

# A nonlinear control approach for relative guidance of commercial aircraft

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#### 1. SUMMARY

The delegation to the flight crew of tasks currently performed by the air traffic controllers provides a new perspective to significantly increase air traffic control capacity. Then, merging and station keeping operations have appeared to be of interest even if they rise new technical challenges. Indeed, these unusual maneuvers in the field of Civil Aviation seem difficult to be performed manually and since in this case they will result in an increase of the flight crew workload, new on-board automated functions should be developed. This paper investigates the design of an auto pilot mode dedicated to merging and station keeping maneuvers behind a leading aircraft. The proposed relative flight guidance law is based on a non linear sliding mode control technique. Simulation results are displayed and discussed.

#### 2. KEY WORDS

Relative Guidance, Flight Control Laws, Non Linear Control.

#### **3. INTRODUCTION**

The increase of air traffic urges the design of new devices to improve significantly air traffic control capacity [1]. So, to meet this challenge, new concepts such as "relative guidance" are presently of interest to contribute to the delegation of tasks to the crew [2]. The relative guidance function should provide an aircraft the ability to establish and to maintain a given time or distance behind a designated aircraft (see figure 1). Nowadays no automatic control mode is available on-board civil aircraft to perform this task and few studies have been published in this field [3, 4].



Figure -1 In-bound traffic at Roissy Charles -De-Gaulle

This communication focuses on the design of a nonlinear controller which enables an aircraft to perform merging maneuvers and to maintain station keeping behind a designated aircraft. The investigated approach is based on the inversion of a simplified non linear model of the relative dynamics coupled with a sliding mode control technique. Preliminary evaluations are performed using simulation models of two wide body aircraft.

# 4. NOMENCLATURE

- TK : along track distance
- XTK : cross track distance
- $\psi$ : heading angle of the trailing aircraft
- V : airspeed of the trailing aircraft
- L : subscript added to all variables related to the leading aircraft
- $\chi$ : track angle of the trailing aircraft
- $W_x$ ,  $W_y$ : winds peed components
- x,y : coordinates of the center of gravity of the trailing aircraft
- $\rho$  : relative range
- $\mu$  : relative bearing
- $\tau_V$  : time constant of the controlled airspeed
- $\underline{X}, \underline{y}, \underline{u}$ : state, output and control vectors
- s<sub>1</sub>, s<sub>2</sub> : sliding mode surface variables
- $k_1, k_2, a_{11}, a_{22}, s_{1max}, s_{2max}$ : control parameters

#### 5. RELATIVE FLIGHT DYNAMICS

#### **5.1 ASSUMPTIONS AND DEFINITIONS**

Some simplifying assumptions are made in this study: the maneuvers are performed at constant altitude over a considered locally flat and non rotating Earth. The wind speed vector is supposed to be the same for both aircraft. The Auto throttle controls the engines so that the

airspeed dynamics can be approximated by a first order linear model (time constant  $\tau_{v}$ ). Heading is assumed to be controlled through coordinated turns while roll and bank angle dynamics are neglected. Then, the variables characterizing the relative position of the trailing aircraft with respect to the leading aircraft are represented in the figure below:



*Figure-2 Relative guidance parameters* 

### **5.2 STATE SPACE REPRESENTATION OF RELATIVE DYNAMICS**

The range and the relative bearing of the two aircraft can be derived from the along track distance TK and the cross track distance XTK by:

$$\rho = \sqrt{TK^2 + XTK^2}$$
 and  $\mu = \psi + arctg(XTK/TK)$  (1)

while 
$$\chi = arctg((V \sin(\psi) + W_x)/(V \cos(\psi) + W_y))$$
 (2)

These relations translate the variables manipulated by air traffic controllers and pilots (i.e. "merge 5 NM behind, 1 NM left") into variables processed by the relative guidance device. The state vector retained here to describe the relative dynamics of the two aircraft is chosen as follows:

$$\underline{X} = \begin{bmatrix} \rho, & \mu, & V, & \psi \end{bmatrix}^T$$
(3)

This leads to the following non linear affine state space representation of the relative guidance kinematics, where  $\underline{u}$  denotes the control vector :

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$$\underline{X} = f(\underline{X}) + g(\underline{X})\underline{u} \tag{4}$$

with 
$$\underline{u} = \begin{bmatrix} V_c, & \mathbf{\phi} \end{bmatrix}^T$$
 (5)

where 
$$f(\underline{X}) = \begin{bmatrix} V_L \cos(\psi_L - \mu) - V \cos(\psi - \mu) \\ V_L \sin(\psi_L - \mu) - V \sin(\psi - \mu) / \rho \\ -V / \tau_V \\ 0 \end{bmatrix} \text{ and } g(\underline{X}) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 / \tau_V & 0 \\ 0 & g / V \end{bmatrix}$$
(6)

The output vector, representative of the relative position of the aircraft, is chosen as :

$$\underline{y} = [\rho, \ \mu]^T \tag{7}$$

The control objective retained in this study is to lead the track variables to the respective desired values  $TK_d$  and  $XTK_d$  while the track angle of the trailing aircraft  $\chi$  becomes equal to the one of the leader. Through relations (1), this objective can be fully expressed with variables  $\rho$ ,  $\mu$  and  $\chi$ . The resulting transient and permanent regimes should be compatible with the handling qualities of the aircraft and the required degree of comfort for a passenger aircraft.

#### **5.3 THE RELATIVE GUIDANCE CONTROL LAW**

Following non linear control theory [5], the relative degree of theses outputs is obtained through their derivation with respect to time until terms of the input vector appears. Here we get:

$$\underline{\underline{y}} = \begin{bmatrix} \dot{\rho} \\ \mu \end{bmatrix} = \Delta_0(\underline{X}) + \Delta(\underline{X}) \cdot \underline{u}$$
(8)

with

and

$$\Delta(\underline{X}) = \begin{bmatrix} -\cos(\psi - \mu)/\tau_V & g \cdot \sin(\psi - \mu) \\ -\sin(\psi - \mu)/(\rho\tau_V) & -(g/\rho) \cdot \cos(\psi - \mu) \end{bmatrix}$$
(10)

Note that in the above relations, the time rates of  $\psi_L$  and  $V_L$  have been neglected. Here the relative degrees of the outputs are both equal to two and the whole dynamics is covered by their dynamics. Then, there is no ground left for inner dynamics at this level of detail for the considered model of the relative flight dynamics. Moreover, as the rank of the matrix  $\Delta(x)$  is in normal conditions ( $\rho > \rho_{min} > 0$ ) equal to 2, this last relation can be inverted to get a non linear control law such as:

$$\underline{u} = \Delta(\underline{X})^{-1} \underline{\ddot{y}} + l(\underline{X}) \tag{11}$$

with 
$$\Delta(\underline{X})^{-1} = \begin{bmatrix} -\tau_V \cos(\psi - \mu) & -\tau_V \rho \sin(\psi - \mu) \\ (1/g) \sin(\psi - \mu) & -(\rho/g) \cos(\psi - \mu) \end{bmatrix}$$
(12)

and  $l(\underline{X}) = -\Delta(\underline{X})^{-1}(\Delta_0(\underline{X}))$  (13)

At this point, to fully determine the control law, a transient dynamics must be specified for the output vector  $\underline{y}$ . In another work [6], the authors have evaluated the case in which the objective of the non linear inverse control law is to get linear decoupled output dynamics. This former approach has shown some strong limitations: high solicitation of the actuators leading to their saturation, generation of complex relative maneuvers and poor robustness with respect to modeling errors. Then the current study investigates the use of a modified sliding mode controller which leads the outputs to adopt progressively first order linear dynamics. Indeed, sliding mode control has been recognized to be a technique which can provide robustness in the control of non linear systems [7].

For the derivation of a sliding mode controller, let us select two switching surfaces as follows:

$$\begin{cases} s_1 = \dot{\rho} + k_1 (\rho - \rho_d) \\ s_2 = \dot{\mu} + k_2 (\mu - \mu_d) \end{cases}$$
(14)

where  $k_1$  and  $k_2$  are positive numbers, and  $\rho_d$  and  $\mu_d$  are the desired range and relative bearing derived from the desired along and cross track distances. Observe that their derivatives take

the form: 
$$\begin{bmatrix} \dot{s}_1 \\ \dot{s}_2 \end{bmatrix} = \frac{\ddot{y}}{\underline{y}} + \begin{bmatrix} k_1 \dot{\rho} \\ k_2 \mu \end{bmatrix}$$
(16)

Then, a classical Lyapunov function [8] can be introduced:

$$W = \frac{1}{2}s_1^2 + \frac{1}{2}s_2^2 \tag{17}$$

It is clear that if dW/dt is forced to remain negative,  $\rho$  and  $\mu$  will be inclined to follow the above (relations (14)) first order sliding mode dynamics towards  $\rho_d$  and  $\mu_d$ . For that, let us choose a control law such that the derivatives of the switching surfaces satisfy:

$$\begin{cases} \dot{s}_1 = -a_{11}sat(s_1) \\ \dot{s}_2 = -a_{22}sat(s_2) \end{cases}$$
(18)

where a<sub>ii</sub> are positive parameters and the saturation functions are given by :

$$sat(s_i) = \begin{cases} s_i \ \forall |s_i| \le s_{i\max} \\ sgn(s_i) s_{i\max} \ \forall |s_i| > s_{i\max} \end{cases}$$
(19)

Then the relative guidance control law adopts the following expression:

$$\underline{u} = \Delta(\underline{X})^{-1} \begin{bmatrix} -a_{11} \cdot sat(s_1) - k_1 \dot{\rho} \\ -a_{22} \cdot sat(s_2) - k_2 \dot{\mu} \end{bmatrix} + l(\underline{X})$$
(20)

#### 5.4 CASE STUDY

Different scenarios have been designed in order to evaluate the performances of the proposed control law. One of them is displayed here. This scenario considers an approach and includes the following features over a time period of 700 seconds for two wide body aircraft:

The leading aircraft starts at  $x_0 = 0$  NM,  $y_0 = 0$  NM, with an initial conventional airspeed of 200kts and a heading of 90 degrees. The bank angle of the leading aircraft remains at zero, except between 220 sec and 320 sec, when the leader changes its heading by about 175 degrees (near a half circle) through setting at 20 the controlled bank angle . Furthermore, the controlled conventional airspeed of the leading aircraft is set to 160 kts for t > 600 sec, which results in a deceleration. The trailing aircraft starts at  $x_0 = -8$  NM,  $y_0 = +4$  NM, with initial conventional airspeed and heading of 200 kts and 90 degrees respectively. The requested separation for the trailing aircraft is 90 seconds behind the leading aircraft. The whole maneuver is supposed to take place within a wind of 20 kts blowing from North. During the maneuver, the inputs have been limited to  $\pm 20^{\circ}$  for  $\varphi$  with a maximum rate of  $\pm 5^{\circ}$ /sec and to values between 140 kts and 250 kts for V<sub>c</sub> with a maximum rate of  $\pm 1$  kts/sec. The time constant  $\tau_V$  has been chosen equal to 40 seconds. The values of the parameters of the control law have been taken as follows:

$$\begin{cases} a_{11} = 0.01 \operatorname{sec}^{-1}, a_{22} = 0.1 \operatorname{sec}^{-1} \\ k_1 = 0.05 \operatorname{sec}^{-1}, k_2 = 0.05 \operatorname{NM} / \operatorname{sec} \\ s_{1 \max} = 0.5 \operatorname{NM} / \operatorname{sec}, s_{2 \max} = 0.05 \operatorname{NM} / \operatorname{sec} \end{cases}$$
(21)

The trajectories of the leading and the trailing aircraft in the horizontal plane as well as the normal load factor are represented hereafter :



*Figure-3*: *Aircraft movement in the Figure-4*: *Load factor(minus 1) versus time horizontal plane (axes in NM)* (*in sec*)

The evolutions with time of current delay and range between aircraft are shown in the following figures:





Figure-5 : Current delay (in sec) between aircraft

Figure-6 : Current range (in NM) between aircraft



Finally, the figures below show the evolution w.r.t. time of the control variables :

Figure-7 : Controlled and actual airspeed (in kts) versus time (in sec)

*Figure-8* : *Controlled and actual bank angle (in degrees) versus time (in sec)* 

#### 6. CONCLUSION

From the above preliminary results, it appears that non linear control theory provides theoretical grounds to perform the automation of relative guidance maneuvers. A possible control structure is displayed in figure 9. However many difficulties remain. They are mainly related with the availability of reliable data exchange between aircraft and accurate relative navigation estimates, the development of new ATC officer's and Pilot's interfaces, the definition of new ground and on board procedures and the on board integration of this new flight guidance mode with the flight management system, the TCAS and the classical flight control and guidance modes.



*Figure-9: A general relative guidance control structure* 

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