2 (y - y_c) = 0 & (18-2) \end{aligned}$$

$$\text{Then : } \phi_c = \arcsin\left(\frac{m}{F} \sin \psi d_x - \cos \psi d_y\right) \quad (19-1)$$

$$\text{and } \theta_c = \arcsin\left(\frac{m}{F} (\cos \psi d_x + \sin \psi d_y) / \cos(\phi_c)\right) \quad (19-2)$$

$$\text{where } d_x = (k_x - 2 \zeta_x \omega_x) \dot{x} - \omega_x^2 (x - x_c) \quad (20-1)$$

$$d_y = (k_y - 2 \zeta_y \omega_y) \dot{y} - \omega_y^2 (y - y_c) \quad (20-2)$$

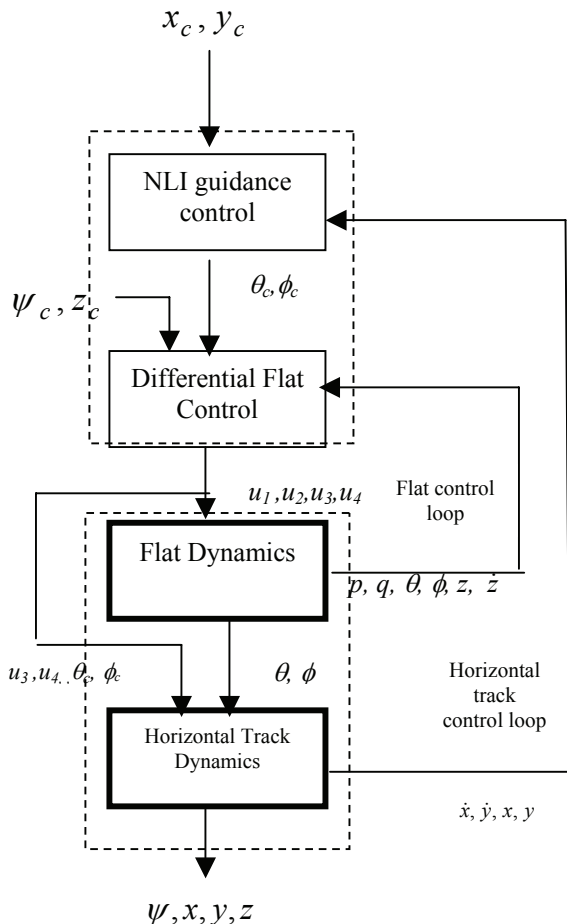


Fig 2. Proposed flat-NLI control structure

#### IV. SUPERVISION OF THE FLAT CONTROL LAW

The function of the control law supervision, is to turn feasible the control laws derived from the differential flatness approach.

Relation (19-1) is meaningful when :

$$-1 \leq \frac{m}{F} (\sin \psi d_x - \cos \psi d_y) \leq 1 \quad (21)$$

In general  $F/m \approx g$ , then this condition is in general satisfied. A sufficient condition for (21) to be satisfied is:

$$\begin{aligned} ((k_x - 2 \zeta_x \omega_x) \dot{x} - \omega_x^2 (x - x_c))^2 + ((k_y - 2 \zeta_y \omega_y) \dot{y} - \omega_y^2 (y - y_c))^2 & \\ = (F/m)^2 & (22) \end{aligned}$$

In that case defining angle  $b_y$  such as:

$$b_y = \arcsin(d_y m / F) \quad (23)$$

$$\text{then : } \text{either } \phi_c = \psi - b_y \text{ or } \phi_c = \psi - b_y + \pi \quad (24)$$

To guarantee that :

$$-\pi / 2 < -\phi_{\max} \leq \phi_c \leq \phi_{\max} < \pi / 2 \quad (25)$$

the parameters  $\zeta_y, \omega_y^2$  and  $y_c$  must be chosen such as:

$$\begin{aligned} s_{\min} \leq ((k_y - 2 \zeta_y \omega_y) \dot{y} - \omega_y^2 (y - y_c)) m / F & \\ \text{and} & \\ ((k_y - 2 \zeta_y \omega_y) \dot{y} - \omega_y^2 (y - y_c)) m / F \leq s_{\max} & (26) \end{aligned}$$

$$\text{where } s_{\min} = \min\{\sin(\psi + \phi_{\max}), \sin(\psi - \phi_{\max})\} \quad (27-1)$$

$$\text{and } s_{\max} = \max\{\sin(\psi + \phi_{\max}), \sin(\psi - \phi_{\max})\} \quad (27-2)$$

Introducing upper bounds for the derivative of  $y$  as well as to the current error in  $y$ , sufficient conditions for the satisfaction of (26) are:

$$s_{\min} \leq -( |k_y - 2 \zeta_y \omega_y| \dot{y}_{\max} + \omega_y^2 \Delta y_{\max}) m / F \quad (28-1)$$

$$( |k_y - 2 \zeta_y \omega_y| \dot{y}_{\max} + \omega_y^2 \Delta y_{\max}) m / F \leq s_{\max} \quad (28-2)$$

Then considering  $b_x$  such as:

$$b_x = \arccos(d_x m / F) \quad (29)$$

$$\text{then : } \text{either } \phi_c = \psi - b_x \text{ or } \phi_c = \psi + b_x \quad (30)$$

To guarantee again that :

$$-\pi / 2 < -\phi_{\max} \leq \phi_c \leq \phi_{\max} < \pi / 2 \quad (31)$$

parameters  $\zeta_x, \omega_x^2$  and  $x_c$  must be chosen such as:

$$\begin{aligned} c_{\min} \leq (k_x - 2 \zeta_x \omega_x) \dot{x} - \omega_x^2 (x - x_c) m / F & \\ \text{and} & \\ (k_x - 2 \zeta_x \omega_x) \dot{x} - \omega_x^2 (x - x_c) m / F \leq c_{\max} & (32) \end{aligned}$$

where

$$c_{\min} = \min\{\cos(\psi + \phi_{\max}), \cos(\psi - \phi_{\max})\} \quad (33-1)$$

and

$$c_{\max} = \max\{\cos(\psi + \phi_{\max}), \cos(\psi - \phi_{\max})\} \quad (33-2)$$

Introducing upper bounds for the derivatives of  $x$  as well as to current errors in  $x$ , sufficient conditions for the satisfaction of (32) are:

$$c_{\min} \leq -( |k_x - 2 \zeta_x \omega_x| \dot{x}_{\max} + \omega_x^2 \Delta x_{\max}) m / F \quad (34-1)$$

$$( |k_x - 2 \zeta_x \omega_x| \dot{x}_{\max} + \omega_x^2 \Delta x_{\max}) m / F \leq c_{\max} \quad (34-2)$$

Relation (19-2) can be written as:

$$\sin \theta_c \cos \phi_c = \cos(\psi - b) \quad (35)$$

where  $b$  is such as :

$$\sin b = d_y m / F \quad \text{and} \quad \cos b = d_x m / F \quad (36)$$

To guarantee that

$$-\pi / 2 < -\theta_{\max} \leq \theta_c \leq \theta_{\max} < \pi / 2 \quad (37)$$

the parameters  $\zeta_x, \omega_x, x_c, \zeta_y, \omega_y$  and  $y_c$  must be chosen such as:

$$(\cos \psi ((k_x - 2 \zeta_x \omega_x) \dot{x} - \omega_x^2 (x - x_c)) m / F + \quad (38-1)$$

$$(\sin \psi ((k_y - 2 \zeta_y \omega_y) \dot{y} - \omega_y^2 (y - y_c)) m / F \leq \sin \theta_{\max} \cos \phi_{\max}$$

and

$$(\cos \psi ((k_x - 2 \zeta_x \omega_x) \dot{x} - \omega_x^2 (x - x_c)) m / F + \quad (38-2)$$

$$(\sin \psi ((k_y - 2 \zeta_y \omega_y) \dot{y} - \omega_y^2 (y - y_c)) m / F \geq -\sin \theta_{\max} \cos \phi_{\max}$$

with the stability conditions:

$$\zeta_x > 0, \omega_x > 0, \zeta_y > 0, \omega_y > 0 \quad (38-3)$$

A sufficient condition for (38-1) and (38-2) is:

$$(|k_x - 2 \zeta_x \omega_x| \dot{x}_{\max} + \omega_x^2 \Delta x_{\max}) m / F + \quad (39)$$

$$(|k_y - 2 \zeta_y \omega_y| \dot{y}_{\max} + \omega_y^2 \Delta y_{\max}) m / F \leq \sin \theta_{\max} \cos \phi_{\max}$$

Finally parameters  $\zeta_x, \omega_x^2, x_c, \zeta_y, \omega_y^2$  and  $y_c$  must be chosen such that relations (22), (26), (32), (38-1), (38-2) and (34-3) are satisfied or introducing the sufficient conditions relations (22), (34-1), (34-2), (58-1) and (39).

Introducing upper bounds to  $|\dot{\theta}|, |\dot{\phi}|$  and  $|\dot{\psi}|$ :  $\dot{\theta}_{\max}, \dot{\phi}_{\max}$  and  $\dot{\psi}_{\max}$ , to insure that constraints:

$$-F_{\max} \leq u_i \leq F_{\max} \quad i = 1, 2 \quad (40-1)$$

and

$$0 \leq u_i \leq 2 F_{\max} \quad i = 3, 4 \quad (40-2)$$

are satisfied the positive parameters  $\zeta_\theta, \zeta_\phi, \zeta_\psi, \zeta_z, \omega_\theta, \omega_\phi, \omega_\psi$ , and  $\omega_z$  must be chosen such as (sufficient conditions) :

$$|2I_{yy} \zeta_\theta \omega_\theta - k_q| \dot{\theta}_{\max} + I_{yy} \omega_\theta^2 \theta_{\max} \leq a F_{\max} \quad (41-1)$$

$$|2I_{xx} \zeta_\phi \omega_\phi - k_p| \dot{\phi}_{\max} + I_{xx} \omega_\phi^2 \phi_{\max} \leq a F_{\max} \quad (41-2)$$

$$|k_z - 2\zeta_z \omega_z m| \dot{z}_{\max} + m \omega_z^2 \Delta z_{\max} + I_{zz} |k_r - 2\zeta_\psi \omega_\psi| \dot{\psi}_{\max} \quad (41-3)$$

$$+ I_{zz} \omega_\psi^2 \Delta \psi_{\max} \leq 2F_{\max} \cos \theta_{\max} \cos \phi_{\max} - mg$$

$$|2\zeta_z \omega_z - k_z| \dot{z}_{\max} + \omega_z^2 \Delta z_{\max} + |2I_{zz} \zeta_\psi \omega_\psi - k_r| \dot{\psi}_{\max} / k \quad (41-4)$$

$$+ I_{zz} \omega_\psi^2 \Delta \psi_{\max} / k \leq g$$

where  $\Delta \psi_{\max}$  and  $\Delta z_{\max}$  are upper bounds for current desired changes in heading and flight height.

To insure that relations:

$$\begin{cases} |\dot{\phi}| \leq \dot{\phi}_{\max}, & |\dot{\theta}| \leq \dot{\theta}_{\max}, & |\dot{\psi}| \leq \dot{\psi}_{\max} \end{cases} \quad (42-1)$$

$$\begin{cases} |\dot{x}| \leq \dot{x}_{\max}, & |\dot{y}| \leq \dot{y}_{\max}, & |\dot{z}| \leq \dot{z}_{\max} \end{cases} \quad (42-2)$$

$$\begin{cases} |x - x_c| \leq \Delta x_{\max}, & |y - y_c| \leq \Delta y_{\max} \end{cases} \quad (42-3)$$

$$\begin{cases} |z - z_c| \leq \Delta z_{\max}, & |\psi - \psi_c| \leq \Delta \psi_{\max} \end{cases} \quad (42-4)$$

are satisfied while tracking trajectory:

$$(x_{cc}(t), y_{cc}(t), z_{cc}(t), \psi_{cc}(t)), \quad t \geq 0$$

$x_c(t), y_c(t), z_c(t)$  and  $\psi_c(t)$  can be chosen such as:

$$x_c(t) = x_{cc}(t) - \alpha (x_{cc}(t) - x(t)) \quad (43-1)$$

$$\text{and} \quad y_c(t) = y_{cc}(t) - \alpha (y_{cc}(t) - y(t)) \quad (43-2)$$

with  $0 < \alpha < 1$ .

With this choice, the instantaneous dynamics in  $x$  (or  $y$ ) are such as:

$$\ddot{x} + 2\zeta(\alpha) \omega(\alpha) \dot{x} + \omega(\alpha)^2 (x - x_{cc}) = 0 \quad (44-1)$$

$$\text{with} \quad \zeta(\alpha) = \zeta_x / \sqrt{1-\alpha} \quad \text{and} \quad \omega(\alpha) = \sqrt{1-\alpha} \quad (44-2)$$

so that convergence can be expected. The same approach can be adopted with  $z$  and  $\psi$ :

$$z_c(t) = z_{cc}(t) - \beta (z_{cc}(t) - z(t)) \quad (45-1)$$

and

$$\psi_c(t) = \psi_{cc}(t) - \beta (\psi_{cc}(t) - \psi(t)) \quad (45-2)$$

with  $0 < \alpha \leq \beta < 1$ .

## V. CASE STUDIES

### A. Heading Control at Hover

The objective is to hover at an initial position of coordinates  $x_0, y_0, z_0$  while acquiring a new orientation  $\psi_f$ . In this case we get the guidance control laws:

$$u_3 = \frac{1}{2} (m g - (I_{zz} \dot{r} + k_r r) / k) \quad (46-1)$$

$$u_4 = \frac{1}{2} (m g + (I_{zz} \dot{r} + k_r r) / k) \quad (46-2)$$

with the following reference values for the attitude angles:

$$\theta_c = 0 \quad \text{and} \quad \phi_c = 0 \quad (47)$$

Here the heading acceleration is given by:

$$\ddot{\psi} = -2 \zeta_\psi \omega_\psi \dot{\psi} - \omega_\psi^2 (\psi - \psi_c) \quad (48)$$

Starting from an horizontal attitude ( $\theta(0)=0, \phi(0)=0$ ), attitude inputs  $u_1$  and  $u_2$  given by relation (14-1) and (14-2) remain equal to zero. Then, figures 3 and 4 display some simulation results:

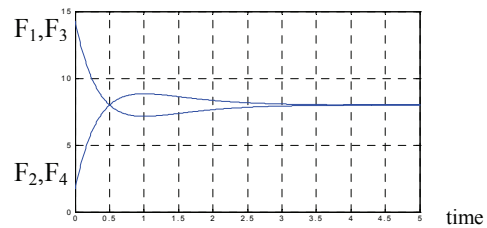


Fig 3. Hover control inputs

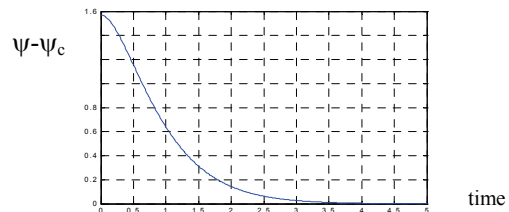


Fig 4. Heading response during hover

### B. Trajectory Tracking Case

Here the rotorcraft is tracking the helicoidal trajectory of equations:

$$\begin{cases} x_c(t) = \rho \cos vt \\ y_c(t) = \rho \sin vt \\ z_c = z_0 + \gamma t \\ \psi_c(t) = vt + \pi/2 \end{cases} \quad (49)$$

where  $\rho$  is a constant radius and  $\gamma$  is a constant path angle. In this case the guidance control laws tend towards:

$$u_1 = k_q \sin \phi_c / a \quad (50-1)$$

$$u_2 = (k_2 \sin \phi_c - k_p) \cos \phi_c v / a \quad (50-2)$$

$$u_3 = (((m + k_z \gamma / g) / (\cos \theta_c \cos \phi_c)) - k_r v / k) / 2 \quad (50-3)$$

and

$$u_4 = (((m + k_z \gamma / g) / (\cos \theta_c \cos \phi_c)) + k_r v / k) / 2 \quad (50-4)$$

Here, with  $k_x = k_y$ , the permanent reference values for the attitude angles are such as:

$$\theta_c = \arctg\left(\frac{k_x v}{mg + k_z \gamma}\right) \quad (51.1)$$

and

$$\phi_c = -\arctg\left(\frac{m v^2}{\sqrt{k_x^2 v^2 + (mg + k_z \gamma)^2}}\right) \quad (51.2)$$

In figures 5 to 7 simulation results are displayed where at initial time the rotorcraft is hovering:

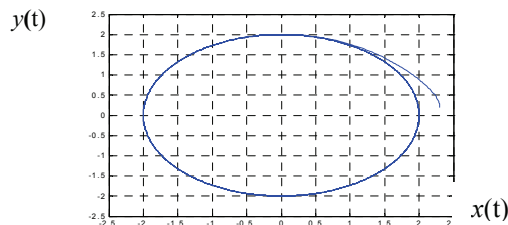


Fig 5. Evolution of rotorcraft horizontal track

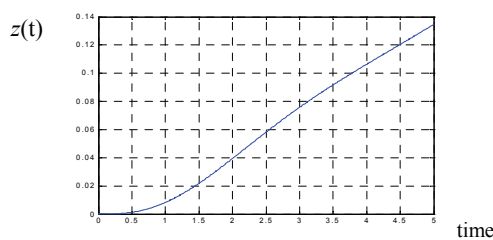


Fig 6. Evolution of rotorcraft altitude

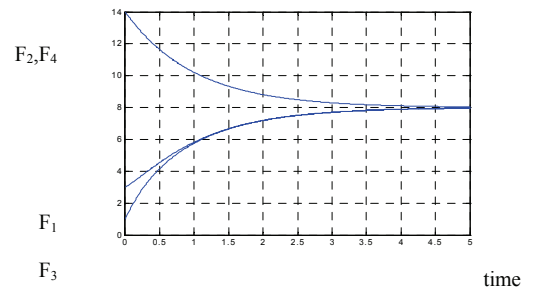


Fig 7. Rotorcraft trajectory tracking

### VI. CONCLUSION

In this communication the theoretical applicability of the differential flatness control technique to rotorcraft trajectory tracking has been investigated. It appears that this approach leads to the design of a two level control structure based on analytical laws. Considering the structure of the rotorcraft flight dynamics, other promising non linear control techniques are non linear inverse control [4] and back stepping control [10],[11].

A supervision layer has been designed to tackle with the limitations of the actuators of the rotorcraft and to avoid the uncontrolled effect of actuator saturation: sufficient conditions have been established with respect to the convergence dynamics while a scaling scheme is proposed to define a current reference trajectory.

However, the robustness of these control laws with respect to the different aerodynamic effects which have been taken as negligible should be investigated. Since only very intricate theories are available to approach this problem, real flight tests appear, at this stage, to be unavoidable.

### APPENDIX

The rotor engine dynamics are characterized by the relation between the input voltage  $V_a$  and the angular rate  $\omega$ . A possible model of rotor dynamics is given by:

$$\dot{\omega}(t) = -\frac{1}{\tau} \omega(t) - K_Q \omega(t)^2 + (K_{V_a} / \tau) V_a(t) \quad (A.1)$$

with  $\omega(0) = \omega_0$ , where  $\tau$ ,  $K_Q$  and  $K_{V_a}$  are given positive parameters and where the voltage input is such as:

$$0 \leq V_a \leq V_{\max} \quad (A.2)$$

with a negligible time response for the voltage generator.

The step response ( $V_a = \text{constant}$ ) of the rotor is solution of the scalar *Riccati* equation:

$$\dot{\omega}(t) = -\frac{1}{\tau} \omega(t) - K_Q \omega(t)^2 + (K_{V_a} / \tau) V_a \quad (A.3)$$

with  $\omega(0) = \omega_0$ .

A particular solution  $\omega_l$  of the associated differential equation is such as:

$$\omega_l = \frac{1}{2\tau K_Q} (\sqrt{1 + 4K_{V_a} K_Q \tau V_a} - 1) \quad (A.4)$$

In the general case, the solution of (A.3) can be written as

$$\omega(t) = \omega_1 + \frac{1}{\frac{1}{\omega(0) - \omega_1} + K_Q \tau' (1 - e^{-t/\tau'})} e^{-t/\tau'} \quad (\text{A.5})$$

with

$$\tau' = \tau / \sqrt{1 + 4K_V K_Q \tau V_a} \quad (\text{A.6})$$

and  $\lim_{t \rightarrow +\infty} \omega(t) = \omega_1 \quad (\text{A.7})$

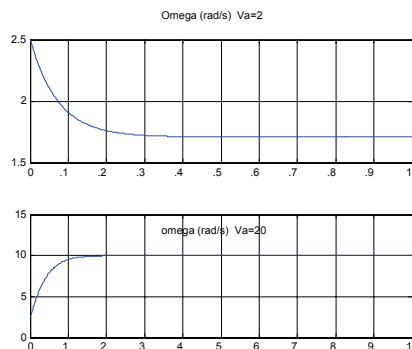


Fig. A1- Two examples of rotor step response

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